

# Sovereign Illiquidity and Recessions

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## Abstract

Motivated by the striking increase in sovereign spreads and the subsequent recession in Europe during 2011, I examine the importance of sovereign debt liquidity in a New Keynesian environment with wage rigidities and financial frictions à la [Kiyotaki and Moore \(2012\)](#). My main findings imply that, independently of credit risk, a decrease in the liquidity of government bonds has significant detrimental effects on output, employment, investment, and equity prices. Therefore, this framework suggests that ECB policies taken in 2012 aimed at introducing liquidity seem to be the desired measures, at least temporally until conventional monetary policy became effective again.

JEL Classifications: E44, G10.

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# 1 Introduction

Since the formation of the European Monetary Union, sovereign bond yields for members have moved tight together. This trend began its diversion following the Great Recession where—by late 2008—bond yields began to show some dispersion across countries due to the increase in sovereign debt levels and overall uncertainty. At its heart, the Eurozone debt crisis stemmed from both liquidity and credit risk concerns in the market which led to a sharp spike in sovereign bond yields not previously observed in the periphery and even threatened the core of the Eurozone by late 2011. It was only after the launch of the Long-Term Refinancing Operations (LTRO) and Outright Monetary Transactions (OMT) programs, which aimed at injecting liquidity into the economy, and to a greater extent after Mario Draghi’s “whatever it takes” comment in July 2012, that the markets alarm diminished. As a result, sovereign bond yields dropped to sustainable levels in most countries by late 2012.

Figure 1 shows 10-year sovereign spreads to the German Bund (i.e., the German treasury bond) for periphery countries such as Spain and Portugal—on the first panel—, and for core countries with stronger fundamentals such as Netherlands and Finland<sup>1</sup>—on the third panel. Interest rates for most European countries have risen with respect to Germany since 2011. This has had a large impact across most European countries. As we can see in the second and fourth panel, output deviations from trend for both: periphery countries—with weak fundamentals—and core countries—with strong fundamentals—faced a recession.

In periods of market stress, extreme market movements from equity to safer bonds are often referred to as “flights to safety”. These episodes may however be related not only to flights to quality but also to flights to liquidity. In periods of market turbulence, the liquidity of bonds, i.e. the capacity to undo positions at reasonable costs, is an important concern for bond market investors. Abrupt changes in bond prices may therefore be the result of market participants suddenly requesting higher premiums for holding the less liquid assets, in addition to or even regardless their credit risk. Although default risk has been the main factor explaining spreads and debt capacity in quantitative literature of sovereign debt, the importance and role of liquidity in explaining recent crises captured the attention of researchers, practitioners and policy makers alike.

Should policy makers address their attention to the liquidity of sovereign markets? Does the drying up of these markets have any relevant implication for the real economy? Motivated by the Eurozone debt crisis, I study the macroeconomic implications of this flight to liquidity—modeled as a drop in the perceived liquidity in sovereign bonds market with respect to a perfectly liquid bond—in a framework with financial frictions and nominal rigidities. My main findings imply that independently

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<sup>1</sup>Periphery countries include Spain, Portugal, Ireland, Slovenia, and core countries include Austria, Belgium, France, Netherlands and Finland.

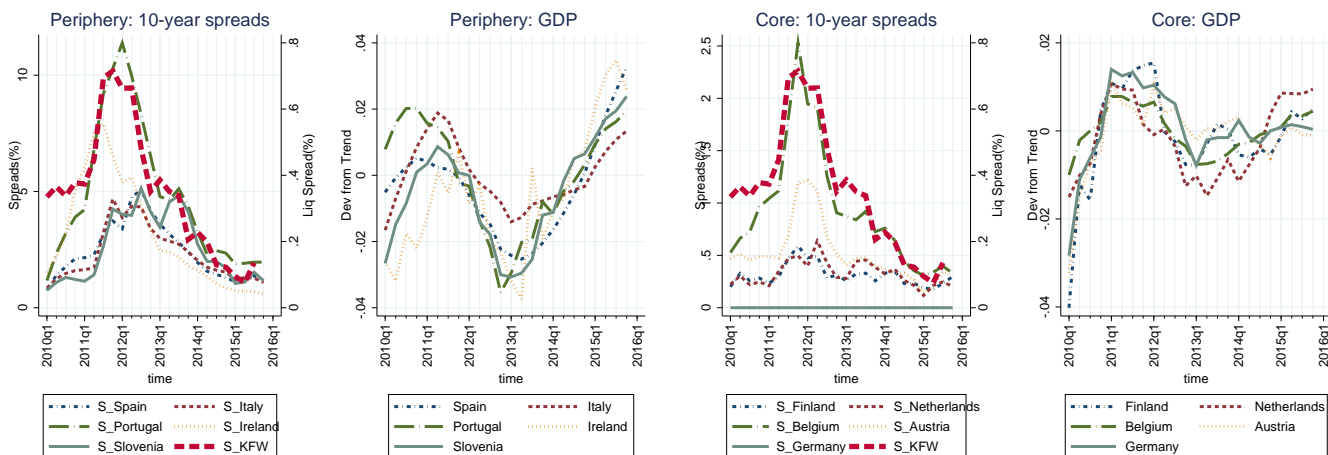


Figure 1: **Illiquidity, Spreads and Recessions**

Note: 10-year sovereign spreads to the German Bund and GDP deviations from trend for periphery countries (Spain, Portugal, Ireland, Slovenia) and core countries (Austria, Belgium, France, Netherlands and Finland). Thick dashed lines in plots for spreads represent the KfW spread to the German Bund. Higher KfW entails higher levels of illiquidity. Source: Eurostat, ECB and Schwarz (2015) for KfW data.

of credit risk, a negative shock to the otherwise liquid government bond has significant detrimental effects on output, employment, investment, and equity prices. This fall in output is the consequence of a reduction in investment as well as consumption. Thus, this framework suggests that ECB policies taken in 2012 aiming at introducing liquidity seem to be the desired measures, at least temporarily until conventional monetary policy became effective again<sup>2</sup>.

Liquidity has become a key focus of international policy debates over recent years. This reflects the view that liquidity, its drivers and consequences are of major importance for international financial stability. The concept of liquidity, however continues to be used in a variety of ways. In this paper, I focus on market liquidity, that is the easiness at which an asset can be sold and transformed into cash. In normal times, sovereign bonds are among the most liquid assets in the market. However, in turbulent times, sovereign bonds can become harder to sell. I examine the importance of liquidity in these markets and the macroeconomic effects of fluctuations in sovereign bonds' liquidity in contrast to other sources of liquidity shocks, such as private equity markets.

Measuring market liquidity is not an easy task. There are several different measures and there is a vast empirical literature explaining the differences between each of these. I will be working with a widely used and accepted European measure: *KfW spread* to the German Bund (i.e., the German's treasury bonds). The Kreditanstalt für Wiederaufbau (KfW) is a government sponsored agency of Germany which issues bonds which are fully guaranteed by the German Federal Government. Therefore, the

<sup>2</sup>ECB reached the Zero Lower Bound by July 2012 (ECB Statistics).

credit risk between these bonds and the German Bund are identical. Their liquidity discrepancy comes mainly from the difference in their market size. German Bund's market is almost three times as big as the KfW's market. In Figure 1 I plot together with sovereign spreads, the KfW spread to the German Bund. Higher values for the KfW spread implies that the KfW is paying a premium compared to the Bund due to its lower liquidity. KfW and sovereign spreads are highly correlated. For example, the correlation between KfW and Finland's credit spread is 0.8 (and 0.85 for Netherlands)<sup>3</sup>.

I study the effects of liquidity shocks to sovereign markets in an economy with financial frictions embedded in a standard New Keynesian framework—with price and wage rigidities. These financial frictions will not only affect borrowing and the resaleability in the equity market as in [Kiyotaki and Moore \(2012\)](#) (KM), but also I introduce a new friction that affects the liquidity of sovereign bonds.

Financial frictions à la KM constrain the financing opportunities of investment in new capital. These, not only affect the possibility of external financing through borrowing but also the possibility of fully liquidating current assets in order to fund the production of new physical capital. Sovereign bonds play a crucial role in relaxing these constraints by providing liquidity. Although government bonds offer lower returns than equity, in equilibrium they are held by households not only due to lower risk but also because of the liquidity they provide. Shocks affecting the resaleability of these bonds, that is their liquidity in secondary markets, can have important macroeconomic effects. Funding ability of entrepreneurs become tighter when government liquidity deteriorates triggering a drop in investment and future capital formation. In real business cycles without nominal rigidities, most of the effect on output comes from drop in investment whereas consumption increases. Nominal and wage rigidities allow me to solve this contrafactual implication. In addition, wage rigidity causes labor demand to drop enough to negatively affect capital's productivity, and therefore equity prices.

The main findings of this work convey that liquidity of sovereign markets is meaningful. First, a negative shock to the liquidity of sovereign bonds is recessionary, generating a significant drop in investment, consumption and employment. In addition, my model speaks to the fall in stock prices, which has been evident during the European debt crisis. Second, other sources of liquidity shocks, although recessionary, have different macroeconomic implications. In compliance with European increase in spreads in 2011, illiquidity of bonds leads to an increase in the return these must pay in order for households to hold them. On the contrary, a fall in the resaleability of private equity—which is also recessionary—affects the relative price of private and public assets in the opposite direction. During the Great Recession and the European Debt Crisis, illiquidity of public bonds in Europe reached similar levels, although sovereign spreads have not increased as much during the first event. Allowing for variation in these two shocks (liquidity of equity and government bonds) is crucial to explain

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<sup>3</sup>[De Santis \(2014\)](#) finds that flight to liquidity benefiting the German Bund is behind the pricing of *all* Euro area spreads and, specifically, is the *only* factor explaining the sovereign spreads for Finland and the Netherlands.

why spreads behaved differently during these two events. Lastly, I provide some sensitivity analysis studying different levels of financial development and show that more financially developed economies, although richer, are more vulnerable to the consequences of sovereign liquidity shocks.

The structure of the paper is as follows. Section 1.1 discusses the related literature. Section 2 introduces the basic framework, and Section 3 presents the results. Finally, Section 4 concludes.

## 1.1 Related literature

My work is mainly related to the literature studying the link between liquidity and business cycles, in particular to [Kiyotaki and Moore \(2012\)](#) (KM). In addition to widely used borrowing constraints, they introduce a new friction: a resaleability constraint on equity that limits the possibility of fully liquidating these assets to finance new investment. They examine the macroeconomic implications of an adverse shock to the resaleability (i.e., the liquidity) of these assets. Although eminent, their model has two unappealing features: countercyclicality of consumption and equity prices. [Shi \(2015\)](#) and [Bigio \(2012\)](#) document these contrfactual effects on consumption and price of equity under plausible calibrations of the model, questioning the ability of liquidity shocks to generate meaningful business cycle dynamic. By introducing nominal rigidities, in particular sticky wages, I can correct these unappealing features of the model.

Several authors have introduced KM's financial frictions into a New Keynesian setting (e.g., [Guillen, Cui, et al., 2012](#); [Cui and Radde, 2014](#); [Nezafat and Slavik, 2014](#); [Shi, 2015](#)). Between them, [Ajello \(forthcoming\)](#) studies the importance of wage rigidities in this environment. More interestingly, he uses firm data to estimate firm's financing through external funding and asset liquidation. [Del Negro, Eggertsson, Ferrero, and Kiyotaki \(2011\)](#) (DEFK) study unconventional monetary policy in the presence of the zero lower bound and KM's financial frictions. Several authors have also embedded their economy in a New Keynesian setting but have analyzed other models of financial frictions such as [Gertler and Karadi \(2011\)](#), [Gertler and Karadi \(2011\)](#), [Christiano, Motto, and Rostagno \(2014\)](#) and [Jermann and Quadrini \(2012\)](#). In all these models, the liquidity friction applies on private financial assets and not on public bonds.

I extend DEFK's work by making sovereign bonds market not perfectly liquid. By doing so, I am able to address a different question which has yet not been studied in depth: is the liquidity of sovereign bonds important? Can liquidity of these bonds have macroeconomic effects? [Molteni \(2015\)](#) studies non conventional monetary policy in Ireland in 2012 which aimed at introducing liquidity in a dry market. He argues that this illiquidity is the consequence of repo haircuts on peripheral government bonds which sharply increased during the European crisis. While *fully* restricting the resaleability of equity in his model, he is overestimating the importance of government bonds in providing financing

liquidity in the economy. Moreover, his setting cannot address the pattern differentials of sovereign spreads during 2009 and 2011 to which my framework can speak to. [Jaccard \(2013\)](#) considers a liquidity shock as the destruction of a fraction of the safe asset produced by the financial sectors which provide liquidity services to firms and households to study the consequences of the Great Recession. An implication of his work is that, a destruction of safe assets should generate an increase in its price, which would not be consistent with increase in spreads during 2011 in Europe.

My work has been mainly motivated by empirical papers which study the relationship between sovereign spreads and liquidity in the Eurozone. For Italian data [Pelizzon, Subrahmanyam, Tomio, and Uno \(2013\)](#) document the strong non-linear relationship between changes in Italian sovereign risk and liquidity in the secondary bond market. They conclude that the crisis began with a spike in credit risk that was transmitted into unprecedented levels of illiquidity. They also find significant effects of how liquidity improved after the intervention by the European Central Bank (ECB), through its Long-Term Refinancing Operations (LTRO) and Outright Monetary Transactions (OMT) programs, starting in December 2011. [Darbha and Dufour \(2013\)](#), use various liquidity measures such as Amihud measure, bid ask spreads, market depth and Roll measure among other measures to study their contribution to yield spreads. Liquidity becomes an important explanatory factor of spreads during the crisis period. Measuring liquidity as the spread between a German state guaranteed agency (KfW) bond and the German Bund [De Santis \(2014\)](#) finds that flight to liquidity benefiting the German Bund is behind the pricing of *all* Euro area spreads and, specifically, is the *only* factor explaining the sovereign spreads for Finland and the Netherlands. [Schwarz \(2015\)](#) constructs a new measure of market liquidity and interbank credit to decompose euro-area sovereign bond and term EURIBOR-OIS spreads into credit and liquidity components in order to analyze liquidity during the Great Recession. She finds that liquidity risk premia are large and significant in the 2008 crisis<sup>4</sup>. Some authors argue that this liquidity measure should be particular to *each* country such as [Garcia and Gimeno \(2014\)](#) who use sovereign and national agency bonds spread to construct indicators of liquidity premia for Spain, France and Germany. They show that flight to liquidity flows led to significant inverse moves sovereign bond yields in Euro area core and periphery markets. In contrast, [Monfort and Renne \(2014\)](#) using agency to sovereign spreads for some additional countries such as Austria and Netherlands, argue that the KfW can be used as a *general* sovereign market liquidity in Europe. Their work also suggest that liquidity has played an important role in explaining spreads recently. Motivated by these empirical findings I study the link between illiquidity in sovereign markets, yield spreads, and economic activity.

This paper abstracts from sovereign default risk to focus on a pure liquidity channel. Some authors study the link between these two. [Bocola \(Forthcoming\)](#) develops a quantitative model to estimate the balance sheet and credit risk channels in the pass-through of sovereign risk. [Perez \(2015\)](#) suggests that sovereign default can disrupt the domestic economy via negative balance sheet effect on banks

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<sup>4</sup>I use her data to estimate the model in Section 3.

and lower domestic liquidity. Although compelling, his results rely on a questionable simple linear production function.

What drives liquidity shocks? I follow KM by taking liquidity shock as exogenous. However, [Cui and Radde \(2014\)](#) endogenize private equity liquidity shocks in a KM setting through search frictions on asset markets. [Bigio \(2015\)](#) and [Kurlat \(2013\)](#) suggest that in the context of KM liquidity shocks could be the result of adverse selection in the market for collateral. If entrepreneurs have private information on the quality of their capital, liquidity shocks may arise endogenously as a problem of asymmetric information at the time of selling older projects. Also self-fulfilling prophecies may arise if assets become illiquid because there is fear of them being hard to sell in the future. For example, illiquidity of sovereign bonds could be the consequence of an increase in default risk that would lead to illiquidity in the event of a default. Feedback loop between liquidity and default risk has been studied by [He and Milbradt \(2014\)](#) for corporate bonds and [Passadore and Xu \(2014\)](#) for sovereign bonds. They study the interaction between default and liquidity for bonds that are traded in an over-the-counter secondary market with search frictions. Lastly, market liquidity could be linked to asymmetric evolution of risk aversion. At booms, expectations of bad outcomes are lower therefore agents accept riskier projects/bonds however during recessions expectations of even lower outcomes might dry up markets due to an increase in risk aversion.

On the relationship between finance and economic stability, there are two opposing views. On one hand, financial development lessens volatility by reducing frictions or informational asymmetries; thereby reducing the amplification of cycles that occurs through the financial accelerator (e.g., [Bernanke, Gertler, and Gilchrist, 1999](#); [Hubbard, 1998](#)). Financial development is also said to promote risk-sharing, reducing financial constraints, enhancing the ability of firms and households to absorb shocks, and allows greater consumption smoothing evidenced by [Easterly, Islam, and Stiglitz \(2001\)](#), [da Silva \(2002\)](#), [Fidrmuc and Scharler \(2013\)](#). On the other hand, finance increases economic and financial volatility and the probability of a crisis, by promoting greater risk-taking and leverage, particularly when the financial system is poorly regulated and supervised as suggested by [Rajan \(2005\)](#) and [Gennaioli, Shleifer, and Vishny \(2012\)](#). Others authors reconcile these two views such as [Loayza and Ranciere \(2006\)](#), [Lopez, Spiegel, et al. \(2002\)](#), [Ranciere, Tornell, and Westermann \(Forthcoming\)](#) who support that while financial development is beneficial over the long run, it may exacerbate short-term volatility in isolated episodes. In line with this idea, I show that more financially developed economies although they have higher steady state levels of output, they also face greater fragility.

## 2 Model

### 2.1 Environment

Consider an infinite-horizon economy with discrete time. There are three main sectors in this economy: Households, Production sector and Government. Households face two stages. In the first stage, households' members are either workers or entrepreneurs, bringing labor income and investment decisions home, respectively. In the second stage, each household as a whole optimally chooses consumption and savings taking as given labor income and the investment possibilities. Financial frictions only affect the investment opportunities faced in the first stage. These frictions consist on borrowing constraints and liquidity constraints affecting the resaleability of equity and government bonds. The Production sector has an homogeneous final good, produced with intermediate inputs. These are produced by monopolistic competitors who in turn, rent capital and labor from households, setting their prices à la Calvo (1983). There are also capital good producers, to allow for some curvature in the production of capital. I introduce sticky wages in a standard way, by having Labor Agencies choose wages on a staggered basis, in order to maximize the utility of workers, taking as given the demand for labor. Government collects taxes and issue new debt following a fiscal rule to insure intertemporal solvency. It also sets the Taylor rule for the Nominal—liquidity free—interest rate.

### 2.2 Households

This economy is populated by a continuum of households with measure one, which in turn are composed of a continuum of members. At the beginning of each period, all assets are equally distributed across members. In the first stage, each household member receives a random shock that determines her profession. She can either be a worker with probability  $(1 - \chi)$  or an entrepreneur with probability  $\chi$ . Each member takes individual decisions depending on her type. At the end of the first stage, workers bring aggregate labor income  $W_t H_t$  to the household and entrepreneurs bring aggregate investment possibilities to the household  $I(N_t, B_{t-1})$ . Household's second stage choice variables are consumption  $C_t$ , and savings for the next period  $(N_{t+1}, B_t)$ .

Each household begins the period with assets from the previous period: net equity  $N_t$ , and government bonds  $B_{t-1}$ . Households own physical capital  $K$  which depreciates at rate  $\delta$  and is rented to firms at rate  $r_t^K$ . They can engage in the production of new physical capital through investment at cost  $p_t^I$  and issue equity on the underlying capital which pays dividends and depreciates at its same rate. Net equity  $N_t$ , is an aggregate of i) physical capital  $K_t$  ii) claims on other agent's capital  $N_t^0$  iii) equity issued on own capital sold to other agents  $N_t^I$ . Therefore net equity  $N_t = N_t^0 - N_t^I + K_t$  is



the household's aggregate of equity holdings and capital net of equity issued on own physical capital sold to others. Government one period nominal bonds  $B_t$  provide another means of saving with gross return  $R_t^B$ .

In the second stage, household as a whole uses savings, labor income and optimal investment decision to maximize its expected utility by choosing consumption and savings

$$\max_{[C_t, N_{t+1}, B_t]_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t E_0 [U(C_t, H_t)]$$

subject to its budget constraint

$$C_t + p_t^I I_t + q_t (N_{t+1} - I_t) + \frac{B_t}{P_t} \leq W_t H_t + D_t - \tau_t + \left( r_t^k + (1 - \delta) q_t \right) N_t + \frac{B_{t-1} R_{t-1}^B}{P_t}, \quad (1)$$

and a "cash in advance" constraint for investment

$$I_t \leq \chi I(N_t, B_{t-1}, S_t). \quad (2)$$

Where  $I_t$  is investment in new physical capital,  $q_t$  is the price of equity, and  $P_t$  is the price index for final consumption goods.

On the income side,  $W_t H_t$  is total labor income. Household pay taxes  $\tau_t$  and are owners of the all firms and receive their profits  $D_t$ <sup>5</sup>.  $\frac{B_{t-1} R_{t-1}^B}{P_t}$  represents the gross return of saving through government bonds. Net equity are claims on capital which pay dividend  $r_t^k$  and a fraction  $\delta$  is depreciated each period, therefore  $(1 - \delta) q_t N_t$  is the market value of current equity. On the expenditure side, household can spend on consumption goods, equity desired for next period  $N_{t+1}$ , from which a fraction  $I_t$  is produced by the household at price  $p_t^I$ .

Financial frictions affect first best investment decisions. In the next subsection I will explain where this function  $I(N_t, B_{t-1}, S_t)$  comes from. Meanwhile, I would like to stress that investment today depends on the amount of assets—net equity and government bonds  $(N_t, B_{t-1})$ —the household is bringing from the previous period and the state of the economy given by  $S_t$ , that is the realization of exogenous shocks affecting the financial markets. Moreover, financial frictions affecting the liquidity of assets imply that the price of producing new physical capital  $p_t^I$  and the price of equity  $q_t$  is not equalized in equilibrium. In a frictionless world, investment would be at its first best therefore there would be no wedge between these two prices. However, the presence of financial frictions constrain investment in new capital implying that (2) will be binding in equilibrium, which leads to a wedge between  $p_t^I$  and  $q_t$ , that is  $p_t^I < q_t$ .

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<sup>5</sup> $D_t = D_t^I + D_t^F + D_t^K$ , aggregate profits of all firms, intermediate goods, final goods and capital goods, respectively.

Assets	Liabilities
$q_t N_t^0(j)$ $B_{t-1}(j) R_{t-1}^B$ $\frac{P_t}{D_t(j) - \tau_t(j)}$ $q_t K_t(j)$	$q_t N_t^I(j)$  <b>Net Worth</b> $N_t(j) + B_{t-1}(j) + D_t(j) - \tau_t(j)$

Table 1: **Entrepreneur's  $j$  Balance Sheet**

### 2.2.1 First stage

Before the beginning of each first stage, all household members receive an equal amount of assets  $B_{t-1}(j), N_t^O(j), N_t^I(j), K_t(j)$ , as well as profits from firms minus taxes  $D_t(j) - \tau_t(j)$ . A random realization of a shock determines the profession of each member with  $\chi$  being the probability of becoming an entrepreneur.

**Workers:** members  $j \in (\chi, 1)$  provide labor. Workers do exactly as mandated by labor unions, supplying all the amount of labor demanded by firms at the wage settled by agencies. At the end of the first stage, workers bring labor income to the household:  $W_t H_t = \int_{\chi}^1 W_t(j) H_t(j) dj$

**Entrepreneurs:** members  $j \in (0, \chi)$  cannot work. They are in charge of making investment in new capital decisions. Because financial frictions restrict optimal investment, when facing an investment opportunity entrepreneurs use all their available resources to invest in new physical capital since  $p_t^I < q_t$ . Table 1 presents the entrepreneurs' balance sheet at the beginning of the first stage.

On the assets side, we have claims on other agent's capital  $N_t^O(j)$ , government bonds  $B_{t-1}(j)$ , cash which comes from profits of firms minus taxes  $D_t(j) - \tau_t(j)$ , and physical capital  $K_t(j)$ . On the liabilities side, there is equity issued on own capital sold to other agents  $N_t^I(j)$ .

**Financial Frictions** In an frictionless world, entrepreneurs would borrow as much as possible by issuing new equity, sell their assets and use all these funds to invest in new physical capital, until the wedge between  $p_t^I$  and  $q_t$  vanished. However, this will not be possible. In first place, entrepreneur face **borrowing constraints**. Borrowing is limited to a fraction  $\theta \in (0, 1)$  out of new capital. This implies that entrepreneurs must internally finance the remaining  $(1 - \theta) I_t(j)$ . If borrowing constraints were the only financial friction, entrepreneurs would liquidate all their assets in order to invest. But, this is also be limited due to **liquidity frictions** that restrain the resaleability of assets. **Resaleability constraint** on equity—following KM—entails that entrepreneurs can only sell a fraction  $\phi \in (0, 1)$  of current equity. More precisely

$$N_{t+1}^O(j) \geq (1 - \phi_t)(1 - \delta) N_t^O(j).$$

Which suggests that tomorrow's holdings of other's claims must be at least a fraction  $(1 - \phi_t)$  of non depreciated equity. This friction is a reduced form that captures trading costs and the fact that equity may not always be immediately sold. It can also be interpreted as a liquidity friction in the secondary market. The more liquid the secondary market for equity is, the higher the probability of finding a buyer, and therefore the greater the probability of liquidating the asset.

Similarly, I will introduce a **liquidity constraint for sovereign bonds**. Investors will be able to sell only a fraction  $z_t \in (0, 1)$  of the value of their current bonds.

$$\frac{B_t(j)}{P_t} \geq (1 - z_t) \frac{B_{t-1}(j) R_{t-1}^B}{P_t}. \quad (3)$$

If  $z_t = 1$ , government bonds are completely liquid, however  $z_t$  could be below one whenever these assets cannot be immediately transformed into cash. We can think that this is a reduced form that captures the fact that bonds cannot be sold immediately in secondary markets.

These two liquidity constraints—on equity and sovereign bonds—can be interpreted as follows. The more liquid their secondary markets are, the higher the probability of finding a buyer and reselling these assets. Whenever  $\phi_t$  or  $z_t$  are equal to one, these resources are perfectly liquid and can be immediately used to finance the production of new physical capital.

Entrepreneur's  $j$  investment decision must satisfy her budget constraint

$$p_t^I I_t(j) + q_t (N_{t+1}(j) - I_t(j)) + \frac{B_t(j)}{P_t} \leq D_t(j) - \tau_t(j) + \left( r_t^k + (1 - \delta) q_t \right) N_t(j) + \frac{B_{t-1}(j) R_{t-1}^B}{P_t},$$

constraints on equity

$$N_{t+1}(j) \geq (1 - \theta) I_t(j) + (1 - \phi_t) (1 - \delta) N_t(j), \quad (4)$$

and liquidity of government bonds

$$\frac{B_t(j)}{P_t} \geq (1 - z_t) \frac{B_{t-1}(j) R_{t-1}^B}{P_t}$$

Borrowing and resaleability constraints on equity can be summarized in Equation (4) which should be read as follows. The amount of net equity the entrepreneur will have for tomorrow  $N_{t+1}(j)$  must be at least the down-payment the entrepreneur must make in order to finance new investment  $(1 - \theta) I_t(j)$ , plus the amount of equity that did not depreciate and that could not be sold in the market. If  $\theta = 1$  the entrepreneur would be able to finance the entire investment by selling equity. When  $\theta$  is less than

1 the entrepreneur is forced to retain  $1 - \theta$  fraction of investment as own equity and use her own fund to partly finance the investment cost<sup>6</sup>.

Putting together the last three constraints and aggregating across all households entrepreneurs  $I_t = \int_0^X I_t(j) dj$  delivers optimal investment decision:

$$I_t \leq \frac{(r_t^k + (1 - \delta) \phi_t q_t) N_t + z_t \frac{B_{t-1} R_{t-1}^B}{P_t} + D_t - \tau_t}{(p_t^I - \theta q_t)} \quad (5)$$

In the numerator we have the total amount of funds (cash/liquid assets) the entrepreneur can achieve given his resources. For each unit of liquidity the entrepreneur can transform them into physical capital at a cost  $(p_t^I - \theta q_t)$  which is the internal financing cost of each unit of new investment. Note how liquidity shocks affect the investment possibilities of the entrepreneur. Unfavorable shocks to either  $\phi_t$  or  $z_t$  can depress investment (ceteris paribus) in a similar manner, however, they will have different general equilibrium implications due to differentials in the relative prices between equity and bonds.

## 2.2.2 Second Stage

In this stage, the household as a whole chooses consumption and savings for next period given labor income and taking into account that these decisions will affect investment in future periods. Let me now specify the household's utility function.

Household problem is as follows

$$\max_{[C_t, N_{t+1}, B_t]_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t E_0 \left[ \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{1}{1+\nu} \int_{\chi}^1 H_t(j)^{1+\nu} dj \right] \quad (6)$$

$$C_t + p_t^I I_t + q_t (N_{t+1} - I_t) + \frac{B_t}{P_t} \leq W_t H_t + D_t - \tau_t + \left( r_t^k + (1 - \delta) q_t \right) N_t + \frac{B_{t-1} R_{t-1}^B}{P_t} \quad (7)$$

$$I_t \leq \chi \frac{(r_t^k + (1 - \delta) \phi_t q_t) N_t + z_t \frac{B_t R_{t-1}^B}{P_t} + D_t - \tau_t}{p_t^I - \theta q_t}; (\lambda_t^e) \quad (8)$$

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<sup>6</sup>Liquidity shocks, could be micro-funded through through search frictions on asset markets [Cui and Radde \(2014\)](#); information asymmetries such as [Bigio \(2015\)](#) and [Kurlat \(2013\)](#). Also, liquidity shocks could be the consequence of a default risk, as [He and Milbradt \(2014\)](#) and [Passadore and Xu \(2014\)](#), have studied the interaction between default and liquidity for corporate and sovereign bonds respectively, that are traded in an over-the-counter secondary market with search frictions.

given  $B_0; K_0$ .

Where  $\sigma$  is risk aversion and  $\nu$  is the inverse Frisch elasticity.  $H_t(j)$  is  $j$ -specific type of labor supplied by member  $j$ , and household total labor income is given by  $W_t H_t = \int_{\mathcal{X}}^1 W_t(j) H_t(j) dj$ .  $\lambda_t^e$  is the shadow value of investment, that is by how much the household values relaxing the investment constraint.

### 2.2.3 Household Optimization

There is no intratemporal decision for households, since workers just supply all labor demanded by firms at wages settled by labor agencies. Intertemporal decisions affecting savings portfolio are not standard given the liquidity constraints affecting investment.

Euler equations for Bonds and Equity are given respectively by

$$C_t^{-\sigma} = \beta E_t \left\{ C_{t+1}^{-\sigma} \left[ \frac{R_t^B}{\pi_{t+1}} + \lambda_{t+1}^e z_{t+1} \frac{R_t^B}{\pi_{t+1}} \right] \right\} \quad (9)$$

$$C_t^{-\sigma} = \beta E_t \left\{ C_{t+1}^{-\sigma} \left[ \frac{[r_{t+1}^k + (1-\delta)q_{t+1}]}{q_t} + \lambda_{t+1}^e \frac{[r_{t+1}^k + (1-\delta)q_{t+1}\phi_{t+1}]}{q_t} \right] \right\} \quad (10)$$

With the liquidity premium given by

$$\lambda_t^e = \chi \frac{q_t - p_t^I}{p_t^I - \theta q_t} \quad (11)$$

The first term on both Euler equations are the standard returns to holding bonds  $\frac{R_t^B}{\pi_{t+1}}$  and equity  $\frac{[r_{t+1}^k + (1-\delta)q_{t+1}]}{q_t}$ . The second term is the liquidity premium which only corresponds to the expected liquid part of these assets. Households value each asset not only for its standard return but also for the liquidity it provides since this relaxes the constraint on investment. In the case of bonds, the fraction of liquid assets is  $z_{t+1}$ , and for equity the premium applies to the resalable fraction of future equity  $\phi_{t+1} (1-\delta) q_{t+1}$ .

Note that although government bond's return might be lower than equity, households will still have this asset in their portfolio not only due to lower risk, but also due to the liquidity they provide. A reduction in the liquidity of government bonds will reduce their demand. Because these assets are less liquid, and households value them less, they will demand higher returns in order to have them their

portfolio.<sup>7</sup>

How should we interpret the liquidity premium? For every extra dollar the household has, only a fraction  $\chi$  is going to entrepreneurs. The numerator,  $q_t - p_t^I$  measures the benefit of an extra unit of investment. Since households can produce capital at cost  $p_t^I$  and sell it at price  $q_t$  the larger this wedge is the greater the household values an extra unit. The denominator  $\frac{1}{p_t^I - \theta q_t}$  is the extent to which the investment constraint is relaxed per extra unit of liquidity. To sum up, the liquidity premium can be interpreted by how much the household values an extra unit of the asset given that the liquid part of the asset relaxes the investment constraint by  $\lambda_t^e$ . If the investment constraint is not binding, ( $\lambda_{t+1}^e = 0$ ) then the Euler equation is standard, and the return to savings should be equal to the discounted expected change in marginal utility. However, in the presence of financial frictions, the price of assets will be higher. Not only should they account for standard consumption smoothing desire, but it also they will be valued for alleviating future investment funding needs.

## 2.3 Production

The production sector in this economy is composed by final goods, produced using intermediate inputs, which in turn hire labor and capital provided by households. In addition, there are capital goods producers and labor agencies who will determine wages.

### 2.3.1 Final Good Producers

There is an homogeneous final good  $Y_t$ , produced under perfect competition, using  $Y_{it}$  as inputs:

$$Y_t = \left[ \int Y_{it}^{\frac{1}{1+\lambda_F}} di \right]^{1+\lambda_F}$$

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<sup>7</sup>Note that the Euler equation for equity can be rewritten as follows

$$q_t = E_t \left\{ \beta \frac{\mu_{t+1}}{\mu_t} \left[ (1 + \lambda_{t+1}^e) r_{t+1}^k + (1 - \delta) (1 + \phi_{t+1} \lambda_{t+1}^e) q_{t+1} \right] \right\}, \quad (12)$$

with  $\mu_t$  the marginal utility of consumption at time  $t$ . The price of equity today depends on the expected present discounted value of future dividends  $\left( \sum_{j=1}^{\infty} r_{t+j}^k \right)$ , since  $q_{t+1}$  can be written as a function of expectations for  $r_{t+2}^k, q_{t+2}, \lambda_{t+2}^e, \phi_{t+2}$  and so on. Combination of (11) and (12) suggests the trade off affecting equity prices. On one hand, a tightening of the liquidity constraint (*ceteris paribus*) will deliver higher equity prices, contrary to real business cycles evidence. On the other hand, low enough rental price of capital in near future periods reduces equity prices.

were  $\lambda_F \geq 0$  is the elasticity of substitution between these inputs. Taking as given the output price  $P_t$  and the intermediate inputs' price  $P_{it}$ , the demand for each intermediate good  $i$  is given by

$$Y_{it} = \left( \frac{P_{it}}{P_t} \right)^{-\frac{1+\lambda_F}{\lambda_F}}.$$

The zero profit condition for competitive final goods producers implies that the price must be such that

$$P_t = \left( \int_0^1 P_{it}^{-\frac{1}{\lambda_F}} di \right)^{-\lambda_F}.$$

### 2.3.2 Intermediate Good producer

Monopolistically competitive intermediate good producer  $i$ , uses labor  $H_{it}$  and capital  $K_{it}$  according to the following constant return to scale technology

$$Y_{it} = K_{it}^\gamma H_{it}^{1-\gamma},$$

with  $\gamma \in (0, 1)$ . Let  $w_t$  and  $r_t^K$  be the price of labor and capital, optimal capital to output ratio for firm  $i$  is given by

$$\frac{K_{it}}{H_{it}} = \frac{K_t}{H_t} = \frac{\gamma}{1-\gamma} \frac{w_t}{r_t^K}$$

which is common across all firms due to their same production technology. Given that the capital to labor ratio is independent of firm specific variables, the marginal cost  $mc_{it}$  is not firm specific as well

$$mc_{it} = mc_t = \left( \frac{r_t^K}{\gamma} \right)^\gamma \left( \frac{w_t}{1-\gamma} \right)^{1-\gamma}.$$

Intermediate good producers face price stickiness à la [Calvo \(1983\)](#). Let  $(1 - \zeta_p)$  the probability that the firm will be able to reset its price next period, then a firm chooses its price  $P_{it}$  in order to solve the following problem

$$\max_{P_{it}} \sum_{s=0}^{\infty} (\zeta_p)^s E_t [Q_{t,t+s} Y_{i,t+s} (P_{i,t} - mc_{t+s})]$$

$$Y_{t+s}^i = \left( \frac{P_{it}}{P_t} \right)^{-\frac{1+\lambda_F}{\lambda_F}}$$

were  $Q_{t,t+s}$  is the household's discount factor<sup>8</sup>.

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<sup>8</sup> $Q_{t,t+s} = \beta^s \frac{u'(C_{t+s})}{u'(C_t)} \frac{P_t}{P_{t+s}}$

The solution to this problem is given by  $P_{i,t}^*$  such that

$$\sum_{s=0}^{\infty} (\zeta_P)^s E_t \left[ Q_{t,t+s} Y_{t+s} (P_{t+s})^{(1+\lambda_F)/\lambda_F} \left[ \frac{P_{ti}^*}{P_{t+s}} - (1 + \lambda_t) m c_{t+s} \right] \right] = 0.$$

By the law of large numbers, the fraction of firms who actually change their price at time  $t$  coincides with the probability of being able to reset the price. Therefore, price evolves in the following way

$$P_t^{-1/\lambda_F} = (1 - \zeta_P) P_t^{*-1/\lambda_F} + \zeta_P P_{t-1}^{-1/\lambda_F}.$$

### 2.3.3 Capital Goods Producers

Capital producers operate in a competitive market transforming consumption into investment goods. These firms make new capital using final good as input, and face adjustment costs from deviation of steady state values. They sell new capital to entrepreneurs at price  $p_t^I$ . Their problem consists on choosing the amount of investment goods produced  $I_t$  to maximize profits  $D^I$ , taking  $p_t^I$  as given

$$\max_{I_t} \left[ p_t^I - \left( 1 + S \left( \frac{I_t}{I} \right) \right) \right] I_t.$$

When evaluated in steady state, the adjustment cost function and its first derivative are zero  $S(1) = S'(1) = 0$ , while the second derivative is positive  $S''(1) > 0$ .

Optimality implies that  $I_t$  will be such that the marginal cost of production is equal to the marginal benefit.

$$p_t^I = 1 + S \left( \frac{I_t}{I} \right) + S' \left( \frac{I_t}{I} \right) \frac{I_t}{I}.$$

## 2.4 Labor Agencies

In order to introduce wage stickiness, let each type of labor  $j \in [\chi, 1]$  be a labor variety with elasticity of substitution across varieties given by  $\lambda_\omega$ . Labor agencies combine  $j$ -specific labor into a composite  $H_t$

$$H_t = \left[ \left( \frac{1}{1-\chi} \right)^{\frac{\lambda_\omega}{1+\lambda_\omega}} \int_{\chi}^1 H_t(j)^{\frac{1}{1+\lambda_\omega}} dj \right]^{1+\lambda_\omega}$$

where the first term  $\left( \frac{1}{1-\chi} \right)^{\frac{\lambda_\omega}{1+\lambda_\omega}}$  is a constant that I have added just to simplify calculus and future notation. Firms hire composite labor input at wage  $W_t = \left[ \frac{1}{1-\chi} \int_{\chi}^1 W_t(j)^{-\frac{1}{\lambda_\omega}} dj \right]^{-\lambda_\omega}$  which is the



aggregate wage index that comes from the following zero profit condition for labor agencies

$$W_t H_t = \int_{\mathcal{X}} W_t(j) H_t(j) dj.$$

This implies that, given wages, the demand for j-specific labor is  $H_t(j) = \frac{1}{1-\mathcal{X}} \left[ \frac{W_t(j)}{W_t} \right]^{-\frac{1+\lambda_w}{\lambda_w}} H_t$ .

Labor agencies representing workers of type j set wages  $W_t(j)$  for each type  $j \in [\mathcal{X}, 1]$  on a staggered basis with probability  $(1 - \zeta_w)$  of resetting wages each period. Their objective is to maximize the expected utility of the household, taking as given the demand for j-specific labor.

$$\begin{aligned} \max_{W_t(j)} \sum_{s=0}^{\infty} (\beta \zeta_w)^s E_t \left[ \frac{C_{t+s}^{1-\sigma}}{1-\sigma} - \frac{1}{1+\nu} \int_{\mathcal{X}} H_{t+s}(j)^{1+\nu} dj \right] s.t. \\ H_t(j) = \frac{1}{1-\mathcal{X}} \left( \frac{W_t(j)}{W_t} \right)^{-\frac{1+\lambda_w}{\lambda_w}} H_t, \end{aligned} \quad (13)$$

and household budget constraint given by Equation(7).

Optimal  $W_t^*(j)$  is given implicitly by the F.O.C. of this problem

$$E_t \left[ \sum_{s=0}^{\infty} (\beta \zeta_w)^s H(j)_{t+s} u_c(C_{t+s}, H_{t+s}) \left( \frac{W_t^*(j)}{P_{t+k}} - (1 + \lambda_w) MRS_{t+s} \right) \right] = 0.$$

Where  $MRS_{t+s} = -\frac{u_n(C_{t+s}, H_{t+s})}{u_c(C_{t+s}, H_{t+s})}$  denotes the marginal rate of substitution between consumption and hours in period  $t + s$  for the household resetting the wage in period t. Given that all agencies face identical problem, we will focus on a symmetric equilibrium in which all labor agencies set the same wage  $W_t^*(j) = W_t^*$ . Letting  $w_t = \frac{W_t}{P_t}$ , the evolution for the wage index can be described in terms of real wage and inflation

$$w_t^{-1/\lambda_w} = (1 - \zeta_w) w_t^{*-1/\lambda_w} + \zeta_w \frac{w_{t-1}^{-1/\lambda_w}}{\pi_t}.$$

## 2.5 Government

### 2.5.1 Fiscal Policy

In every period, the government issues new one period nominal bonds  $B_t$  and collects lump sum taxes from households  $\tau_t$  in order to repay previously issued debt  $B_{t-1}$  and promised interests  $R_{t-1}^B$ . The government's budget constraint is

$$\tau_t = \frac{B_{t-1} R_{t-1}^B}{P_t} - \frac{B_t}{P_t}$$

In order to ensure intertemporal solvency, taxes will follow a simple fiscal rule written in deviations from steady state

$$\tau_t - \tau = \psi_\tau \left( \frac{R_{t-1}^B B_t}{P_t} - \frac{r^B b}{\pi} \right),$$

with  $\psi_\tau > 0$ . This rule implies that taxes in period  $t$  will be proportional to the debt position at the beginning of the period.

## 2.5.2 Monetary Policy

Monetary policy in this economy is given by a Taylor rule for the liquidity free nominal interest rate<sup>9</sup>

$$R_t^{Nom} = R \pi_t^\psi \quad (14)$$

Although during the period analyzed the nominal interest rate set by ECB did not reach the Zero Lower Bound<sup>10</sup>, including this constraint would amplify the macroeconomic effects.

## 2.6 Equilibrium and Solution Method

An equilibrium in this economy is defined as a sequence of prices  $[\pi_t, p_t^I, q_t, r_t^K, w_t, w_t^*, R_t^B, R_t^{Nom}]_{t=0}^{t=\infty}$  and a sequence of allocations  $[N_{t+1}, B_t, C_t, K_t, I_t, H_t, \tau_t, Y_t]_{t=0}^{t=\infty}$  such that, given prices and exogenous evolution for  $z_t$ , households optimize, final good, intermediate good and capital good producers optimize, labor agencies solve problem (13), government follows its fiscal and monetary rules, and markets clear:

1. Composite labor supply equals the aggregate amount of labor demanded by firms  $H_t = \int H_t(i) di$
2. The total capital amount supply of households is equal to the aggregate demand for capital from firms  $K_t = \int K_t(i) di$
3. Supply of government bonds is equal to demand from households  $B_t^s = B_t^d$
4. Aggregate resource constraint holds  $Y_t = C_t + [1 + S(\frac{I_t}{Y})] I_t$
5. Law of motion for capital given aggregate investment decision is  $K_{t+1} = (1 - \delta) K_t + I_t$

<sup>9</sup>By liquidity free nominal interest rate I mean, an "ideal" interest rate that would not be affected by liquidity shocks.  $R^{LF}$  such that it satisfies  $C_t^{-\sigma} = \beta E_t \left\{ C_{t+1}^{-\sigma} \left[ \frac{R_t^{LF}}{\pi_{t+1}} + \lambda_{t+1}^e \frac{R_t^{LF}}{\pi_{t+1}} \right] \right\}$

<sup>10</sup>ECB nominal interest rate reached ZLB by July 2012

Parameters	Value	Description
$\bar{z}$	0.65	Bonds' liquidity, consistent with Liq. Spread of 39bps.
$\bar{\phi}$	0.19	Resalebility of equity
$\theta$	0.19	Borrowing constraint tightness
$\chi$	0.05	Prob of receiving an investment opportunity
$\beta$	0.99	Discount factor
$\sigma$	1.00	Risk aversion
$\nu$	1.00	Inverse Frisch elasticity
$\lambda_p$	0.10	Elasticity of substitution among goods varieties
$\lambda_w$	0.10	Elasticity of substitution among labor varieties
$\zeta_p$	0.75	Price stickiness, one year duration
$\zeta_w$	0.75	Wage stickiness, one year duration
$S''(1)$	1.00	Investment adjustment cost parameter
$\pi$	1.00	Steady state gross inflation rate
$\gamma$	0.40	Capital share
$\psi_\tau$	0.10	Tax sensitivity
$\psi_\pi$	1.50	Taylor rule penalty on target inflation
$B/Y$	0.50	Debt securitites to GDP ratio

Table 2: **Calibration**

6. Finally, the amount of equity issued for next period has to be equal to the underlying capital available in the economy, therefore  $K_{t+1} = N_{t+1}$ . Note this entails  $N_{t+1}^0 = N_{t+1}^I$

I approximate the model's equilibrium conditions up to the first order around a steady state in which funding constraints are binding ( $q > p^I$ ). Full set of equilibrium conditions in their linear forms are found in the Appendix [A.3](#).

## 3 Results

### 3.1 Estimation and Calibration

I calibrate the model at a quarterly frequency for a non stochastic steady state consistent with the European economy. Table 2 presents all calibrated parameters.

**Financial Frictions and Sovereign Liquidity** The most important parameters are those determining the financial frictions and in particular the one which characterizes the liquidity of government bonds. I estimate its steady state value for  $\bar{z} = 0.65$ , to match the average spread of the KfW bond,

which is 39 bps.<sup>11</sup> Sovereign liquidity spread in my model is the difference between a (partially liquid) government bond and an "ideal" perfectly liquid bond<sup>12</sup> with gross return  $R^{LF}$ . The rationale in using the spread between bonds issued by the German KfW bank and the German Bund is that, being fully guaranteed by the German Federal Government, the bonds issued by such agency have the same credit risk as the treasuries. Therefore, yield spreads provide evidence of the liquidity premium in the bond market. The evolution of  $z_t$  is given by an AR(1) process, with its persistence  $\rho_z = 0.82$ , and standard deviation  $\sigma_z = 0.2$  to match KfW spread's persistence and standard deviation.

Parameters determining frictions in the equity market are in line with what is mostly used in the KM literature. Following [Del Negro, Eggertsson, Ferrero, and Kiyotaki \(2011\)](#) I set the constraint on borrowing  $\theta = 0.185$  and the resaleability parameter in steady state  $\bar{\phi} = 0.185$ <sup>13</sup>. The opportunity to invest  $\chi = 0.05$  is also taken from DEFK. This parameter can be interpreted as the fraction of firms that adjust their capital in a period. The estimate of this value on the annual basis ranges from 0.20 [Doms and Dunne \(1998\)](#) to 0.40 [Cooper, Haltiwanger, and Power \(1999\)](#).

**Business cycles** Risk aversion  $\sigma$  is equal to one and the discount factor  $\beta = .99$ . The inverse Frisch elasticity of labor supply  $\nu$  equals to one as well. Capital share  $\gamma$  is 0.4 The annual depreciation is 10% ( $\delta = 0.025$ ) and I set  $S''(1) = 1$  so that the price elasticity of investment is consistent with instrumental variables estimates in [Eberly \(1997\)](#). Common in the literature, I calibrate symmetrically the degree of monopolistic competition in labor and product markets steady state markups  $\lambda_p = \lambda_w$  to 10%. I set  $\zeta_p = 0.75$  which implies an average price durations close to one year, this is consistent with evidence provided by [Alvarez, Dhyne, et al. \(2006\)](#) for Europe. As regards wage rigidity, following [Smets and Wouters \(2003\)](#) I set  $\zeta_w = 0.75$  such that wages are on average resettled annually<sup>14</sup>. Steady state debt to GDP is obtained from Eurostat, to match average sovereign debt securities to GDP between 2000 and 2014, which is 50% on average for European economies<sup>15</sup>. I set the feedback coefficient on inflation  $\psi_\pi$  in the interest rate rule to be 1.5, value commonly used in the literature. Tax sensitivity  $\psi_\tau$  is set equal to 0.1 to allow government taxes to adjust smoothly.

<sup>11</sup>KfW spread is a widely used measure of liquidity in Europe. I use data on KfW made available by Krista Schwarz's, [Schwarz \(2015\)](#).

<sup>12</sup> $R^{LF}$  such that Equation(9) holds.

<sup>13</sup>The exogenous process for  $\phi_t$  will be also AR(1) when replicating DEFK. Its persistence and size of the shock are taken from DEFK.

<sup>14</sup>[Le Bihan, Montornès, and Heckel \(2012\)](#) using quarterly firm data suggest the probability of resetting wages each quarter is on average 0.38%.

<sup>15</sup>Long term plus short term government debt securities to GDP, Eurostat.

### 3.2 Equilibrium Response to Sovereign Liquidity Shocks

Should policy makers worry about liquidity of sovereign bonds? Can fluctuations in this market explain increase in spreads and lower economic activity in Europe? I answer these questions through the lens of this model. First, I examine the effects of a negative shock to the liquidity on government bonds, when this is the only shock that the economy faces. Later, I allow this shock to be correlated with the liquidity of equity to address to other questions.

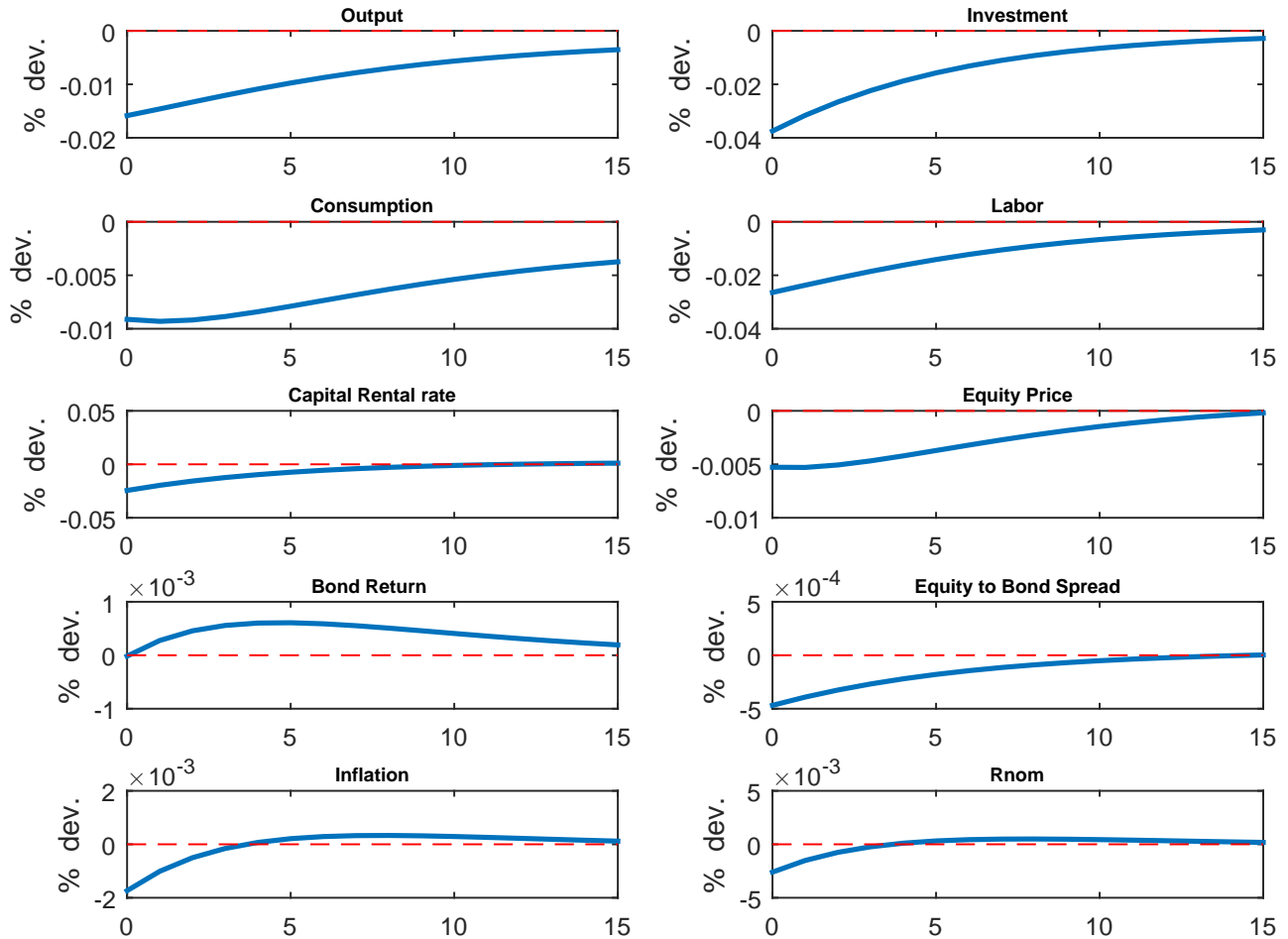


Figure 2: Negative shock to liquidity of Sovereign Bonds ( $z$ )

Figure 2 presents the response of the model to a negative shock—of size  $\sigma_z$ —to the liquidity of government bonds. After this shock,  $z_t$  follows the process a AR(1) process with persistence  $\rho_z$ . On the vertical axis of each plot we have percentage deviations from steady state levels for each variable and the horizontal axis is the number of quarters after the shock. A drop in the liquidity of government bonds reduces investment and generates meaningful fall in consumption and output on impact, which combined with wage rigidity, harmfully affects labor. The decline in consumption is

mainly an equilibrium consequence of the adverse income effect faced by households due to significant lower employment and deterioration of the value of capital.

Lower aggregate demand at the time of the shock can only be satisfied by decreasing the amount of labor hired—since capital has been determined in the previous period. Because wages are sticky they do not react much to a reduction in labor demand. Thus, the amount of labor hired in equilibrium drops on impact and gradually recovers while wages slowly adjust. A fall in labor negatively affects the marginal productivity of capital reducing its rental price. Given that equity prices are the present discounted value of expected dividends—rental rate of capital—low expected returns to capital for coming periods diminishes the price of equity (Equation (12)). More interestingly, the drop in the liquidity of sovereign bonds increases the return these must pay in order for households to want to hold them. This result is consistent with the increase in yields for non German sovereign bonds observed in Europe in 2011. Also, in compliance with the steady reduction in the interest rate set by the ECB, the safe asset (which is in zero net supply) nominal interest rate falls while trying to mitigate the negative effect on prices.

### 3.3 Economic Analysis and Robustness

#### 3.3.1 Great Recession vs European Debt Crisis

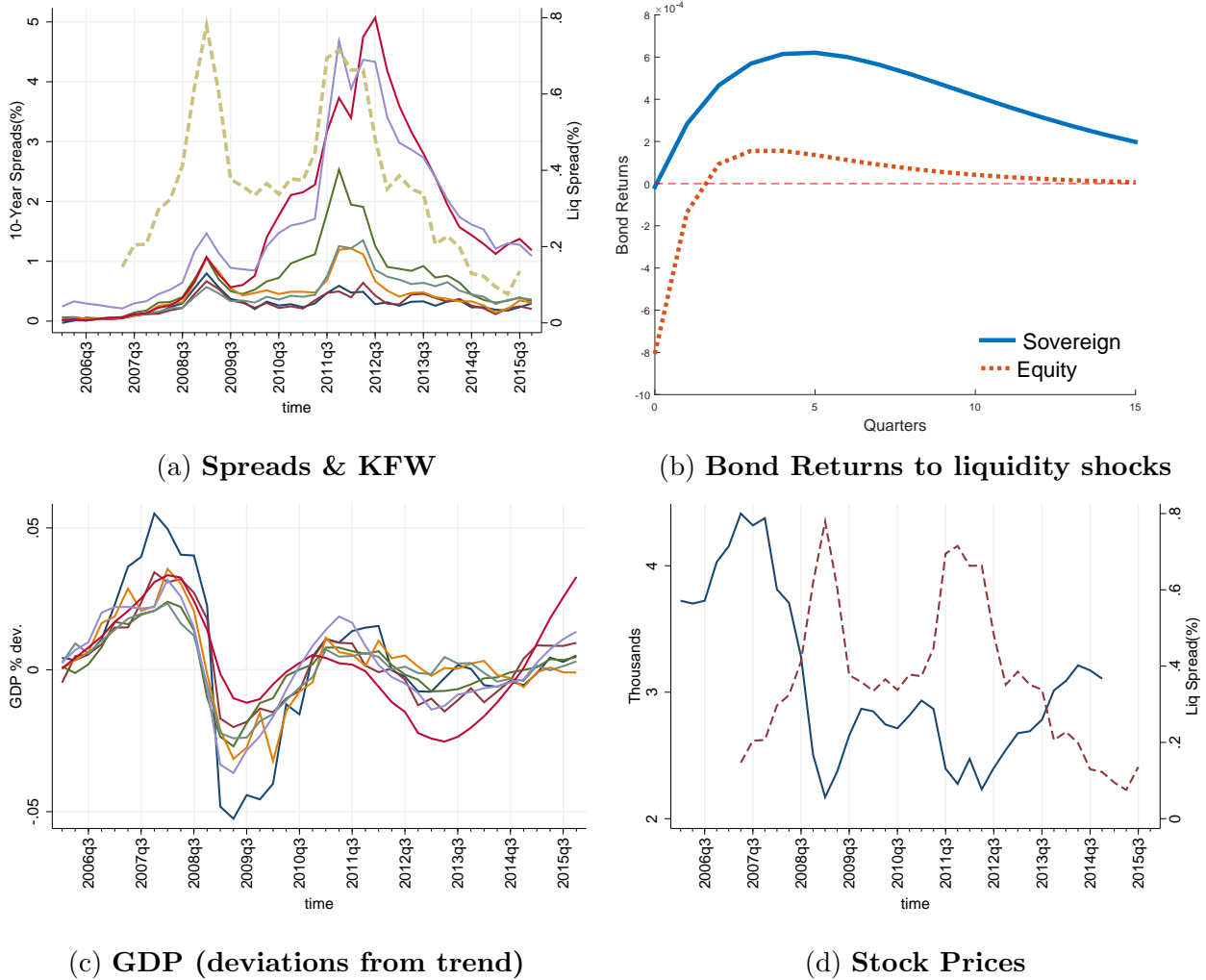


Figure 3: Great Recession vs European Debt Crisis

Note: Figure 3a plots solid lines for 10-year spreads to German bonds for Finland, Netherlands, Austria, Belgium, France, Spain and Italy, whereas the dashed line is KfW spread (liquidity). Figure 3b plots IRF for Government bonds return under a sovereign liquidity shock (solid) and a private equity shock (dashed). Figure 3c plots GDP deviations from trend for the same countries mentioned above. Figure 3d has Euro Stoxx 50 (solid) and KfW(dashed). Source: Eurostat, ECB, Schwarz (2015), Yahoo Finance.

Strikingly, during the Great Recession and the European Debt Crisis, the illiquidity of government bonds reached similar levels, as we can observe from the dashed line (KfW) in Figure (3a). However sovereign spreads (solid lines) did not seem to have followed the same pattern. In fact, not only have spreads behaved differently in these two events, but also other macroeconomic variables such as output

(Figure 3c) and stock prices (Figure 3d) have suffered a much larger fall in 2008 than in 2011. How can we explain this similar illiquidity of sovereign bonds measured by the KfW spread but at least twice as big a recession, and very little increase in sovereign spreads during the Great Recession?

Del Negro, Eggertsson, Ferrero, and Kiyotaki (2011) show the importance of a negative shock to the liquidity of private equity to explain the macroeconomic dynamics during the Great Recession in the United States and the need of unconventional monetary policy to navigate this crisis. Furthermore, I believe that explaining the Great Recession by only looking at sovereign illiquidity in a world in which interbank transactions froze is not adequate. I replicate their analysis of a fall in the resaleability on equity  $\phi_t$ , under no unconventional monetary policy nor zero lower bound. Figure A.1 in the Appendix shows my replication. As it can be inferred from the investment decision Equation(5) liquidity of equity works through the same channel as that of sovereign bonds, by reducing investment and thus leading to a recession. The main difference is the effect on the relative prices of equity to public bonds. Lower resaleability of equity induces households to move out from private equity into government bonds generating an increase in their price.

Sovereign and private equity liquidity shocks have similar qualitative<sup>16</sup> effects on macroeconomic variables, however, they affect the relative price of equity and government bonds in opposing directions. In Figure (3b), I plot returns to government bonds under a liquidity shock to public bonds in blue and to private equity in orange dotted line. A negative shock to liquidity of public bonds increases its returns whereas a shock to the liquidity of private equity reduces bonds returns on impact due to the flight from equity to more liquid government bonds.

I can accommodate my model to both: the Great Recession and the European Debt Crisis by admitting private equity and sovereign liquidity shocks to have taken place during the first event whereas only the latter would have been at place during 2011. As explained previously, a drop in the liquidity of private equity increases the relative price of government bonds, whereas a drop in sovereign liquidity has the opposite effect. Therefore, by allowing these two shocks to interact during the Great Recession, qualitatively, I can explain why during this period although sovereign bonds were as illiquid as in 2011 we did not observe large increase in returns to government bonds. During the 2011, however, only sovereign liquidity shocks were at place, delivering large increase in sovereign spreads and slower economic activity.

### 3.3.2 Financial Development vs Fragility

Does financial development increase or reduce fragility to liquidity shocks? One view is that financial development lessens volatility by reducing frictions/informational asymmetries Bernanke, Gertler, and

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<sup>16</sup>Although not necessarily quantitative



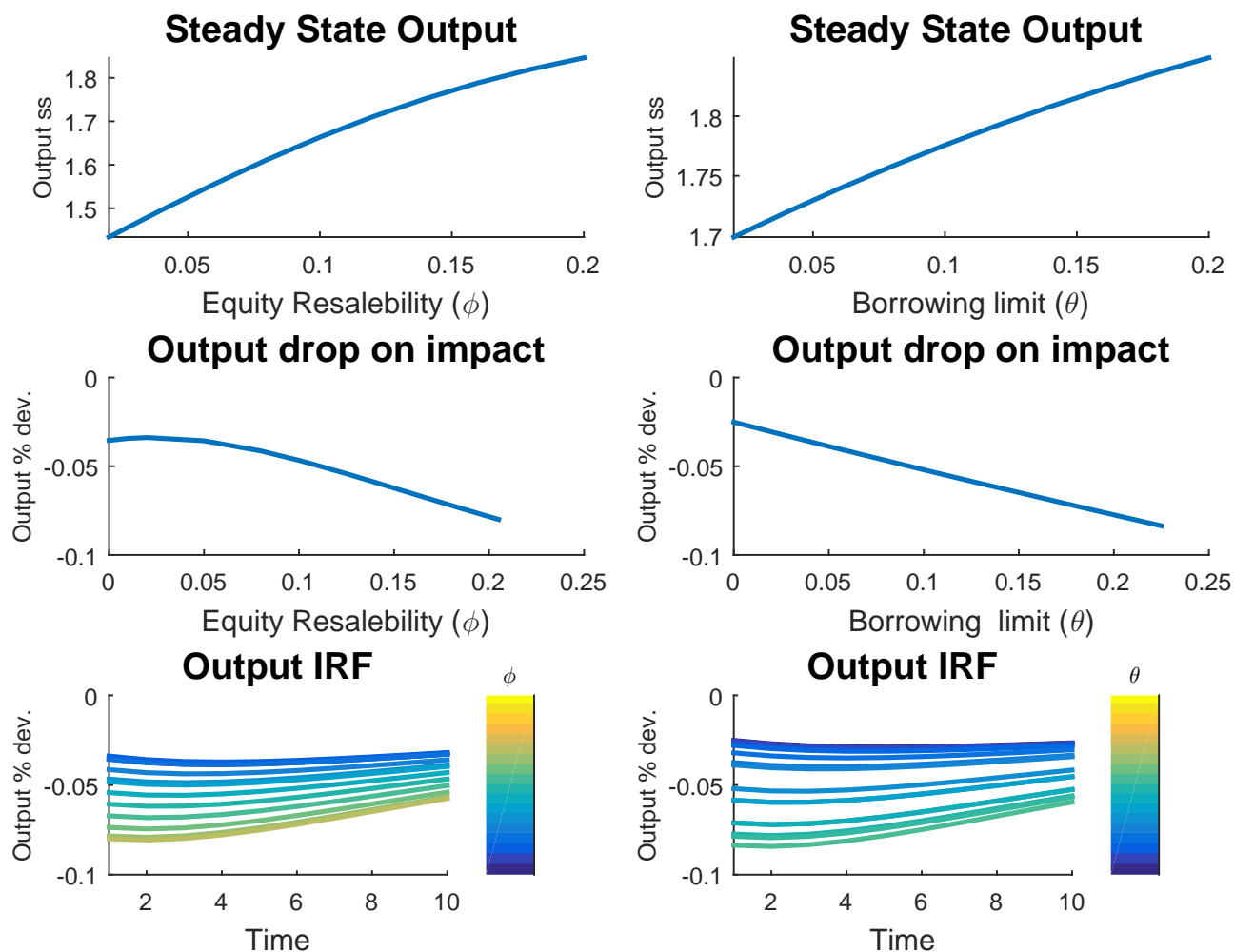


Figure 4: **Financial Development: Output Response to Liquidity Shocks**

Gilchrist (1999), promote risk sharing, allowing for better consumption smoothing Hubbard (1998), da Silva (2002). The opposing view is that finance increases economic and financial volatility by promoting greater risk-taking and leverage, suggested by Rajan (2005) and Gennaioli, Shleifer, and Vishny (2012). This section aims at studying this question in the framework analyzed.

Financial development in this setting can be interpreted as the easiness of finding external funding, or liquidating assets. The higher the amount that can be externally financed—given by greater values for  $\theta$ —the greater the level of financial development. This can be interpreted as lower monitoring costs, more efficient contracts and lower incidence of moral hazard affecting the lending/borrowing in this economy. Similarly, higher values for steady state private equity liquidity—given by greater values for  $\bar{\phi}$ —resemble lower transaction costs, larger markets and more efficient trading platforms which in turn can also be understood as higher levels of financial development.

The main question in this section is whether economies are more fragile to sovereign liquidity shocks

depending on their level of financial development. Should countries more financially developed face higher or lower costs due to sovereign liquidity crisis?

In Figure 4 I study the impact of negative sovereign liquidity shocks, in economies with diverse degree of financial development given by either different levels of resaleability of private equity (left panel) or of borrowing limits (right panel). The top panel, plots steady state levels of output for economies differentiated by their degree of financial development. In this framework, greater levels of financial development are associated with richer economies. In middle panel, I plot the effect that a sovereign liquidity shock has on output on impact for distinct levels of financial development. In the bottom panel I plot the evolution of output under a negative shock to the liquidity of public bonds for different values of  $\bar{\phi}$  (left), and  $\theta$  (right), where lighter colors represent higher levels of financial development. These figures suggest that economies with higher levels of financial development face deeper recessions when exposed to sovereign liquidity shocks. Thus, countries in which financial frictions do not allow for efficient allocations, should bear in mind that sovereign liquidity can have significant effects depending on their degree of financial development. [Loayza and Ranciere \(2006\)](#), [Lopez, Spiegel, et al. \(2002\)](#), [Ranciere, Tornell, and Westermann \(Forthcoming\)](#) show compelling evidence to support that while financial development is beneficial over the long run, it may exacerbate short-term volatility in isolated episodes. In line with this idea, I show that in this framework more financially developed economies although they have higher steady state levels of output, they also face greater fragility. Therefore, economies with relative high levels of financial development should be aware of how important sovereign liquidity is, and address the drying up of these markets with immediacy.

### 3.3.3 Do we need Nominal Rigidities?

Introducing price and wage rigidities improves upon the unappealing results KM's financial frictions deliver as pointed out by [Shi \(2015\)](#) and [Bigio \(2012\)](#). KM's financial frictions in a real economy have counterfactual effects on consumption and equity prices, in addition to almost insignificant effects on macroeconomic variables. Figure 5 presents the impulse response functions to a one standard deviation shock to sovereign liquidity under an economy without nominal rigidities. Vertical axis are percentage deviations of the variables from their steady-state levels and the horizontal axis is the number of quarters after the shock. In a real economy, there is no contemporaneous effect on total output. Although a negative shock to the liquidity of government bonds reduces total investment at time  $t = 0$  and also in future periods this does not translate into a fall in output on impact. Output will gradually decline over time due to lower capital and labor in the following periods. Lower investment at the time of the shock implies lower capital in future periods. Keeping the demand for capital fixed this translates into higher capital rental price for the following periods. Given that the price of equity depends on expected future dividends paid by capital,  $q$  increases as well. In addition, the tightening

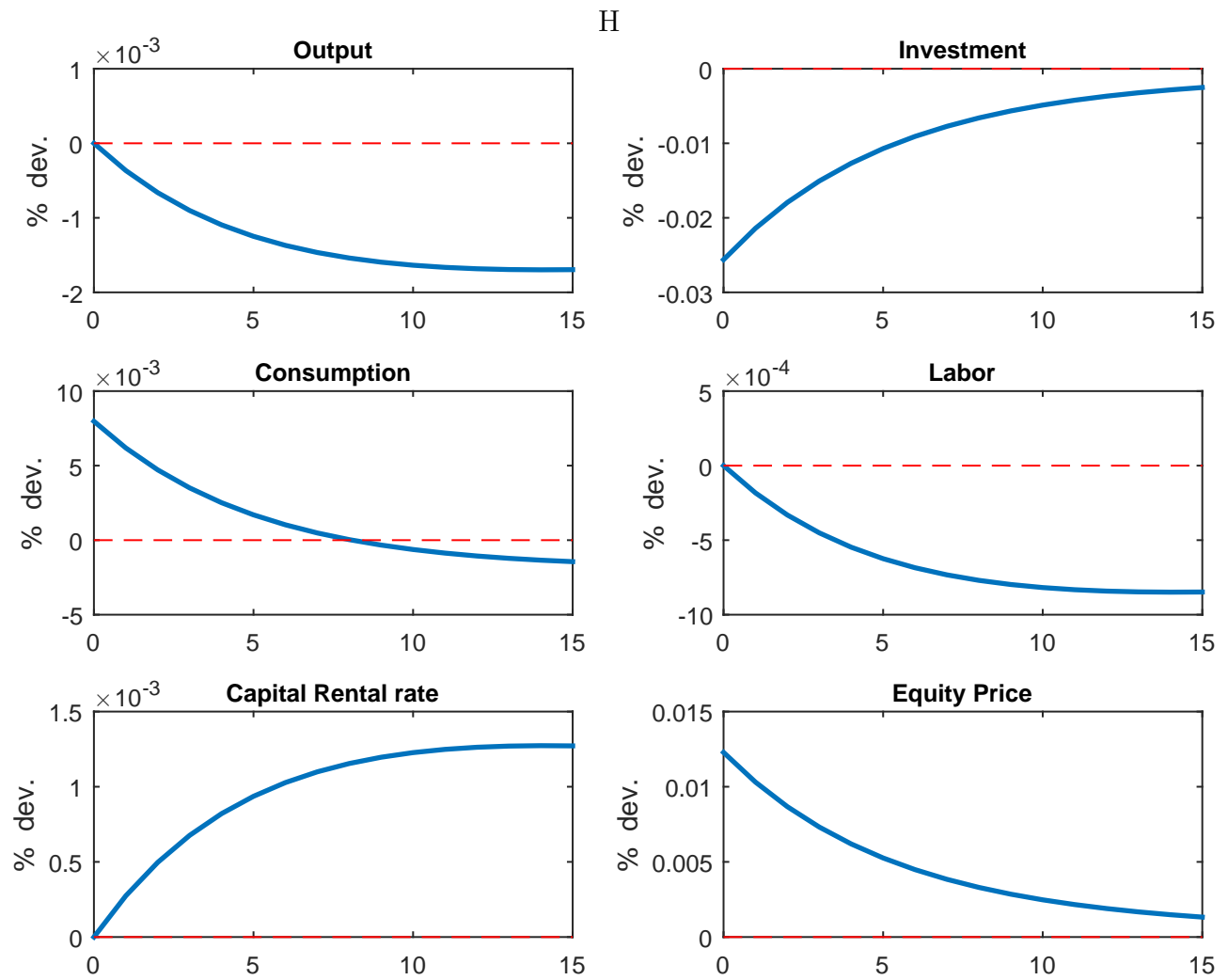


Figure 5: Real Economy, negative shock to liquidity of Sovereign Bonds ( $z$ )

of the liquidity constraint ( $\lambda_t^e$ ) affects equity price in the same direction.

Introducing price and wage rigidities improves upon these unappealing results. In compliance with the data, we can use this setting to understand the real effects of a reduction in sovereign liquidity.

## 4 Conclusion

Should policy makers address their attention to the liquidity of sovereign markets? Does the drying up of these markets have any meaningful implications for the real economy?

In this paper, I introduce financial frictions à la KM in an otherwise standard New Keynesian environment with price as well as wage rigidities, and study the relevance of fluctuations in sovereign market liquidity. Extending these frictions to allow for some illiquidity in sovereign markets, I find that disturbances in this market can have significant real effects. Thus, the drying up of sovereign markets should be carefully addressed, in particular for relatively high financially developed economies.

Financial frictions constrain the funding ability of entrepreneurs. These, not only affect the possibility of external financing through borrowing but also the possibility of fully liquidating current assets in order to fund the production of new physical capital. Sovereign bonds play a crucial role in relaxing these constraints by providing liquidity.

The main findings of this work convey that liquidity of sovereign markets is meaningful. First, a negative shock to the liquidity of sovereign bonds is recessionary, generating a significant drop in investment, consumption and employment. My model also speaks to the fall in stock prices, which has been evident during the European debt crisis. Second, other sources of liquidity shocks, although recessionary, have different macroeconomic implications. In compliance with European increase in spreads in 2011, illiquidity of bonds leads to an increase in the return these must pay in order for households to hold them. On the contrary, a fall in the resaleability of private equity—which is also recessionary—affects the relative price of private and public assets in the opposite direction. During the Great Recession and the European Debt Crisis, illiquidity of public bonds in Europe reached similar levels, but sovereign spreads did not increase as much during the first event. Allowing for variation in these two shocks (public and private liquidity) is crucial to explain why spreads behaved differently during these two events. Lastly, I provide some sensitivity analysis studying different levels of financial development and show that more financially developed economies although richer are more vulnerable to the consequences of sovereign liquidity. This suggests that economies with relative high levels of financial development should be aware of how important sovereign liquidity is, and address the drying up of these markets with immediacy.

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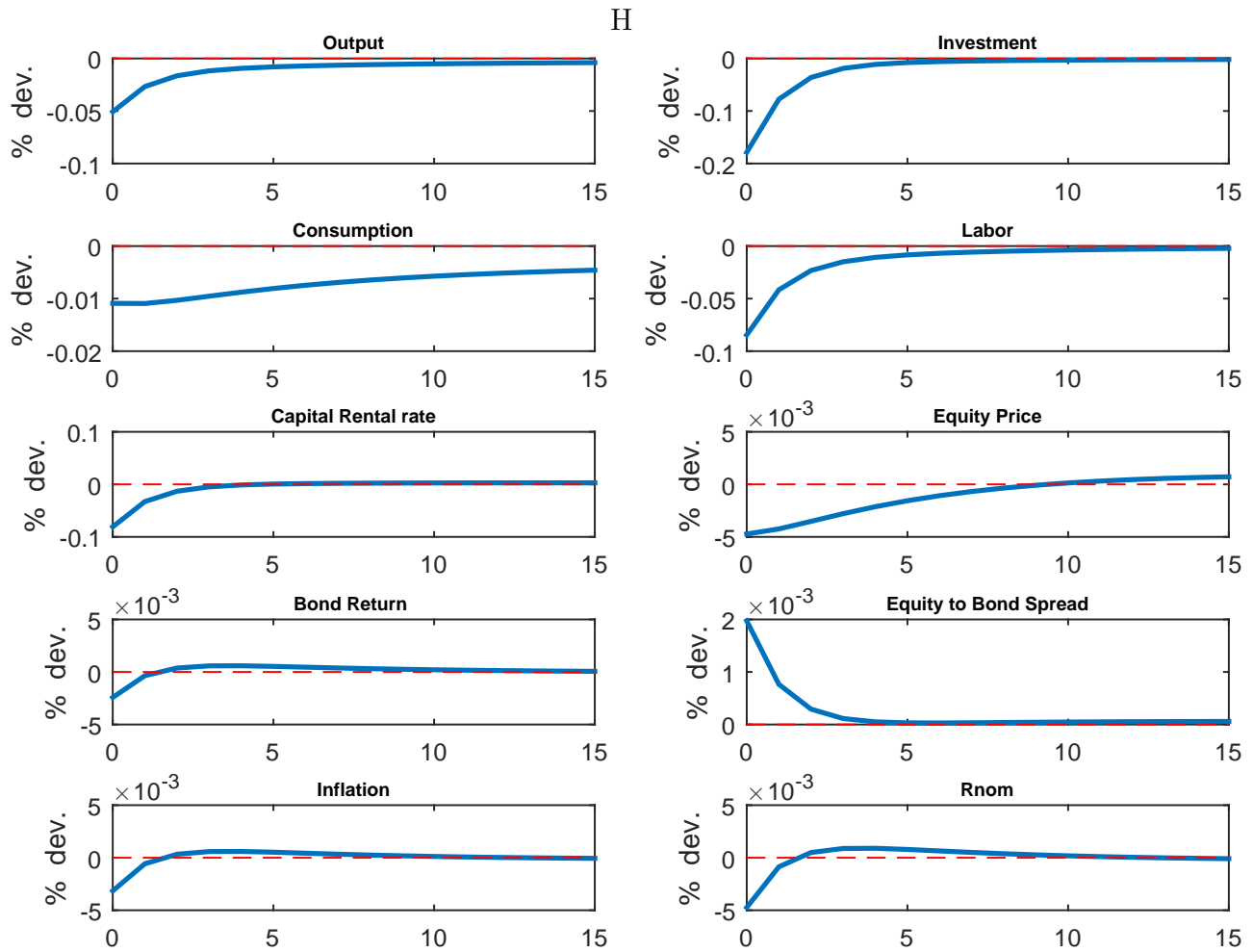


Figure A.1: Negative shock to liquidity of Equity ( $\phi$ )

## A Appendix

### A.1 Figures



## A.2 Steady State

From capital/labor ratio in Steady State we get that

$$\frac{K}{H} = \frac{\gamma}{1-\gamma} \frac{w}{r^K}$$

Marginal cost:

$$mc = \left(\frac{r}{\gamma}\right)^\gamma \left(\frac{w}{1-\gamma}\right)^{1-\gamma} = \frac{1}{1+\lambda_F} \quad (15)$$

Incorporating these two equation into the ss in  $Y = K^\gamma H^{1-\gamma}$ , delivers

$$\frac{Y}{K} = \frac{1+\lambda_f}{\gamma} r^K$$

Real wage as a function of rental rate ( manipulating 15)

$$w = (1-\gamma) \left(\frac{1}{1+\lambda_F}\right)^{\frac{1}{1-\gamma}} \left(\frac{\gamma}{r^K}\right)^{\frac{\gamma}{1-\gamma}} \quad (16)$$

In ss, the real wage is equal to a markup over the marginal rate of substitution between labor and consumption (wage=markup/mrs)

$$w = (1+\lambda_w) \frac{(H/(1-\chi))^\nu}{C^{-\sigma}}$$

From the steady state version of the Euler equation for bonds, using that  $p^I = 1$

$$\beta^{-1} = R^B \left(1 + z\chi \frac{q-1}{1-\theta q}\right)$$

Steady state for tax, from the govt budget constraint

$$\tau = b(R^B - 1)$$

Where  $b$  are real government bonds ( over  $\pi = 1$ )

From law of motion for capital

$$I = \delta K$$

Using the eq for investment , replacing all variables in ss, and combining with the eq for  $\tau$

$$I = \frac{\chi}{1 - \theta q} \left( R^B + (1 - \delta) \phi q + b + \frac{\lambda}{1 + \lambda} Y \right)$$

$$\frac{I}{k} (1 - \theta q) \frac{1}{\chi} = \left( r^k + (1 - \delta) \phi q \right) + \frac{b}{k} (z R^B - R^B + 1) + \frac{\lambda}{1 + \lambda} \frac{Y}{k} \quad (17)$$

Recall  $N = K$  in ss.

From Euler equation for bonds / liquid and not liquid:

Government bonds not completely liquid  $R^B$  :

$$\beta^{-1} = \frac{R^B}{\pi} + \lambda^e z \frac{R^B}{\pi}$$

Liquidity free  $R^{LF}$  :

$$\beta^{-1} = \frac{R^{LF}}{\pi} + \lambda^e \frac{R^{LF}}{\pi}$$

Combining these two last equations I get an expression for the Liquidity Spread defined as the difference between these two rates. LS is taken from the data to match KfW liquidity spread.  $LS = r^B - r^{LF}$ ; where small case  $r$  is the net return. This implies that Liq spread:  $\lambda^e (1 - z)$ , where  $\lambda^e = \chi \frac{q-1}{1-\theta q}$ . Therefore,

$$\chi \frac{q-1}{1-\theta q} (1 - z) = LS$$

From Euler for bonds:

$$\chi \frac{q-1}{1-\theta q} z = 1 - r^{LF} \beta$$

Using Euler equation for equity, can get a relationship between  $r^k$  and  $q$

$$\beta^{-1} = \frac{r^k + (1 - \delta) q}{q} + \chi \frac{q-1}{1-\theta q} \left( \frac{r^k}{q} + (1 - \delta) \phi \right)$$

**Four equations and four unknowns:**  $q, r^k, z, r^{LF}$

Using Liq Spread and  $\frac{b}{Y}$  as given

$$\delta - (\delta \theta + (1 - \delta) \phi \chi) q = r^k \left( 1 + \frac{1 + \lambda_f}{\gamma} \frac{b}{Y} (z r^{LF} - r^{LF} + 1) + \frac{\lambda}{\gamma} \right)$$

$$1 = \beta \left[ \frac{[r^k + (1 - \delta) q]}{q} + \lambda^e \frac{[r^k + (1 - \delta) q \phi]}{q} \right]$$

$$\chi \frac{q-1}{1-\theta q} (1-z) = LS$$

$$\chi \frac{q-1}{1-\theta q} z = 1 - r^{LF} \beta$$

Now, having  $r^k$  I can back up wages from (16), capital to labor ratio as well as output to capital ratio.

So,

$$Y = \frac{1 + \lambda_f r^K}{\gamma} K$$

$$I = \delta K$$

$$Y = C + I$$

$$C = \frac{1 + \lambda_f r^K}{\gamma} K - \delta K$$

Now using

$$w = (1 + \lambda_w) \frac{(H/(1-\chi))^\nu}{C^{-\sigma}}$$

$$H = K \frac{1 - \gamma r^K}{\gamma} \frac{1}{w}$$

and

$$C = \frac{1 + \lambda_f r^K}{\gamma} K - \delta K$$

solve for K since I already have  $r^k$  and  $w$ .

### A.3 Log-linear approximation

List of log-linearized conditions characterizing the equilibrium where  $\hat{x}_t = \log(\frac{x_t}{x})$ ; where  $x$  is the steady state value of  $x_t$ . The equilibrium is characterized by 18 equations, with six expectation terms  $E_t w_{t+1}; E_t \pi_{t+1}; E_t q_{t+1}, E_t r_{t+1}^k, E_t p_{t+1}^I, E_t C_{t+1}$

1. Log linearized for bonds

$$\begin{aligned} & \sigma E_t \hat{c}_{t+1} - \hat{R}_t^B + E_t \hat{\pi}_{t+1} - \chi z \beta r \frac{q-1}{1-\theta q} E_t (\hat{z}_{t+1}) \\ & - r \beta q \chi z \frac{1-\theta}{(1-\theta q)^2} E_t (\hat{q}_{t+1}) + r \beta q \chi z \frac{1-\theta}{(1-\theta q)^2} E_t (\hat{p}_{t+1}^I) \\ = & \sigma \hat{c}_t \end{aligned}$$

2. Log linearized for equity

$$\begin{aligned}
 & \sigma E_t [\hat{c}_{t+1}] - \beta \frac{r^k}{q} \left( 1 + \chi \frac{q-1}{1-\theta q} \right) E_t [\hat{r}_{t+1}^k] - \beta (1-\delta) \chi \frac{q-1}{1-\theta q} \phi E_t [\hat{\phi}_{t+1}] \\
 & - \beta \left( (1-\delta) + (1-\delta) \chi \frac{q-1}{1-\theta q} \phi + \chi (r^k + (1-\delta) \phi q) \frac{1-\theta}{(1-\theta q)^2} \right) E_t [\hat{q}_{t+1}] \\
 & + \beta \chi \left[ \frac{r^k}{q} + (1-\delta) \phi \right] \frac{q(1-\theta)}{1-\theta q} E_t (\hat{p}_{t+1}^I) \\
 = & \sigma \hat{c}_t - \hat{q}_t
 \end{aligned}$$

3. Loglinearization for investment

$$\begin{aligned}
 & \delta (1-\chi) \hat{p}_t^I + (1-\theta q) \delta I_t - [\theta \delta + \chi (1-\delta) \phi] q \hat{q}_t \\
 & - \chi (1-\delta) \phi q \hat{\phi}_t - \chi [r^k + (1-\delta) \phi q] \hat{N}_t - \chi r z \frac{b}{k} \left( \hat{R}_{t-1}^B + \hat{z}_t + \hat{b}_{t-1} - \hat{\pi}_t \right) \\
 & + \chi \frac{\tau}{K} \hat{\tau}_t - \chi \frac{Y}{K} \hat{Y}_t + \chi \frac{(1-\gamma) r^k}{\gamma} (\hat{w}_t + \hat{H}_t) + \chi r^k \hat{k}_t \\
 = & 0
 \end{aligned}$$

4. Wage setting:

$$\begin{aligned}
 & \left( 1 + \nu \frac{1+\lambda_w}{\lambda_w} \right) \hat{w}_t^* - (1-\zeta_w \beta) \nu \frac{1+\lambda_w}{\lambda_w} \hat{w}_t \\
 = & (1-\zeta_w \beta) (\nu \hat{H}_t + \sigma \hat{c}_t) + \zeta_w \beta \left( 1 + \nu \frac{1+\lambda_w}{\lambda_w} \right) E_t (\hat{w}_{t+1}^* - \hat{\pi}_{t+1})
 \end{aligned}$$

5. Wage index

$$\hat{w}_t = (1-\zeta_w) \hat{w}_t^* + \zeta_w (\hat{w}_{t-1} - \hat{\pi}_t)$$

6. Capital / labor ratio

$$\hat{K}_t = \hat{w}_t - \hat{r}_t^k + \hat{H}_t$$

7. Marginal cost

$$\widehat{m}c_t = (1-\gamma) \hat{w}_t + \gamma \hat{r}_t^k$$

8. Sticky prices

$$\hat{\pi}_t = \frac{(1-\zeta_p \beta)(1-\zeta_p)}{\zeta_p} \widehat{m}c_t + \beta E_t (\hat{\pi}_{t+1})$$

9. Capital producers

$$\hat{p}_t^I = S''(1) \hat{I}_t$$

10. Aggregate market clearing

$$\hat{Y}_t = \frac{I}{Y} \hat{I}_t + \frac{C}{Y} \hat{c}_t$$

11. Capital Law of motion

$$\hat{K}_{t+1} = \delta \hat{I}_t + (1 - \delta) \hat{K}_t$$

12. Aggregate production function

$$\hat{Y}_t = \gamma \hat{K}_t + (1 - \gamma) \hat{H}_t$$

13. Capital and equity

$$\hat{K}_{t+1} = \hat{N}_{t+1}$$

14. Government

$$\tau \hat{\tau}_t = b \left( \hat{R}_{t-1}^B + \hat{b}_{t-1} - \hat{\pi}_t \right) - b \hat{b}_t$$

15. Taylor rule for interest rate

$$\hat{R}_t^{Nom} = \psi \hat{\pi}_t$$

16. Rule for transfers

$$\frac{\tau}{K} \hat{\tau}_t = \psi_\tau \frac{rb}{k} \left( \hat{R}_{t-1}^B - \hat{\pi}_t + \hat{b}_{t-1} \right)$$

17. Exogenous evolution for government liquidity

$$\hat{z}_t = \rho_z \hat{z}_{t-1} + \varepsilon_t^z$$

18. Log linearized for Liquidity Free bonds

$$\sigma E_t \hat{c}_{t+1} - \hat{R}_t^{LF} + E_t \hat{\pi}_{t+1} - r^{LF} \beta q \chi \frac{1 - \theta}{(1 - \theta q)^2} E_t (\hat{q}_{t+1} - \hat{p}_{t+1}^I) = \sigma \hat{c}_t$$