Education Success Uncertain: Further Investigation

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Abstract

Governments may influence product, literacy and unemployment rates through educative reforms and subsidies. This paper presents a model that analyzes qualitatively the effects on society from changes in educative policies. The framework is based on credit market imperfections, indivisibility in human capital investment and uncertainty in the outcome of education. A major conclusion is that educative reforms may lead to an increase in unemployment rate of educated workers in the society.

1 Introduction

The theoretical framework developed in "Caramp, Espinosa, Melero and Szenig" (2009) was able to explain qualitatively Argentinean experience in the early 1990's. During this period, Argentina launched a widespread liberalization process involving, among others things, large privatizations and financial and trade reforms. Between 1990 and 1994 Argentina experienced a rapid increase in Total Factor Productivity (TFP) and Real GDP per capita compared to its past. There was also an increase of the skill premium; i.e.the extra-wage received by skilled labour. Unemployment rate increased from 7% in 1992 to 13% in 1998, and Gini coefficient went from 0.422 to 0.493, representing a major economic inequality.

In that paper, the Argentinean experience had been addressed, leaving aside further possible studies. In this paper, topics such as how governments may affect product and inequality through changes in interest rates and educative reforms costs and quality will be examined.

The theoretical framework combines two literatures. On the one hand, a literature that studies unemployment and wage inequality leaded by Acemoglu (1999, American Economic Review). He studies how a skill biased technological change can generate a sufficiently large gap between productivity of skilled and unskilled workers such that there is a boost in skill-premium and unemployment rate is studied. This result is due to a shift from a pooling to a separating equilibrium, individuals' characteristics are exogenous and they differ in the level of human capital they were born with. Unemployment is a result of the particular labour market structure, i.e. a firm matches a worker and hiring takes place if the worker's human capital level suits the firm's requirements.

On the other hand, there is a literature that studies wealth distribution. Galor & Zeira (1993, Review of Economic Studies) present an overlapping-generations framework with parental altruism to show how capital market imperfections and a non-convexity in the human capital technology generate inequality in the short and in the long run. In this setting individual decisions are endogenous. In particular, agents decide whether to study or not: education improves individuals' productivity but due to capital market imperfections the cost of education may be higher than its benefits. There is no source of uncertainty in this framework.

These two literatures are combined. Unlike Acemoglu, human capital acquisition is endogenous allowing studying the interaction between the technological change and the adjustments of individual decisions to the new environment.

I study a small open economy populated by overlapping-generations. There is a single consumption good that can be produced by technologies that differ in their human capital requirements. Individuals of each generation live for two periods. When they are young, each one of them receives a bequest from her parent. She has to decide whether to invest in human capital or not while the result of education is uncertain. With a given probability she becomes a fully skilled worker, if not she is unskilled. Educated workers' productivity is higher than educated and skilled workers are more productive than unskilled.

Immediately after this decision, she assists to a job interview. In this economy there will be two types of firms. One type will hire only non-educated people and other will hire educated workers only. Depending on the equilibrium properties, there will be two types of firms that look for educated agents: those which will hire individuals who succeeded at school (skilled individuals) and those which will hire those who did not (unskilled or medium individuals). At the time individuals choose which interview they will assist to, they know the firm's type, but educated agents do not know their own level of human capital. Through the interview process, individuals get to know their type. If this type matches the firm's requirements, it hires her. If not, the individual stays unemployed.

Following Galor & Zeira (1993), there are enforcement and supervision costs individual borrowers and, hence, the borrowing interest rate is higher than the lending rate. The amount of bequest she receives determines whether she studies or not. This is an indivisible decision, there is a non-convexity in the human capital technology making the decision of studying dichotomy. But the effect of wealth distribution is not only seen in the short run, as the different levels of human capital determine the distribution of wealth through time. Hence, the initial distribution determines how big the educated and non-educated sectors are, and therefore what is the long-run equilibrium in the economy.

The results are driven by three features in the model. First, credit markets are imperfect, as the interest rate for borrowers is higher than the one of lenders. This imperfection makes the cost of education be different across individuals, affecting wealth distribution and the level of economic activity in the short run. The second important assumption is that there is a technological non-convexity, namely, investment in human capital is indivisible. This makes inherited distribution of wealth affects the economy not only in the short run but in the long run as well. Finally, the last assumption is that the outcome of education is uncertain. This generates unemployment and has long run implications in inequality after different shocks are applied to the model.

The paper is organized as follows. Section 2 presents and studies the model. Section 3 discuss different exercises of comparative statistics, different shocks have been included in the model and analyze their effect on output, unemployment and inequality. Finally, Section 4 concludes.

2 The Model

2.1 Technology, Preferences and Markets

I study a small open economy populated by overlapping-generations. Only one good is produced and it can be used either for consumption or investment in physical or human capital. This good can be produced by technologies that differ in their human capital requirements. Following Acemoglu (1999, American Economic Review), production technology of sector ℓ is described by:

$$y_\ell = k_\ell^{1-\alpha}\eta_\ell^\alpha$$

where k_ℓ and η_ℓ are, respectively, physical and human capital levels employed by each of the firms of this sector. In this economy there will be four levels of human capital attainable by individuals, and thus, five possible sectors: the skilled sector (s), the unskilled sector (u), the medium sector (m), the non-educated sector (ne) and the educated or pooling sector (p). The existence and characteristics of the latter deserve a special treatment and will be addressed later on.

In this economy, individuals live for two periods and have only one parent and one child. This assumption means that there is no population growth. At each period of time, two generations coexist. Each cohort is a continuum of individuals of measure 1. In the second period of her life, an individual has a child and leaves her a bequest. Following Galor & Zeira, it is assumed, for the sake of simplicity, that an individual born in period t does not derive utility from comsumption in her first period of life:

$$U\left(c, x_{t+1}\right) = \left(c_2\right)^\beta \left(x_{t+1}\right)^{1-\beta}$$

where c_2 is second period consumption, x_{t+1} denotes the bequest left to her child (who belongs to generation $t+1$), and $0 < \beta < 1$. Notice that individuals in this economy differ only in the amount they inherit from their parents. Other than that, there are no other parental effects.

There will be as many labour markets as types of workers and equilibrium wages are so that firms benefits are null[1]. Labour contracts are enforceable for two periods. This is not negotiable by workers.

With regard to capital markets, it is assumed that individuals can save any amount at the world interest rate, $r_w > 0$. As for borrowing, evasion is supposed to be costly, and so is keeping track of borrowers. These costs create a capital market imperfection, where individuals can borrow only at the interest rate r_b, which is higher than r_w. Both rates are assumed to be constant over time.

[1]Notice that in the definition of labour markets equilibria effects regarding offer's size are not possible. That is, if for any reason there is a change in the supply of workers of any type, this will not have any impact on equilibrium wages. This assumption is merely a simplification that the current state of the literature [Acemoglu (1999); Autor, Katz & Krueger (1998); Berman, Bound & Machin (1998)] allows to take. Studies based on data from the U.S. and other developed countries show that simple relative supply and demand framework is not clearly applicable to explain successfully the behavior of relative wages between skilled and unskilled labour, since in the last decades there was a decline in the relative wages of less skilled workers despite their increasing scarcity relative to the rapidly expanding supply of skilled labour.

2.2 Consumers

2.2.1 Endogenous Qualification

Endogenous qualification is considered. Namely, individuals are born with the same abilities, but they can decide whether to study or not. If they choose to do so, they must pay $\varepsilon > 0$ units of the consumption good in their first period of life. They become skilled workers with a certain probability, the same for all individuals. If they do not study, they remain unskilled. Formally,

$$\Pr(S = 1|e = 1) = \mu$$

$$\Pr(S = 1|e = 0) = 0$$

where $\mu \in (0, 1)$ and

$$e = \begin{cases} 1 \text{ if the individual studies} \\ 0 \text{ otherwise} \end{cases}$$

$$S = \begin{cases} 1 \text{ if the individual results skilled} \\ 0 \text{ otherwise} \end{cases}$$

In this way, education is necessary but not sufficient for becoming a skilled worker. This creates two major sectors in the economy: the educated and the non-educated sector. Furthermore, within the educated group, there are two groups: skilled and unskilled workers[2]. It is supposed that individuals do not know for sure the results of their training. They only know their probabilities. If an individual results skilled, she obtains a productivity (or human capital level) of η_s. Those who did study but did not succeed, obtain η_u, that is assumed to be higher than the uneducated worker's human capital, η_{ne}, and, reasonably, smaller than η_s. That is, $\eta_s > \eta_u > \eta_{ne}$.

Unskilled workers have a second chance for acquiring more human capital in their second period of life and, thus, to become "medium" workers. If they want to do so, they must pay $\varepsilon_2 > 0$ units of the consumption good. In this second opportunity there is no uncertainty about the result of the process: the outcome will be, with certainty, a productivity of η_m, which is higher than η_u but still lower than η_s.

[2] From now on, "unskilled" will be used to refer to those individuals who did study but did not succeed. Those individuals who did not study will be called "uneducated" or "non-educated".

2.2.2 Individual Optimal Decision

Individual's budget constraints are as follows:

$$I_1 + x_t = b_1 + c_1$$

$$I_2 = b_2 + c_2 + x_{t+1}$$

where I_T and b_T denote current income and net savings (debts) of period of life $T = 1, 2$, respectively. x_t and x_{t+1} denote the bequest inherited from the parent and the bequest left to the child, respectively. Finally, c_T stands for consumption in period $T = 1, 2$. I_1 is composed by wage (if the individual works in the first period) minus education cost ε (if she decides to study). I_2 will be composed by wage plus (minus) interests of her savings (debts). Consider that re-education cost will be taken into account if the individual decides to do so.

From No-Ponzi condition, an optimal decision for the individual will be $b_2^* = 0$. Recalling individual preferences, optimal consumption level in period 1, c_1^*, will be null and, then, $b_1^* = I_1 + x_t$. So

$$x_{t+1}^* = (1 - \beta) I_2$$
$$c_2^* = \beta I_2$$

This leads to the following indirect utility function

$$V = \delta I_2$$

where $\delta \equiv \beta^\beta (1 - \beta)^{1-\beta}$.

So, given a certain level of I_2, the individual will choose x_{t+1} and c_2 in the way that it has been recently shown. But the uncertainty regarding the result of the education process translates into uncertainty about I_2. In this way, it is straightforward to see that individual optimal decision regarding education will be such that maximizes the expected income in period 2. That is,

$$\underset{e}{Max} E[U] \iff \underset{e}{Max} E[V] \iff \underset{e}{Max} E[I_2]$$

Hence, individuals will decide to study $(e = 1)$ if the expected value of I_2 derived from doing so is the highest. This decision will depend, among other things, on equilibrium wages, so economy's equilibria will be studied first and the decision about education later.

2.3 Firms and the Timing of Events

Production technology is operated by risk-neutral firms that have an infinitely long life. At the very beginning of the economy, each firm chooses the level of physical capital it will operate. This decision is costless but irreversible. This means that the firm cannot change its capacity after this decision has been taken.

After the decision of education, individuals enter the labour market to search a suitable firm. In each period, it is of common knowledge which type of worker each firm is looking for (think of free classified ads that last one period only). In each period, if vacant, a firm interviews one agent -and one agent only- and decides whether to hire her or not. If the firm does so, it installs the chosen capacity (with a total cost of ck_ℓ) and pays the worker. The firm, in this way, becomes active. Otherwise, it remains inactive until hiring occurs[3] and the worker will be unemployed for the rest of the period. Staying inactive has no cost for firms (without considering opportunity costs). Furthermore, firms have to install the capacity -already chosen at the beginning of their lives- in each of the periods they are active. Notice that a firm could only be active if there is a worker that has incentives to assist to its interview. In this sense, the incentives of workers will determine in which of all possible equilibria this economy is at each period[4], by determining which types of firms are active and which are not.

Firms are supposed to have a perfect recognition technology. That is, when a worker assists to the interview, the firm can instantly and perfectly tell what type the worker is. This implies that a worker cannot lie about her abilities[5]. Recalling that educated workers in their first period of life do not know their type, the incentives of firms to hire them or not must be analyzed. Consider then a young worker of type $\ell = s, u$ assisting to the wrong interview (that is, with a firm of type u or s, respectively). The firm has two possibilities. If the firm decides to hire the worker it must pay the wage of market ℓ, that is, the worker must be paid according to her skills (think of trade unions standing for workers' rights). It has been shown in the previous paper that this would imply non positive benefits for the firm. It is clear, then, that the firm will choose not to hire the worker, remaining inactive during that period.

If the individual is an educated worker, this first interview reveals to her information about her type. It is assumed, for simplicity, that there are always inactive firms (of any type) in the economy. Hence, there will be no unemployed educated individuals in the second period of life of a cohort. Following this line of reasoning, there will never be unemployment in the non-educated sector, since uneducated workers, naturally, know their type and, in this way, they will always assist to an interview with the right firm. Thus, if in equilibrium there is a nonnegative unemployment rate, it will come from the educated sector of the economy. This result will be of crucial importance for dynamics.

Because of the assumptions made so far, only unskilled workers who were unemployed in the first period will take the re-education possibility as an

[3] The matching process will be described in the next section.

[4] This will depend on parameters conditions.

[5] Since this property of firms is common knowledge, many cases are ruled out. For example, the case of an uneducated individual assisting to an interview with a firm not of type ne.

effective option: those who worked in the first period will not be able to study again because they are working.

2.4 Equilibrium: pooling vs. separating

2.4.1 Pooling

A pooling equilibrium is characterised by pooling firms being active. Pooling firms are those who choose a level of physical capital that is operable by any type of educated worker. It is clear that this type of firm will consider the uncertainty regarding the educative process result, since with probabilities μ and $1-\mu$ a skilled or unskilled worker will be hired, respectively. The problem of the pooling firm is then

$$\underset{k}{Max}\, \pi_p = \left[\mu(k^{1-\alpha}\eta_s^\alpha - ck - w_p^s) + (1-\mu)(k^{1-\alpha}\eta_u^\alpha - ck - w_p^u)\right]$$

The chosen capacity will be

$$k_p = \left[\tfrac{(1-\alpha)}{c}\right]^{1/\alpha} \left[\mu\eta_s^\alpha + (1-\mu)\,\eta_u^\alpha\right]^{1/\alpha}$$

Wages in this case are such that, in each of the two possible states, firm p makes zero profits. Thus, we have

$$w_s^{Pool} = \frac{\theta}{\alpha}\left[\mu\eta_s^\alpha + (1-\mu)\,\eta_u^\alpha\right]^{(1-\alpha)/\alpha}\left[\eta_s^\alpha - (1-\alpha)\left[\mu\eta_s^\alpha + (1-\mu)\,\eta_u^\alpha\right]\right]$$

$$w_u^{Pool} = \frac{\theta}{\alpha}\left[\mu\eta_s^\alpha + (1-\mu)\,\eta_u^\alpha\right]^{(1-\alpha)/\alpha}\left[\eta_u^\alpha - (1-\alpha)\left[\mu\eta_s^\alpha + (1-\mu)\,\eta_u^\alpha\right]\right]$$

where $\theta \equiv \alpha\left[\frac{(1-\alpha)}{c}\right]^{(1-\alpha)/\alpha}$.

As it has been emphasized earlier, firms of type ne will always be active. They will choose a level of capital in order to solve

$$\underset{k}{Max}\, \pi_{ne} = k^{1-\alpha}\eta_{ne}^\alpha - ck - w_{ne}$$

So,

$$k_{ne} = \left[\tfrac{(1-\alpha)}{c}\right]^{1/\alpha} \eta_{ne}$$

It follows that,

$$w_{ne} = \theta\eta_{ne}$$

2.4.2 Separating

Separating equilibria are those where pooling firms are inactive. If a firm of type $j = s, u, m$ is considered, this firm solves

$$\underset{k}{Max}\, \pi_j = k^{1-\alpha}\eta_j^\alpha - ck - w_j$$

The optimal level of physical capital is then

$$k_j = \left[\tfrac{(1-\alpha)}{c}\right]^{1/\alpha} \eta_j$$

and wages are,

$$w_j = \theta\eta_j$$

By the equilibria definition considered, there are more than one kind of separating equilibrium. Namely, there is an equilibrium situation where young educated individuals assist to an interview with a firm of type s and an equilibrium where the interview is with a firm of type u. Notice that a firm of type m can be active only in the first of these equilibria since, as it has already been stated, unskilled workers can re-enter the educative system only if they are unemployed when young, and this occurs if they assist to an interview with a firm of type s.

Following "Caramp, Espinosa, Melero and Szenig" (2009), a particular separating equilibrium where educated workers assist to interviews with firms of type s is going to be study. This is so because pooling equilibrium, since it has an unemployment rate equal to zero, is basically the same as the one studied by Acemoglu, and so are the economics of a shift from a pooling to a separating equilibrium. The main difference with Acemoglu is that individual decisions are not exogenous, and this produces interesting results about wealth distribution and economic activity dynamics in an equilibrium where unemployment rate is nonnegative. Finally, that particular separating equilibrium has been chosen because, in the other case, changes in the parameters could provoke shifts from this equilibrium to the other, making the analysis more complex without adding any relevant result. The conditions needed for the desired equilibrium have been stated in previous research.

That being said, individuals and the dynamics of this particular separating equilibrium are going to be analyzed.

2.5 Education and Equilibrium Dynamics

Once the equilibrium wages have been determined, I_2 can be described for each possible case. Then, individuals can be divided among those who study from those who don't.

Consider then a young individual who inherits an amount of x_t units of consumption good from her parent. If she decides to study and results a skilled worker, she will receive w_s in both periods of life. Regarding educative costs, parameters are such that $w_s+x_t-\varepsilon \geq 0 \; \forall x_t$[6]. That is, at the end of the first period, this worker is a net lender, and so faces interest rate r_w. In this case, $I_2 = w_s + (w_s + x_t - \varepsilon)(1+r_w)$. If the individual decides not to train, she does not pay for education and in the first period receives w_{ne}. As in the previous case, this individual is a net lender and faces r_w. In her second period she receives w_{ne} once again. Then $I_2 = w_{ne} + (w_{ne} + x_t)(1+r_w)$. Unskilled individuals pay for education but in the first period of their lives they are unemployed. It is clear to see then that if $x_t - \varepsilon \geq 0$ they are net lenders and face r_w, and if $x_t - \varepsilon < 0$ they are net borrowers and interest rate r_b is faced. Once in their second period of life they have the possibility of re-entering the educative system. Since first period costs are sunk, they will choose to train once more if and only if $w_m - \varepsilon_2 > w_u$. This is the case assumed. In this way, $I_2 = w_m - \varepsilon_2 + (x_t - \varepsilon)(1+r_w)$ if $x_t \geq \varepsilon$ and $I_2 = w_m - \varepsilon_2 + (x_t - \varepsilon)(1+r_b)$ otherwise. Summing up, considering equilibrium wages,

$$I_2 = \begin{cases} \theta\eta_s + (\theta\eta_s + x_t - \varepsilon)(1+r_w) & \text{if } e=1, S=1 \\ \theta\eta_{ne} + (\theta\eta_{ne} + x_t)(1+r_w) & \text{if } e=0 \\ \theta\eta_m - \varepsilon_2 + (x_t - \varepsilon)(1+r_w) & \text{if } e=1, S=0, x_t \geq \varepsilon \\ \theta\eta_m - \varepsilon_2 + (x_t - \varepsilon)(1+r_b) & \text{if } e=1, S=0, x_t < \varepsilon \end{cases}$$

It is neccessary to analyze $E[I_2]$. Recall that the probability of becoming skilled is μ if the individual studied and 0 if she did not. Hence,

$$E[I_2|e=1, x_t \geq \varepsilon] = \\ = (x_t - \varepsilon)(1+r_w) + [\theta\eta_s(2+r_w)\mu + (\theta\eta_m - \varepsilon_2)(1-\mu)]$$

$$E[I_2|e=1, x_t < \varepsilon] = \\ = (x_t - \varepsilon)[(1+r_w)\mu + (1+r_b)(1-\mu)] + [\theta\eta_s(2+r_w)\mu + (\theta\eta_m - \varepsilon_2)(1-\mu)]$$

$$E[I_2|e=0] = x_t(1+r_w) + \theta\eta_{ne}(2+r_w)$$

The same conditions among parameters have been asked as in the quoted paper. It has been asked that, at least in the short run, some people decide to study and some to remain uneducated. It is clear that if rich individuals (i.e., those who inherit $x_t \geq \varepsilon$) do not study, no one does. The following condition has to be satisfied:

$$E[I_2|e=1, x_t \geq \varepsilon] \geq E[I_2|e=0]$$

[6] This condition is immediately satisfied if it is asked that medium individuals to die with no debt.

That is, expected income for an individual with the sufficient resources to pay the educative cost must be higher if she studies than if she does not. This leads to

$$\varepsilon \leq \frac{\left[(2+r_w)\left(\theta\eta_s\mu-\theta\eta_{ne}\right)+\left(\theta\eta_m-\varepsilon_2\right)(1-\mu)\right]}{(1+r_w)}$$

This generates the dynamics between both groups and creates unemployment since, as it has been said before, the uneducated sector will never have a nonnegative unemployment rate.

The following condition must also be satisfied:

$$E\left[I_2|e=1,x_t<\varepsilon\right]|_{x_t=0}<E\left[I_2|e=0\right]|_{x_t=0}$$

This means that an individual with $x_t=0$ must prefer not to study. It is clear that if this were not the case, all individuals would train since $E\left[I_2\right]$ is monotonically increasing in x_t. From this condition the following lower bound rises

$$\varepsilon > \frac{\left[(2+r_w)\left(\theta\eta_s\mu-\theta\eta_{ne}\right)+\left(\theta\eta_m-\varepsilon_2\right)(1-\mu)\right]}{\mu\left(1+r_w\right)+(1-\mu)\left(1+r_b\right)}$$

From these two conditions, a lower and an upper bound for ε are obtained.

Let now x^* denote the threshold training cost. This is the cost that leaves someone indifferent between studying and remaining uneducated. That is,

$$E\left[I_2|E=1,x_t<\varepsilon\right]|_{x^*}\equiv E\left[I_2|E=0\right]|_{x^*}$$

The following threshold is obtained

$$x^*=\frac{\varepsilon\left[(1+r_w)\mu+(1+r_b)(1-\mu)\right]-\left[(2+r_w)(\theta\eta_s\mu-\theta\eta_{ne})+\left(\theta\eta_m-\varepsilon_2\right)(1-\mu)\right]}{(r_b-r_w)(1-\mu)}$$

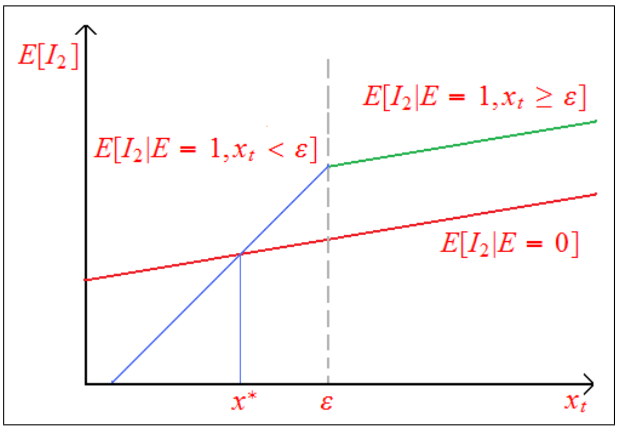

Figure 1. Determination of x^*.

Now, two groups of individuals are clearly difference, those who will study and who will not. Namely, individuals who inherit an amount $x_t\geq x^*$ from their parents will choose to train. Otherwise, if $x_t < x^*$, they will not enter the educative system.

Recalling the expressions obtained for I_2, the inheritance dynamics in the space (x_t, x_{t+1}) is as follows

$$x_{t+1}(x_t) = \begin{cases} (1-\beta)\left[\theta\eta_s + (\theta\eta_s + x_t - \varepsilon)(1+r_w)\right] & \text{if } E=1, S=1 \\ (1-\beta)\left[\theta\eta_{ne} + (\theta\eta_{ne} + x_t)(1+r_w)\right] & \text{if } E=0 \\ (1-\beta)\left[\theta\eta_m - \varepsilon_2 + (x_t - \varepsilon)(1+r_w)\right] & \text{if } E=1, S=0, x_t \geq \varepsilon \\ (1-\beta)\left[\theta\eta_m - \varepsilon_2 + (x_t - \varepsilon)(1+r_b)\right] & \text{if } E=1, S=0, x_t < \varepsilon \end{cases}$$

To guarantee the existence of positive steady state levels of bequests for each case the following conditions must be satisfied

$$(1-\beta)(1+r_w) < 1$$

$$(1-\beta)(1+r_b) > 1$$

The first restriction assures that the function $x_{t+1}(x_t)$ crosses the 45 degrees line in the first quadrant for the cases of individuals of type s, ne and m with $x_t \geq \varepsilon$. The second one, assures the same but for the case of individuals of type m with $x_t < \varepsilon$. The steady state level of bequest for individuals of type ℓ, x_ℓ, is defined as follows:

$$x_{t+1}^{\ell}(x_\ell) \equiv x_\ell$$

This means that in steady state, if there were no uncertainty about the result of education, each individual leaves to her child the same amount of bequest than the one she inherited from her parent. By this definition the following steady state levels are obtained

$$x_{ne} = (1-\beta)\frac{\theta\eta_{ne}(2+r_w)}{(1-(1-\beta)(1+r_w))}$$

$$x_s = (1-\beta)\frac{[\theta\eta_s(2+r_w) - \varepsilon(1+r_w)]}{(1-(1-\beta)(1+r_w))}$$

$$\overline{x}_m = (1-\beta)\frac{[\theta\eta_m - \varepsilon_2 - \varepsilon(1+r_w)]}{(1-(1-\beta)(1+r_w))}$$

$$\underline{x}_m = (1-\beta)\frac{[\theta\eta_m - \varepsilon_2 - \varepsilon(1+r_b)]}{(1-(1-\beta)(1+r_b))}$$

Since the function $x_{t+1}(x_t)$ for medium workers crosses the 45 degrees line twice, as it has been asked, there are two possible steady states for this case, $\overline{x}_m$ and $\underline{x}_m$, with $\overline{x}_m > \underline{x}_m$.

The steady state equilibrium of this economy is defined as the equilibrium where the proportion of a generation which decides not to study remains constant over time. For this matter, it is useful to study bequests' dynamics over time and their implications in the amount of educated individuals. For this purpose, the following questions will be addressed:

- May uneducated individuals' children study?
- Which proportion of medium workers' children study?
- Do all skilled workers' children study?

The following figures shed light on the answers. For uneducated individuals, there are two possibilities depicted in Figures 2 and 3.

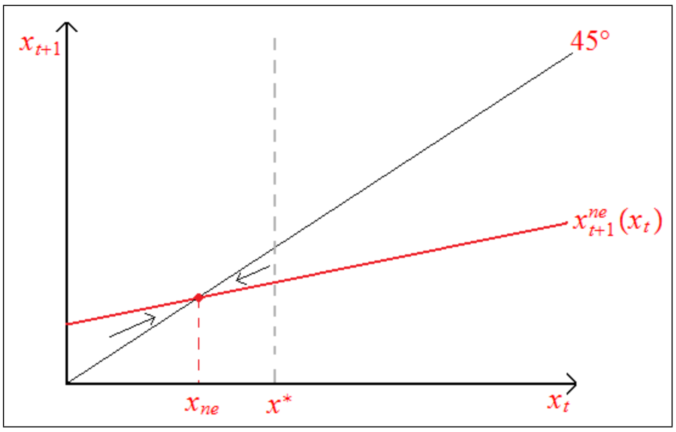

Figure 2. A situation for uneducated workers where $x_{ne} < x^*$.

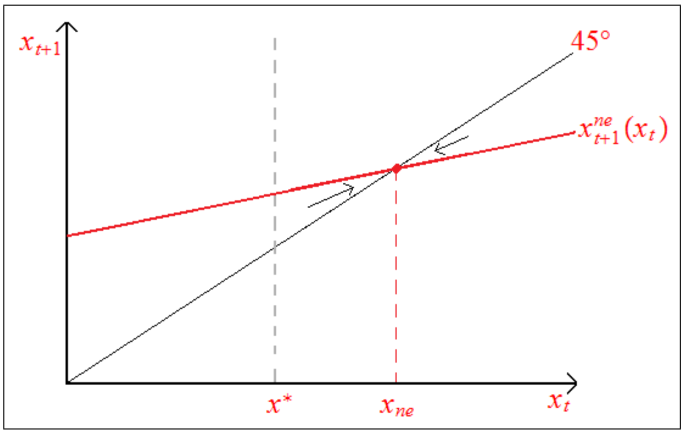

Figure 3. A situation for uneducated workers where $x^* < x_{ne}$.

In Figure 2 $x_{ne} < x^*$. In this case, in bequests' steady state, there is a nonnegative mass of individuals who choose not to study. Furthermore, all offspring of uneducated individuals who were born in $t = 0$ prefer to stay uneducated.

The case of Figure 3 is the opposite: uneducated individuals earn as much as to make some of their offspring educated individuals. Since beyond x^* the function $x_{t+1}^{ne}(x_t)$ is not relevant, we cannot assert that once an uneducated worker leaves to her child a bequest higher than x^* all of her offspring will remain educated. This will depend on the particular cases of function $x_{t+1}^{m}(x_t)$.

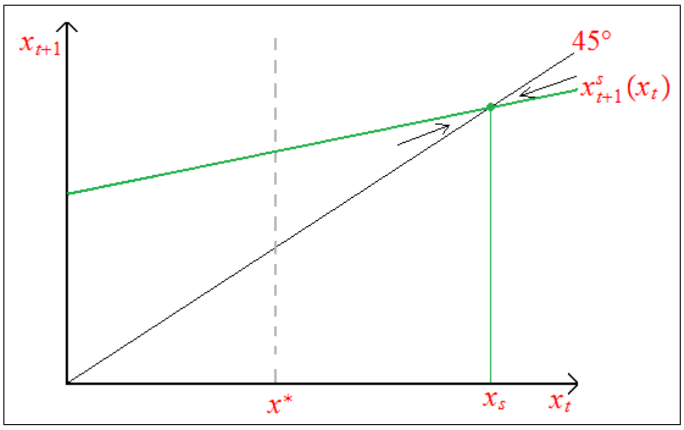

Figure 4. Skilled workers' case.

Figure 4 depicts the case for skilled workers. If there were no uncertainty, a skilled worker leaves a bequest level such that all of their offspring also choose to study.

The possible cases for medium workers are displayed in the next three figures.

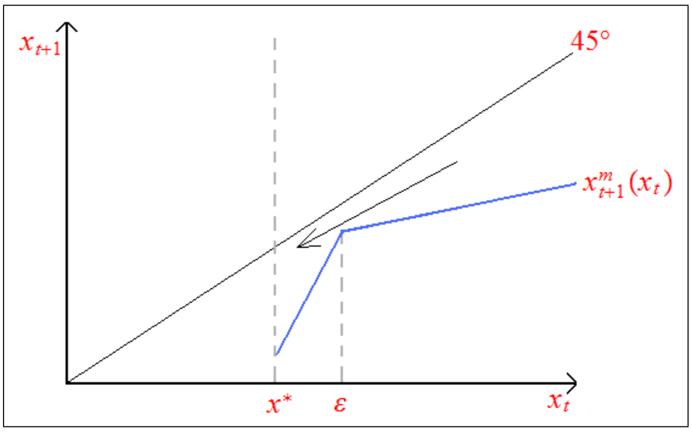

Figure 5. Medium workers could be intended to end up in the uneducated group.

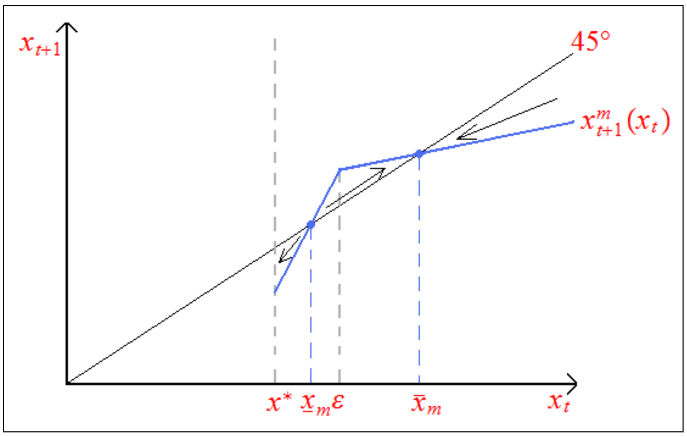

Figure 6. The two steady state levels are relevant.

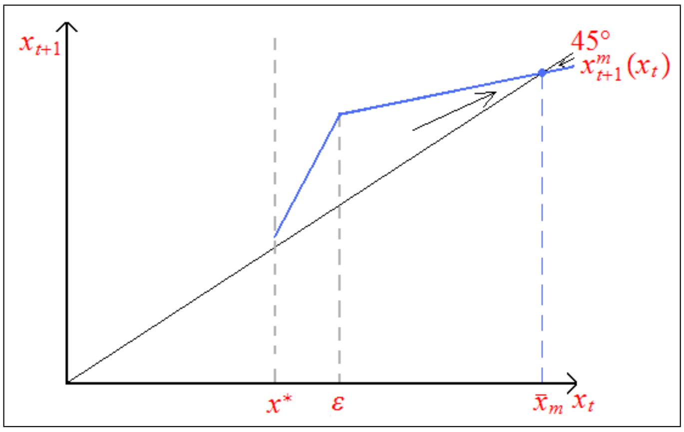

Figure 7. The case is similar to that of skilled workers'.

In Figure 5, medium workers' offspring intends to move to the uneducated sector of the economy. As in Figure 3, there can be forces from the non-educated sector that make individuals to go back to the educated group. In this way, it is not clear whether all offspring of a medium worker -who leaves a bequest such that her child does not study- will remain as uneducated.

In Figure 6 both medium steady state levels $\overline{x}_m$ and $\underline{x}_m$ are relevant. As can be shown in the figure, while the former represents a stable steady state value, the latter is an unstable one. Hence, if there were no uncertainty it can be assert that medium workers who inherit an amount at least as high as $\underline{x}_m$ earn as much as to make all of their offspring educated workers. As for this type of workers who inherit $x_t \in [x^*, \underline{x}_m]$, the analysis is the same as the one of Figure 5.

Finally, in Figure 7, medium workers' dynamics behave as skilled workers. This means that in absence of uncertainty these individuals will end up at $\overline{x}_m$, always choosing to study.

Now, on the one hand, if Figures 3, 4 and 5 were joined, in the steady state all individuals will choose to study. The result is the same if Figures 3, 4 and 6 were joined and, in a more clear way, with Figures 3, 4 and 7. On the other hand, if Figures 2, 4 and 5 were put together, in the steady state all individuals will decide to remain as uneducated. This case seems unreasonable[7]. The case chosen will be the economy which results from Figures 2, 4 and 6. This is so because this economy will have nonnegative steady state proportions of both educated and uneducated workers. To assure this to be the case the following conditions[8] must be asked:

$$x_{ne} \leq x^* \leq \underline{x}_m \leq \overline{x}_m$$

[7]The situations represented in Figures 2 and 7 cannot happen at the same time since $x^m_{t+1}(x^*) \leq x^{ne}_{t+1}(x^*)$.

[8]If this condition was met, the condition for the upper bound of ε is also satisfied. Intuitively, this is clear: if in the long run there are people who decide to study, there must have been people in the short term who decided to do so.

Before proceeding, it is important to emphasize that the equilibrium described below results from a number of conditions among parameters. There are several sets of parameters that satisfy them.

Graphically, the resulting economy is as follows

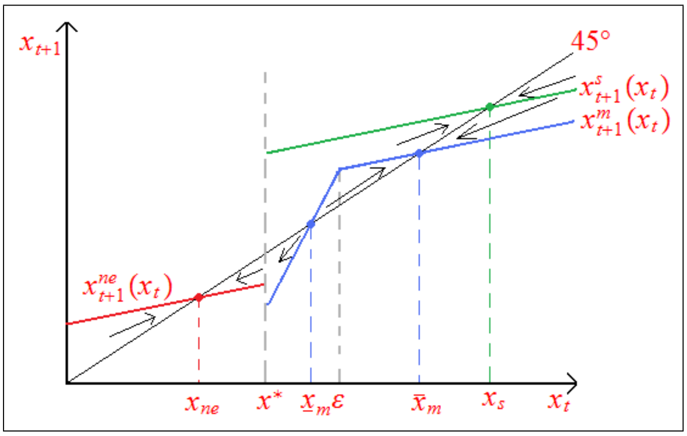

Figure 8. The Economy.

Figure 8 states that for the case of each individual who receives a bequest lower than x^*, not only she, but all of her offsprings will not study. This assures that in steady state a positive proportion of individuals will choose not to train. This is because the bequest that a non-educated worker leaves will never be higher than x^*.

It is also straightforward to see that all individuals who receive a bequest larger than $\underline{x}_m$ will educate, and also all of her offspring. This observation does not depend on the result of the educative process.

It is also shown that it may be possible that the bequest that any skilled worker leaves will always be higher than $\underline{x}_m$. This would mean that a skilled worker's dynasty will always study and it will be assumed that this is actually the case.

However, it is not clear what is happening with medium workers that inherit an amount $x_t \in [x^*, \underline{x}_m]$. For the study of the steady state equilibrium, the dynamics of these individuals must be analyzed in detail.

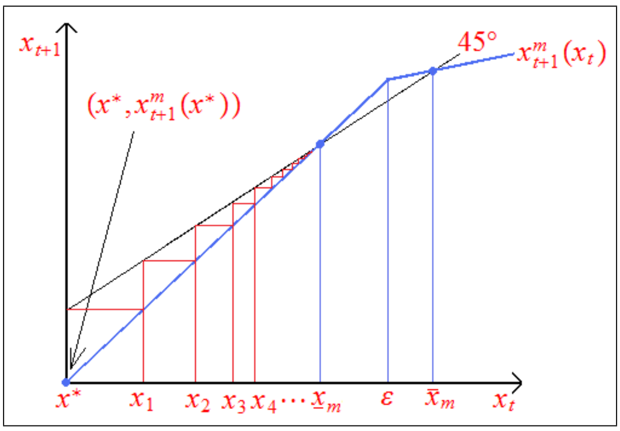

Figure 9. Medium workers who inherit amounts $x_t \in [x^*, \underline{x}_m]$. Amounts x_n with $n \in \mathbb{N}$ represent critical levels.

Figure 9 helps to clarify this situation. x_1 will denote the level inherited by a medium worker such that she leaves her child a bequest of x^*. x_2 will denote the level inherited by a medium worker such that she leaves her child a bequest of x_1, and so on. This levels will be indexed by n. It can be seen that a medium worker who inherits some level between x^* and x_1, will leave her child an amount lower than x^*. In this way, all of their offspring will be non-educated individuals. For these dynasties of medium workers, education lasts one period only.

All unskilled parents that have received a bequest between x_1 and x_2, will leave their children an inheritance between x^* and x_1. Thus, the young generation will study and if they become skilled workers, all of their offspring will also train. If they become medium, none of their offspring will educate. In this case, for these dynasties, education lasts at least for two periods.

Generalizing this results, it can be said that for dynasties of medium workers who inherit an amount between x_{n-1} and x_n, education lasts at least for n periods.

By a recursive method, the studied interval can be shorten until it gets close to $\underline{x}_m$.

$$x_n = a^n x^* + b\left(\sum_{i=0}^{n-1} a^i\right) \qquad \forall n \in \mathbb{N}$$

Where $a = \dfrac{1}{(1-\beta)(1+r_b)}(< 1)$ and $b = \dfrac{\varepsilon(1+r_b) - (\eta_m\theta - \varepsilon_2)}{(1+r_b)}$. It can be proved that $\lim\limits_{n\to+\infty} x_n = \underline{x}_m$. This implies that, under certain assumptions, there is always someone in any of the intervals $[x_{n-1}, x_n]$ whenever $t < +\infty$ and then, if a change in parameters occurs in finite time, this change will have consequences on dynamics.

It follows that in the long run, the economy will be concentrated in two groups: the non-educated individuals, at x_{ne}, and the educated ones, scattered in the interval $[\overline{x}_m, x_s]$. Thus, if a change in parameters occurs in the long run is very likely to have no impact on the long-term proportion of educated individuals.

2.6 Endogenous Variables

One of the endogenous variables that deserves special attention is the proportion of individuals of each generation that decides to remain uneducated, which will be denoted by υ_t. This is the endogenous state variable. Once this variable has been determined, the others are easily obtained. Recalling that in each period those individuals who decide not to train are the ones that inherited an amount smaller than x^*,

$$\upsilon_t = \int_0^{x^*} D_t dx$$

where D_t is the density function of bequests in period t, distributed in the interval $[0, +\infty)$. Given that, the following law of motion applies: $\forall t \geq 1$,

$$\upsilon_{t+1} = \upsilon_t + (1-\mu) \int_{x^*}^{x_1} D_t dx$$

In terms of the initial distribution,

$$\upsilon_t = \int_0^{x^*} D_0 dx + \sum_{n=1}^{t} \left[(1-\mu)^n \int_{x_{n-1}}^{x_n} D_0 dx \right]$$

Where $x_0 = x^*$ and x_n are defined as before for each $n = 1, \ldots, t$. In this way, the proportion of educated individuals in period t, e_t, will be

$$e_t = \frac{1}{2} \left[(1-\upsilon_t) + (1-\upsilon_{t-1}) \right]$$

In terms of economic activity, aggregate output per capita in period t will be

$$\mathcal{Y}_t = \sum_{l \in L} \lambda_\ell^t \, y_\ell$$

where $y_\ell = (k_\ell)^{1-\alpha} (\eta_\ell)^\alpha$ is the equilibrium output level of a firm of sector $\ell \in L = \{s, m, ne\}$ and λ_ℓ^t is the proportion of firms of type ℓ in period t. By recalling that in this economy an active firm of type ℓ corresponds to a worker of the same type, λ_l^t is the proportion of workers of type ℓ in period t. Then,

$$\lambda_{ne}^t = \frac{1}{2} (\upsilon_t + \upsilon_{t-1})$$

$$\lambda_s^t = \frac{1}{2} \left[\mu (1-\upsilon_t) + \mu (1-\upsilon_{t-1}) \right]$$

$$\lambda_m^t = \frac{1}{2} \left[(1-\mu)(1-\upsilon_{t-1}) \right]$$

Finally,

$$\mathcal{Y}_t = \frac{1}{2} \frac{\theta}{\alpha} \left[\begin{array}{c} (\upsilon_t + \upsilon_{t-1}) \eta_{ne} + (\mu (1-\upsilon_t) + \mu (1-\upsilon_{t-1})) \eta_s \\ + ((1-\mu)(1-\upsilon_{t-1})) \eta_m \end{array} \right]$$

Recall that in the separating equilibrium that is being subject of study, unemployed individuals are the young, unskilled workers. That is, a fraction $(1-\mu)$ of individuals of generation t who decide to enter the educative system. Formally, by calling this variable Un_t, unemployment is defined by the following expression

$$Un_t = \frac{1}{2}(1-\upsilon_t)(1-\mu)$$

It follows that a decline in the proportion of non-educated workers has two opposite effects on the product at period t. On the one hand, there will be more skilled workers in the economy. This means that a larger amount of workers will have a higher productivity. On the other hand, the unemployment rate will be higher due to the fact that more workers will be unskilled. The net impact depends on the gap between productivities.

2.7 Steady State Equilibrium

The steady state of this economy is defined as the stage in which υ's growth rate remains constant over time. This would imply that the rest of the endogenous variables will also remain constant. $\gamma^{z}_{t,t+1}$ will denote variable z's growth rate. In this way, $\gamma^{\upsilon}_{ss} = g$, where $g \in \mathbb{R}$. Due to the fact that $\upsilon_t \in [0,1]\ \forall t$ since it is a proportion, then, $g = 0$. It can be shown that this is only achieved when $t \to +\infty$.

Hence, in steady state:

$$\upsilon_{ss} = \int_0^{x^*} D_0 dx + \sum_{n=1}^{+\infty} \left[(1-\mu)^n \int_{x_{n-1}}^{x_n} D_0 dx \right]$$

$$e_{ss} = (1 - \upsilon_{ss})$$

$$\mathcal{Y}_{ss} = \frac{1}{2}\frac{\theta}{\alpha}\left[2\upsilon_{ss}\eta_{ne} + 2\mu\left(1-\upsilon_{ss}\right)\eta_s + \left(1-\mu\right)\left(1-\upsilon_{ss}\right)\eta_m\right]$$

$$Un_{ss} = \frac{1}{2}\left(1-\upsilon_{ss}\right)\left(1-\mu\right)$$

As it is shown in Figure 8 the transition is characterised by the transfer of individuals from the educated to the non-educated sector. This is due to the dynamics of particular dynasties in which people inherit enough to decide to study, work as unskilled but leave their children less than what they received, so that they decide that is more profitable not to study. Hence, the initial distribution of wealth determines how big these two groups are, and therefore what the long-run equilibrium in the economy is.

In this way, during the transition to the steady state, there is a monotonic decrease in aggregate output and unemployment and an increasing proportion of uneducated individuals.

3 Comparative Statistics

In the following section, different exercises of comparative statistics will be done. Productivity boosts in the medium and non-educated sector, as educational reforms, and changes in interest rates will be analyzed.

3.1 Productivity Boosts

Permanent and unanticipated changes in productivity level of medium and non-educated workers were analyzed.

3.1.1 Uneducated Worker's Productivity Boost

A productivity boost for the uneducated sector will be represented by a change from η_{ne} to $\eta_{ne}^{'}$, with $\eta_{ne}^{'} > \eta_{ne}$.This change may occur in the steady state or during the transition. τ will denote the period of alteration. In any case, it is supposed that the change occurs after the decision of education has already been taken by cohort τ. In this way, υ_τ is already determined when the productivity change happens.

For every $t \geqslant 1$

$$\upsilon_{\tau+1} = \upsilon_\tau + (1-\mu)\int_{x^*}^{x_1} D_t dx$$

Then, if υ_τ is determined when the economy acquires the new productivity, the sequences $(\upsilon_\tau(\eta_{ne}))_{t=0}^{+\infty} = (\upsilon_\tau(\eta_{ne}^{'}))_{t=0}^{+\infty}$ will coincie until $t = \tau$.

Thus, if the change occurs in the steady state ($\tau = +\infty$), then $\upsilon_{ss}^{'} = \upsilon_{ss}$, that is, there will be no impact on the proportion of uneducated inidividuals of each cohort (this is so, if the change to the productivity level is not big enough so that the bequest left by a non-educated parent is enough so that her offspring studies). By recalling Figure 8, it can be seen that this result is clear: in the steady state, there are no individuals in the interval $\left[x^*; \underline{x_m}\right)$, then, no one will be affected by the productivity change (in terms of mobility between sectors). Thus, there will not be any differences neither in unemployment nor in the fraction of educated individuals. However, there is in the aggregate output per capita, since each λ_l remain unchanged and the productivity shock means an increase in productivity of the non-educated sector. Finally, x_{ne} rises and then there is a reduction in the dispersion of wealth between educated and uneducated individuals. The society becomes more equal.

Following, the case in which $\tau < +\infty$ will be studied.

When there is an increase in η_{ne}, the threshold training cost increases.

$$\frac{\partial x^*}{\partial \eta_{ne}} = \frac{(2+r_w)\theta}{(r_b - r_w)(1-\mu)} > 0$$

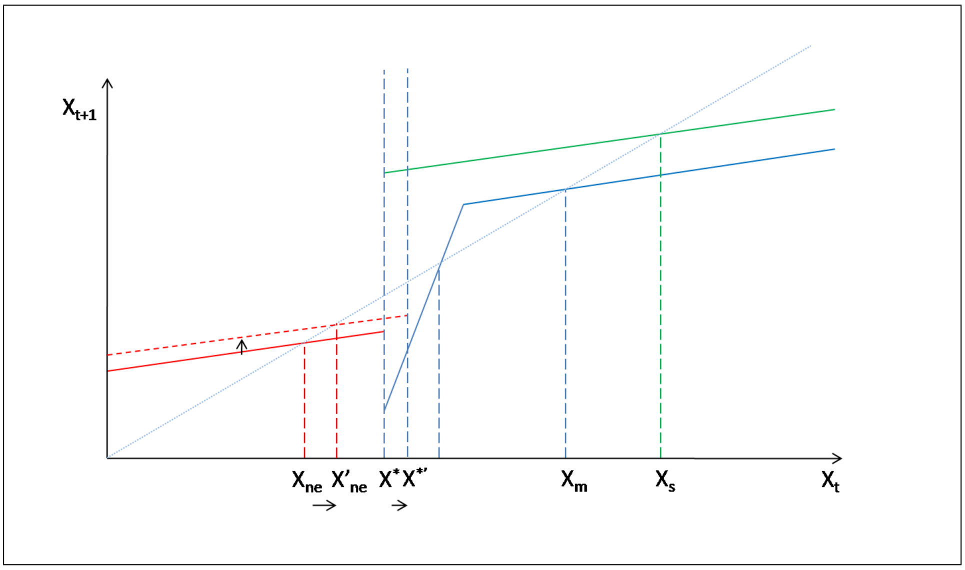

Figure 10. The economy after an increase in η_{ne}.

Furthermore, w_{ne} rises and then $x_{t+1}^{ne}(x_t)$ shifts upwards. For non-educated workers, the impact on wages is immediate. Non-educated workers of generation τ will earn $\theta\eta_{ne}^{'}$ in both periods. However, the non-educated group of period τ is composed by those individuals with $x_\tau < x^*$.

How does the economy change with this productivity boost? Now, those individuals who will be incorporated to the uneducated group in period $\tau+1$ will be those whose parents have a level of bequest such that, considering their respective wages leave their children an amount x_{t+1} smaller than $x^{*\prime}$. It is neccessary to introduce two level of bequests of great importance. The first is denoted by $x_1^{'}$ which is the bequest that must have a medium worker for leaving her child $x^{*\prime}$[9]. The second is the counterpart of $x_1^{'}$ for the case of uneducated parents and will be denoted by $\widetilde{x}$. There are different possible outcomes for υ_{t+1} depending the combination of possible cases of these levels of bequests. However, if the rise is infinitesimal, $x_1^{'} > x^*$ and $\widetilde{x} > x^*$. This implies that,

$$\upsilon_{t+1}^{'} = \upsilon_{t+1} + (1-\mu)\int_{x_1}^{x_1^{'}} D_\tau dx$$

It is straightforward to see, that the uneducated proportion of the society increases. The result derives in the fact that the threshold training cost increases.

In steady state,

$$\frac{\partial \upsilon_{ss}}{\partial \eta_{ne}}|_\tau > 0$$

[9]Notice that this level is the x_1 defined earlier but for the case of $x^{*\prime}$. In this way, it is clear to see that if $x^{*\prime} > x^*$, then $x_1^{\prime} > x_1$, $\frac{\partial x_1}{\partial \eta_{ne}} = a\frac{\partial x^*}{\partial \eta_{ne}} > 0$.

where $\frac{\partial \upsilon_{ss}}{\partial \eta_{ne}}|_{\tau}$ denotes the derivative of υ_{ss} for the case in which the productivity boost occurs in period τ.

A boost in the productivity of non-educated workers increases the proportion of uneducated individuals of each generation in the long run. Basically, this is due to the fact that there is an increase in the threshold training cost and that the change does affect some individuals' decisions. Then, it clearly follows that the educated sector of the economy decreases its proportion and then there will be less unemployment. Formally,

$$\frac{\partial e_{ss}}{\partial \eta_{ne}}|_{\tau} = -\frac{\partial \upsilon_{ss}}{\partial \eta_{ne}}|_{\tau} < 0$$

$$\frac{\partial Un_{ss}}{\partial \eta_{ne}}|_{\tau} = -\frac{1}{2}(1-\mu)\frac{\partial \upsilon_{ss}}{\partial \eta_{ne}}|_{\tau} < 0$$

Regarding economy's output, there are two different forces that hit aggregate production. In the first place, there is an increase in the productivity of uneducated workers, which makes output to go up. In the second place, there is less unemployment but more individuals go from a sector with higher to productivity to one of lower.

$$\frac{\partial \mathcal{Y}_{ss}}{\partial \eta_{s}}|_{\tau} = \frac{1}{2}\frac{\theta}{\alpha}\left[2\upsilon_{ss} - \left(\frac{\partial \upsilon_{ss}}{\partial \eta_{ne}}|_{\tau}\right)\left[2\eta_{s}\mu + (1-\mu)\,\eta_{m} - 2\eta_{ne}\right]\right]$$

The net result on output per capita depends on the gap between productivities. A sufficient condition for aggregate output to increase is $2\eta_{s}\mu + (1-\mu)\,\eta_{m} < 2\eta_{ne}$. This means that the educated sector must not have a sufficiently large productivity in relation to that of the uneducated sector.

Regarding economic inequality, it decreases. To see this, recall that in steady state, a proportion υ_{ss} of each generation will inherit x_{ne} and the other fraction will receive bequests in the interval $[\overline{x}_m; x_s]$. Assuming that the mean educated individual's inheritance is a weighted average of levels $\overline{x}_m$ and x_s - denoted by $\widehat{x}_e$ -, in steady state, the incomes for the second period of life are:

$$I_2^{ne}|_{ss} = \theta\eta_{ne} + (\theta\eta_{ne} + x_{ne})\,(1+r_w)$$
$$I_2^{m}|_{ss} = \theta\eta_{m} - \varepsilon_2 + (\widehat{x}_e - \varepsilon)\,(1+r_w)$$
$$I_2^{s}|_{ss} = \theta\eta_{s} + (\theta\eta_{s} + \widehat{x}_e - \varepsilon)(1+r_w)$$

Then, total income of sector ℓ ($I_2^{S\ell}|_{ss}$) will be:

$$I_2^{Sne}|_{ss} = \upsilon_{ss}\left[\theta\eta_{ne} + (\theta\eta_{ne} + x_{ne})\,(1+r_w)\right]$$

$$I_2^{Sm}|_{ss} = (1-\mu)\,(1-\upsilon_{ss})\left[\theta\eta_{m} - \varepsilon_2 + (\widehat{x}_e - \varepsilon)\,(1+r_w)\right]$$
$$I_2^{Ss}|_{ss} = \mu\,(1-\upsilon_{ss})\left[\theta\eta_{s} + (\theta\eta_{s} + \widehat{x}_e - \varepsilon)(1+r_w)\right]$$

If it is assumed that that weights for $\overline{x}_m$ and x_s do not change with η_{ne} then $sign\left(\frac{\partial \widehat{x}_e}{\partial \eta_{ne}}\right) = sign\left(\frac{\partial x_s}{\partial \eta_{ne}}\right)$

$$\frac{\partial I_2^{ne}|_{ss}}{\partial \eta_{ne}}|_\tau = 2\theta > 0$$

$$\frac{\partial I_2^{m}|_{ss}}{\partial \eta_{ne}}|_\tau = \frac{\partial I_2^{s}|_{ss}}{\partial \eta_{ne}}|_\tau = 0$$

Finally, the ratio of incomes for the (mean) educated and the non educated can be approximated by

$$R = \frac{\mu I_2^s|_{ss} + (1-\mu)\, I_2^m|_{ss}}{I_2^{ne}|_{ss}}$$

$$\frac{\partial R}{\partial \eta_{ne}} < 0$$

Then the result is a decrease in R. Economic inequality between educated and uneducated sectores reduces[10]. It is worthwile to remark that educated workers' wealth do not decrease.

3.1.2 Medium Worker's Productivity Boost

A productivity boost for the medium sector will be represented by an increase of η_m to $\eta_m^{'}$, with $\eta_m^{'} > \eta_m$. This change may occur in the steady state or during the transition. τ will denote the period of alteration. As in the previous case, it is supposed that the change occurs after the decision of education has already been taken by cohort τ. In this way, υ_τ is already determined when the productivity change happens.

In this way, if the change occurs in the steady state ($\tau = +\infty$), then $\upsilon_{ss}^{'} = \upsilon_{ss}$, that is, there will be no impact on the proportion of uneducated inidividuals of each cohort. As it has been said before, there are no individuals in the interval $\left[x^*; \underline{x_m}\right)$, then, no one will be affected by the productivity change (in terms of mobility between sectors). Thus, there will not be any differences neither in unemployment nor in the fraction of educated individuals. However, there is in the aggregate output per capita, since each λ_l remain unchanged and the productivity shock means an increase in productivity of the medium sector. Finally, x_m rises and then there is a reduction in the dispersion of wealth among educated workers, and a increase between the mean educated and uneducated individuals. The society becomes more unequal between uneducated and educated individuals, but more equal among educated ones.

Following, the case in which $\tau < +\infty$ will be studied. When there is an increase in η_m, the threshold training cost decreases.

[10] Notice that for this analysis, it has only been considered old individuals, this is to avoid double counting of bequests.

$$\frac{\partial x^*}{\partial \eta_m} = -\frac{\theta}{(r_b - r_w)} < 0$$

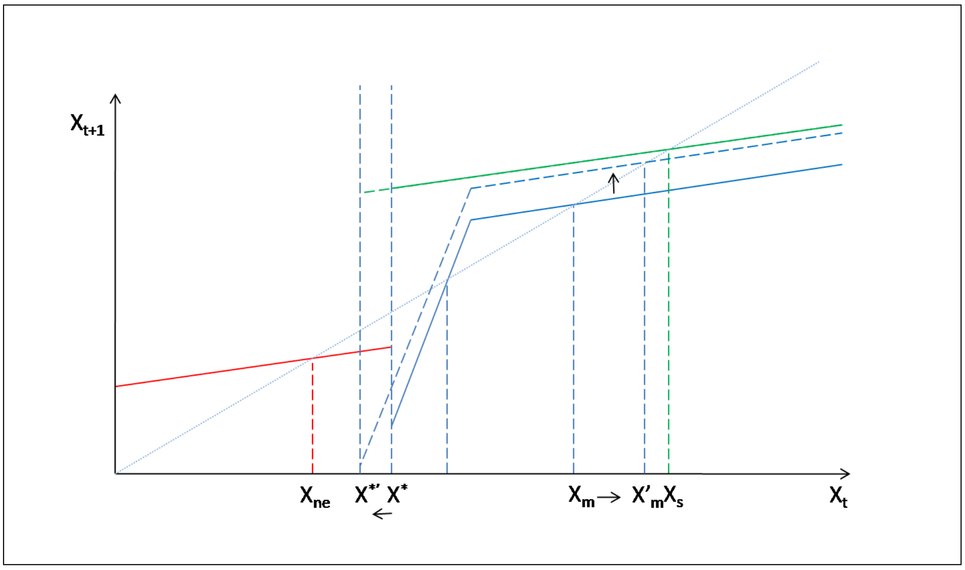

Figure 11. The economy after an increase in η_m.

Furthermore, w_m rises and then $x^m_{t+1}(x_t)$ shifts upwards. For medium workers, the impact on wages is immediate. Medium workers of generation $\tau - 1$ will earn $\theta\eta_m^{'}$ in the second period of their life. However, the educated group of period $\tau + 1$ is composed by those individuals with $x_{\tau+1} > x^*$.

If the alterations that the economy goes through are analyzed, those individuals who will decide to study in period $\tau + 1$ will be those whose parents have a level of bequest such that, considering their respective wages leave their children an amount x_{t+1} higher than $x^{*\prime}$. It is neccessary to introduce two level of bequests of great importance. The first is denoted by $x_1^{'}$ which is the bequest that must have a medium worker for leaving her child $x^{*\prime}$[11]. The second is the counterpart of $x_1^{'}$ for the case of uneducated parents and will be denoted by $\widetilde{x}$. There are different possible outcomes for v_{t+1} depending on the combination of the possible position of the bequests explained before. However, if the rise is infinitesimal, $x_1^{'} > x^*$ and $\widetilde{x} > x^*$. This implies that,

$$v^{'}_{t+1} = v_{t+1} - (1-\mu)\int_{x_1^{'}}^{x_1} D_\tau dx$$

It is straightforward to see that the uneducated proportion of the society decreases. The result derives in the fact that the threshold training cost falls and the medium worker's income increases.

[11] Notice that this level is the x_1 defined earlier but for the case of $x^{*\prime}$. In this way, it is clear to see that if $x^{*\prime} < x^*$, then $x_1' < x_1$, $\frac{\partial x_1}{\partial \eta_m} = a\frac{\partial x^*}{\partial \eta_m} - \frac{\theta}{1+r_b}\sum_{i=0}^{n-1} a^i < 0$.

In steady state,

$$\frac{\partial \upsilon_{ss}}{\partial \eta_m}|_\tau < 0$$

where $\frac{\partial \upsilon_{ss}}{\partial \eta_m}|_\tau$ denotes the derivative of υ_{ss} for the case in which the productivity boost occurs in period τ.

A boost in the productivity of medium workers reduces the proportion of uneducated individuals of each generation in the long run. Basically, this is due to the fact that there is a fall in the threshold training cost and that the change does affect some individuals' decisions. Then, it clearly follows that the educated sector of the economy increases its proportion and then there will be more unemployment. Formally,

$$\frac{\partial e_{ss}}{\partial \eta_m}|_\tau = -\frac{\partial \upsilon_{ss}}{\partial \eta_m}|_\tau > 0$$

$$\frac{\partial Un_{ss}}{\partial \eta_m}|_\tau = -\frac{1}{2}(1-\mu)\frac{\partial \upsilon_{ss}}{\partial \eta_m}|_\tau > 0$$

Regarding economy's output, there are two different forces that hit aggregate production. In the first place, there is an increase in unemployment. In the second place, more individuals are part of the most productive sector of the economy.

$$\frac{\partial \mathcal{Y}_{ss}}{\partial \eta_s}|_\tau = \frac{1}{2}\frac{\theta}{\alpha}\left[(1-\upsilon_{ss})(1-\mu) - \left(\frac{\partial \upsilon_{ss}}{\partial \eta_m}|_\tau\right)[2\eta_s\mu + (1-\mu)\,\eta_m - 2\eta_{ne}]\right]$$

The net result on output per capita depends on the gap between productivities. A sufficient condition for aggregate output to increase is $\eta_s\mu > \eta_{ne}$. This means that the skill sector must have a sufficiently large productivity in relation to that of the uneducated sector.

Regarding economic inequality, it decreases among educated individuals but increases between the educated and uneducated sector of the economy.

$$\frac{\partial I_2^{ne}|_{ss}}{\partial \eta_m}|_\tau = 0$$

$$\frac{\partial I_2^{m}|_{ss}}{\partial \eta_m}|_\tau = \left(\theta + \frac{\partial \widehat{x}_e}{\partial \eta_m}(1+r_w)\right) > 0$$

$$\frac{\partial I_2^{s}|_{ss}}{\partial \eta_m}|_\tau = \frac{\partial \widehat{x}_e}{\partial \eta_m}(1+r_w) > 0$$

Finally, the ratio of incomes for the (mean) educated and the non educated can be approximated by

$$R = \frac{\mu I_2^s|_{ss} + (1-\mu)\, I_2^m|_{ss}}{I_2^{ne}|_{ss}}$$

$$\frac{\partial R}{\partial \eta_m} > 0$$

Then the result is an increase in R. Economic inequality between educated and uneducated sectores increases. It is worthwile to remark that uneducated workers' wealth do not decrease.

There has not been made an analysis to an increase in the productivity of the skill sector because has already been analyzed in "Caramp, Espinosa, Melero and Szenig" (2010), and of the unskill sector (those who do not succeed in the educative process and do not re-educate themselves) because if certain conditions among parameters are satisfied, there will not be any effect in the economy because no worker will end up in this sector.

3.2 Interest Rates

Changes in interest rates may be interpreted as a political decision to stop subsidizing rates to students who decide to educate themselves. Lower active rate will make students face a lower financial cost of studying.

3.2.1 Active Interest Rate

An increase in the active interest rate may be interpreted as a reduction of subsidized loans that students may obtain in the economy to pay their studies. This political decision to stop subsidizing loans to students will make that the active rate of the economy to increase.

A increase in the active interest rate will be represented by a change from r_b to $r_b^{'}$, with $\eta_b^{'} > \eta_b$. In this way, if the change occurs in the steady state ($\tau = +\infty$), then $\upsilon_{ss}^{'} = \upsilon_{ss}$, that is, as in the previous cases, there will be no impact on the proportion of uneducated inidividuals of each cohort. There will not be any differences neither in unemployment nor in the fraction of educated individuals. There are no changes in the aggregate output per capita. Due to the fact that no individual faces the active interest rate in steady state, their incomes will not be affected and inequality remains the same.

If $\tau < +\infty$, the threshold training cost increases.

$$\frac{\partial x^*}{\partial r_b} = \frac{(2 + r_w)(\theta\eta_s - \theta\eta_{ne}) + (1 - \mu)(\theta\eta_m - \varepsilon_2 - \varepsilon(1 + r_w))}{(1 - \mu)(r_b - r_w)^2} > 0$$

This condition is positive because is the one that guarantees that some individuals educate themselves.

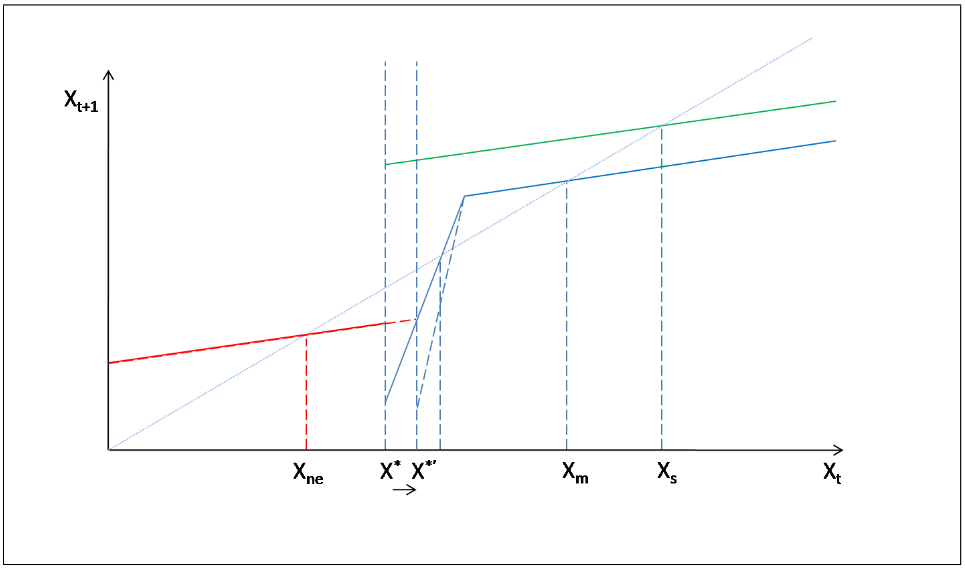

Figure 12. The economy after an increase in r_b.

Furthermore, the income of medium workers rises as the financial cost increases. $x_{t+1}^{m}(x_t)$ shifts downwards and the slope increases.

If we analyze the alterations that our economy goes through, those individuals who will decide to study in period $\tau+1$ will be those whose parents have a level of bequest such that, considering their respective wages leave their children an amount x_{t+1} higher than $x^{*\prime}$. It is neccessary to introduce $x_1^{'}$ level of bequest which is the bequest that must have a medium worker for leaving her child $x^{*\prime}$. It increases because not only the bequest that the parent will leave her falls because of the increase in the financial cost, but also the threshold training cost increases.

$$\upsilon_{t+1}^{'} = \upsilon_{t+1} + (1-\mu)\int_{x_1}^{x_1^{'}} D_\tau dx$$

It is straightforward to see, that the uneducated proportion of the society increases. The result derives in the fact that the threshold training cost increases and a medium worker (those who have $x_\tau < \varepsilon$) leaves her offspring a lower bequest.

In steady state,

$$\frac{\partial \upsilon_{ss}}{\partial r_b}|_\tau > 0$$

An increase in the active interest rate increases the proportion of uneducated individuals of each generation in the long run. Basically, this is due to the fact that there is an increase in the threshold training cost and a decrease in the bequests that a medium worker (those who have $x_\tau < \varepsilon$) leaves to her offspring. This affects some individuals' decisions. Then, it clearly follows that the educated sector of the economy decreases its proportion and then there will be less unemployment.

Regarding economy's output, there are two different forces that hit aggregate production. In the first place, there is a reduction in unemployment. In the second place, less individuals are part of the most productive sector of the economy.

$$\frac{\partial \mathcal{Y}_{ss}}{\partial \eta_s}|_\tau = -\frac{1}{2}\frac{\theta}{\alpha}\left(\frac{\partial \upsilon_{ss}}{\partial r_b}|_\tau\right)[2\eta_s\mu + (1-\mu)\,\eta_m - 2\eta_{ne}]$$

The net result on output per capita depends on the gap between productivities of the educated and uneducated sector. A sufficient condition for aggregate output to decrease is $\eta_s\mu > \eta_{ne}$. This means that the skill sector must have a sufficiently large productivity in relation to that of the uneducated sector.

Regarding economic inequality, it stays the same as in steady state no individual faces the active interest rate of the economy (educated individuals' steady state income are higher than ε).

3.2.2 Pasive Interest Rate

An increase in the pasive interest rate will reduce the spread between active and pasive rates of the economy. An increase in the pasive rate, on the one hand, will encourage individuals to be lenders during their first period of life because of the financial gain they may gain. On the other hand, will make studying cheaper.

A increase in the pasive interest rate will be represented by a change from r_w to $r_w^{'}$, with $r_w^{'} > r_w$.

In this way, if the change occurs in the steady state ($\tau = +\infty$), then $\upsilon_{ss}^{'} = \upsilon_{ss}$, that is, as in the previous cases, there will be no impact on the proportion of uneducated inidividuals of each cohort. There will not be any differences neither in unemployment nor in the fraction of educated individuals. There are no changes in the aggregate output per capita. But in this case, the income of all individuals will increase because they will all have a higher financial gain of their savings (in steady state, all individuals are net lenders).

Regarding the threshold training cost, there are two different forces that affect it.

$$\frac{\partial x^*}{\partial r_w} = \frac{\theta\eta_{ne} - (\theta\eta_s\mu - \varepsilon(1+r_b))}{(1-\mu)(r_b - r_w)^2} + \\ + \frac{(2+r_w)\theta\eta_{ne} - [(2+r_w)\theta\eta_s\mu + (1-\mu)(\theta\eta_m - \varepsilon_2)]}{(1-\mu)(r_b - r_w)^2}$$

On the one hand, the first term of the right compares the financial gains of an educated to a uneducated individual. If it is possitive or not, depends on which first period income is higher. The second term is a negative term.

It reflects that studying is less costly than before, because the gap between the active and pasive rate has reduced. Depending the net effect of these two effects, the threshold training cost may increase or reduce. If the productivity of the skill sector was supposed to be much higher than the one of the uneducated sector, $\theta\eta_{ne} + \varepsilon(1 + r_b) < \theta\eta_s\mu$, the threshold training cost decreases.

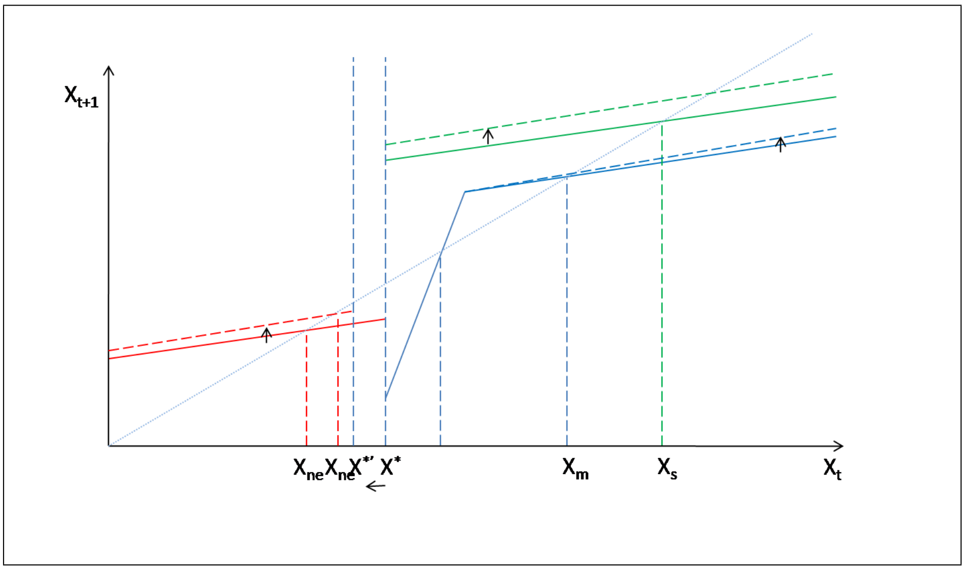

Figure 13. The economy after an increase in r_w.

$x^j_{t+1}(x_t)$, $j = ne,\ m,\ s$, shifts upwards and the slope increases. There is no change to the segment of bequests that an educated parent who did not succeed in the educative process leaves to her offspring. All stable steady state equilibriums rises.[12]

$$sign\left(\frac{\partial x_{ne}}{\partial r_w}\right) = sign\left(\frac{\partial \overline{x}_m}{\partial r_w}\right) = sign\left(\frac{\partial x_s}{\partial r_w}\right) > 0$$

This is so because all individuals obtain a higher gain for their savings of the first period. The unstable equilibrium for medium workers does not change because those workers face the active interest rate of the economy.[13]

If the alterations that the economy goes through are analyzed, those individuals who will decide to study in period $\tau + 1$ will be those whose parents have a level of bequest such that, considering their respective wages leave their children an amount x_{t+1} higher than $x^{*\prime}$[14]. It is neccessary to introduce $x_1^{'}$ level of bequest which is the bequest that must have a medium worker for leaving her child $x^{*\prime}$ and its counterpart for the uneducated worker, $\widetilde{x}$. In the infinitesimal case, only $x_1^{'}$ is relevant. It decreases because the threshold training cost falls, $\frac{\partial x_1}{\partial r_w} = a\frac{\partial x^*}{\partial r_w} < 0$.

[12] The sign of the second derivative is assured by the condition that $\overline{x}_m > \varepsilon$.
[13] For these changes it is not necessary to make any further assumptions.
[14] The assumption that the threshold training cost decreases will be made.

$$\upsilon'_{t+1} = \upsilon_{t+1} - (1-\mu)\int_{x'_1}^{x_1} D_\tau dx$$

It is straightforward to see, that the uneducated proportion of the society decreases. The result derives in the fact that the threshold training cost decreases.

In steady state,

$$\frac{\partial \upsilon_{ss}}{\partial r_w}|_\tau < 0$$

An increase in the pasive interest rate decreases the proportion of uneducated individuals of each generation in the long run. Basically, this is due to the fact that the education cost decreases because of the reduction in the gap between pasive and active rate. Then, it clearly follows that the educated sector of the economy increases its proportion and then there will be more unemployment.

Regarding economy's output, there are two different forces that hit aggregate production. In the first place, there is an increase in unemployment. In the second place, more individuals are part of the most productive sector of the economy.

$$\frac{\partial \mathcal{Y}_{ss}}{\partial \eta_s}|_\tau = -\frac{1}{2}\frac{\theta}{\alpha}\left(\frac{\partial \upsilon_{ss}}{\partial r_w}|_\tau\right)\left[2\eta_s\mu + (1-\mu)\,\eta_m - 2\eta_{ne}\right]$$

The net result on output per capita depends on the gap between productivities of the educated and uneducated sector. A sufficient condition for aggregate output to decrease is $\eta_s\mu > \eta_{ne}$. This condition arises from the assumption needed to assure that $\frac{\partial x^*}{\partial r_w} < 0$.

Regarding economic inequality, it increases among educated workers. Those who succeed in the educative process have a higher financial income than those who do not succeed. Inequality between the educated and uneducated sector also increases. Educated workers have higher bequests and incomes that allows them to obtain a higher financial income from their savings than the uneducated sector. Despite these facts, steady state incomes of all types of workers increases.

$$\frac{\partial I_2^{ne}|_{ss}}{\partial r_w}|_\tau = (\theta\eta_{ne} + x_{ne}) + \frac{\partial x_{ne}}{\partial r_w}(1+r_w) > 0$$

$$\frac{\partial I_2^{m}|_{ss}}{\partial r_w}|_\tau = (\widehat{x}_e - \varepsilon) + \frac{\partial \widehat{x}_e}{\partial r_w}(1+r_w) > 0$$

$$\frac{\partial I_2^{s}|_{ss}}{\partial r_w}|_\tau = (\theta\eta_s + \widehat{x}_e - \varepsilon) + \frac{\partial \widehat{x}_e}{\partial r_w}(1+r_w) > 0$$

3.3 Educative Reforms

In the following section, two types of educative reforms will be analyzed. First, one that represents an increase of educative costs. Government may reduce subsidies to the educative sector. Second, the educative system may become more efficient and more students become skill through the process.

3.3.1 Educative Costs

The educative cost involves two different costs. On the one hand, the first period cost that all students must pay to receive education. On the other hand, the second period cost must be paid only by those students who did not succeed during the process. Both cases will be analyzed.

An increase in the educative cost will discourage individuals to educate themselves due to the higher cost.

An increase in the first period educative cost will be represented by a change from ε to $\varepsilon^{'}$, with $\varepsilon^{'} > \varepsilon$.

In this way, if the change occurs in the steady state ($\tau = +\infty$), then $\upsilon_{ss}^{'} = \upsilon_{ss}$, that is, as in the previous cases, there will be no impact on the proportion of uneducated inidividuals of each cohort. There will not be any differences neither in unemployment nor in the fraction of educated individuals. There are no changes in the aggregate output per capita. But in this case, the income of all educated individuals will decrease because they will all have a higher educative cost.

Regarding the threshold training cost,

$$\frac{\partial x^*}{\partial \varepsilon} = \frac{(1+r_w)\mu + (1+r_b)(1-\mu)}{(1-\mu)(r_b - r_w)} > 0$$

The threshold training cost increases due to the increase of the educative cost. If the shock occurs in finite time there will be alterations to the economy.

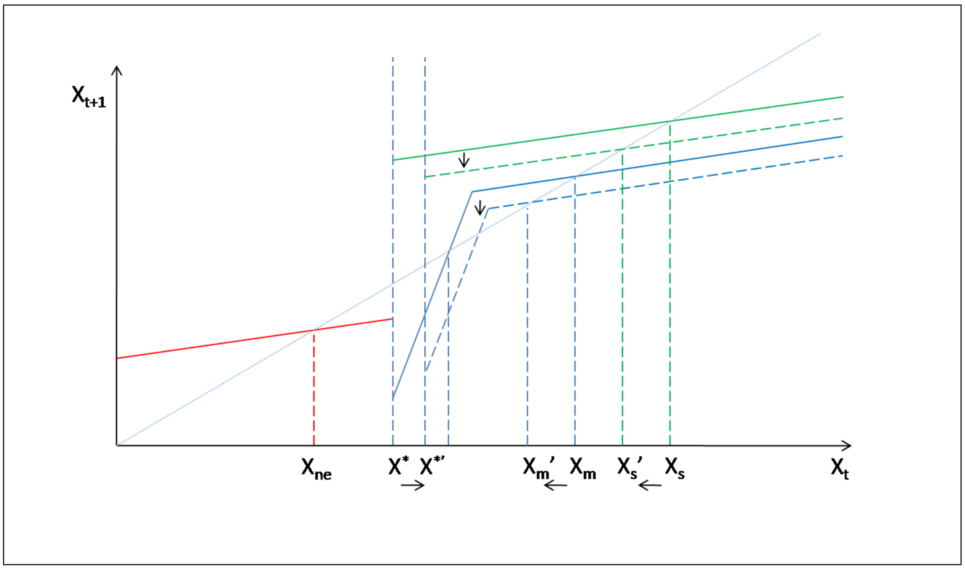

Figure 14. The economy after an increase in ε.

Furthermore, $x^j_{t+1}(x_t)$, $j = s$, m, shifts downwards. Educated individuals will have to pay for higher educative costs, reducing the bequests they leave to their offspring.

Those individuals who will decide to study in period $\tau + 1$ will be those whose parents have a level of bequest such that, x_{t+1} is higher than $x^{*\prime}$. Due to the fact that no uneducated parent leave their offspring the bequest that was needed to educate, it is possible to assure they will not be able to leave them the new level of bequest which is higher. It is neccessary to introduce which is the bequest that must have a medium worker for leaving her child $x^{*\prime}$[15]. $x_1^{'}$ increases due to the increase of the threshold training cost. Following bequests, $(x_j^{'})_{j=2}^{+\infty}$, will not only increase because of the increase in x^*, but also because medium workers leave their offspring less bequests because of the increase in the educative cost. This implies that,

$$\upsilon_{t+1}^{'} = \upsilon_{t+1} + (1-\mu)\int_{x_1}^{x_1^{'}} D_\tau dx$$

It is straightforward to see, that the uneducated proportion of the society increases. The result derives in the fact that the threshold training cost rises and the bequests that medium educated workers leave to their offspring reduces.

In steady state,

$$\frac{\partial \upsilon_{ss}}{\partial \varepsilon}|_\tau > 0$$

An increase in the educative cost increases the proportion of uneducated individuals of each generation in the long run. Then, it clearly follows that the educated sector of the economy increases its proportion and then there will be more unemployment.

Regarding economy's output, there are two different forces that hit aggregate production. In the first place, there is a decrease in unemployment. In the second place, less individuals are part of the most productive sector of the economy.

$$\frac{\partial \mathcal{Y}_{ss}}{\partial \eta_s}|_\tau = -\frac{1}{2}\frac{\theta}{\alpha}\left(\frac{\partial \upsilon_{ss}}{\partial \varepsilon}|_\tau\right)[2\eta_s\mu + (1-\mu)\,\eta_m - 2\eta_{ne}]$$

The net result on output per capita depends on the gap between productivities. A sufficient condition for aggregate output to decrease is $\eta_s\mu > \eta_{ne}$.

Regarding economic inequality, it increases among educated individuals but decreases between the educated and uneducated sector of the economy. Educated individuals' income falls because of the higher educative cost. This reduces the inequality with uneducated individuals whose income does not

[15] Notice that this level is the x_1 defined earlier but for the case of $x^{*\prime}$. In this way, it is clear to see that if $x^{*\prime} > x^*$, then $x_1' > x_1$, $\frac{\partial x_1}{\partial \varepsilon} = a\frac{\partial x^*}{\partial \varepsilon} + \sum_{i=0}^{n-1} a^i > 0$.

change. Both the skill and medium workers' incomes fall in the same magnitude, so it is proportionally higher for the medium worker than for the skill.

$$\frac{\partial I_2^{ne}|_{ss}}{\partial \varepsilon}|_\tau = 0$$

$$\frac{\partial I_2^{m}|_{ss}}{\partial \varepsilon}|_\tau = \frac{\partial I_2^{s}|_{ss}}{\partial \varepsilon}|_\tau = \left(\frac{\partial \widehat{x}_e}{\partial \varepsilon} - 1\right)(1 + r_w) < 0$$

$$\frac{\partial R}{\partial \varepsilon} < 0$$

Then the result is a decrease in R. If the second period educative cost increases, conclusions are quite similar. The threshold training cost increases, $\frac{\partial x^*}{\partial \varepsilon_2} = \frac{1}{(r_b - r_w)}$, but in a lower proportion. The uneducated sector of the economy increases. There is less unemployment but less individuals are part of the most productive sector of the economy. The net effect on output depends on the gap between productivity of both sectors.

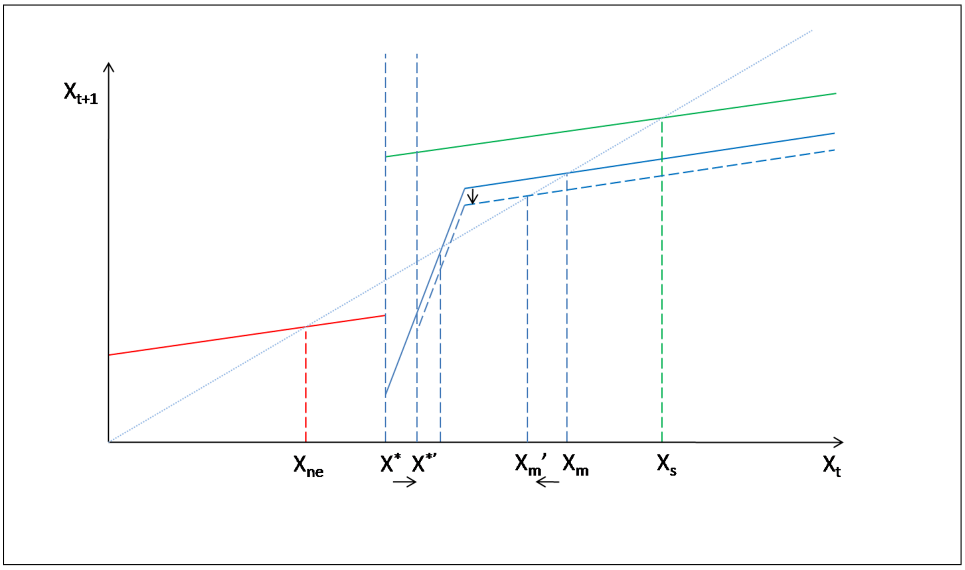

Figure 15. The economy after an increase in ε_2.

Inequality among educated workers increases more than in the previous case. This is so because skill workers only face the reduction of the bequest that they receive from their parents, meanwhile a medium worker faces this effect and an increase of the education that it takes during the second period of their life, $\frac{\partial I_2^s|_{ss}}{\partial \varepsilon_2}|_\tau = \frac{\partial \widehat{x}_e}{\partial \varepsilon}(1+r_w)$ and $\frac{\partial I_2^m|_{ss}}{\partial \varepsilon_2}|_\tau = \frac{\partial \widehat{x}_e}{\partial \varepsilon}(1+r_w) - 1$. Inequality between uneducated and educated workers decreases.

3.3.2 Educative Success

Government may decide to improve the country's education. They may change the programs that are taught in schools and universities. This may lead to a better educative program which rate of success is higher. In other words, more individuals obtain the skills that they were willing to obtain.

An increase in the educative rate of success will encourage individuals to educate themselves.

A increase in the educative rate of success will be represented by a change from μ to $\mu^{'}$, with $\mu^{'} > \mu$.

In this way, if the change occurs in the steady state ($\tau = +\infty$), then $\upsilon^{'}_{ss} = \upsilon_{ss}$, that is, as in the previous cases, there will be no impact on the proportion of uneducated inidividuals of each cohort. Despite this fact, there will be a reduction in unemployment. The changes in the aggregate product will be due to a reduction in unemployment and that more individuals are skill. Incomes of individuals will not change, but the mean income of an educated individual will increase due to the fact that the probability that her parent was skill is higher.

Regarding the threshold training cost,

$$\frac{\partial x^*}{\partial \mu} = -\frac{(2+r_w)(\theta\eta_s - \theta\eta_{ne}) - \varepsilon(1+r_w)}{(1-\mu)^2(r_b - r_w)} < 0$$

The threshold training cost falls due to the increase in the rate of success. If the shock occurs in finite time there will be alterations to the economy.

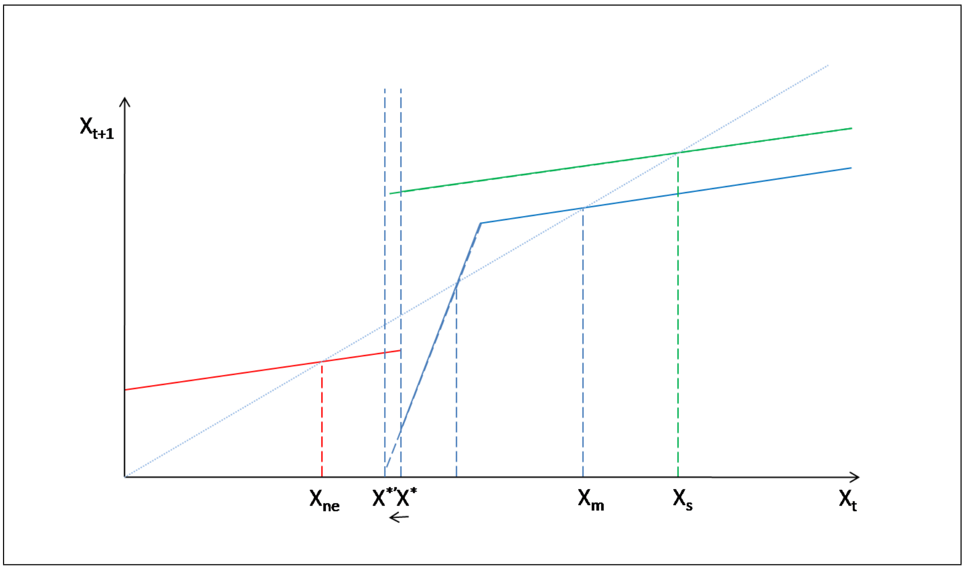

Figure 16. The economy after an increase in μ.

$x^j_{t+1}(x_t)$, $j = s,\ m,\ ne$, do not shift as all individuals' incomes do not change.

Those individuals who will decide to study in period $\tau + 1$ will be those whose parents have a level of bequest such that, x_{t+1} is higher than $x^{*\prime}$.

It is neccessary to introduce $x_1^{'}$ level of bequest which is the bequest that must have a medium worker for leaving her child $x^{*\prime}$ and its counterpart for the uneducated worker, $\widetilde{x}$. In the infinitesimal case, only $x_1^{'}$ is relevant. It decreases because the threshold training cost falls, $\frac{\partial x_1}{\partial \mu} = a\frac{\partial x^*}{\partial \mu} < 0$.

$$\upsilon_{t+1}^{'} = \upsilon_{t+1} - (1-\mu)\int_{x_1^{'}}^{x_1} D_\tau dx$$

It is straightforward to see, that the uneducated proportion of the society decreases. The result derives in the fact that the threshold training cost falls

In steady state,

$$\frac{\partial \upsilon_{ss}}{\partial \mu}|_\tau < 0$$

An increase in the success rate of the educative process decreases the proportion of uneducated individuals of each generation in the long run. Then, it clearly follows that the educated sector of the economy increases. The effect on unemployment it is not straightforward to see. Two opposite forces affect unemployment. On the one hand, unemployment rate among educated workers fall due to the increase in μ. On the other hand, more individuals are deciding to educate themselves which increases the proportion of individuals that might be unemployed if they do not success in the educative process. Formaly,

$$\frac{\partial Un_{ss}}{\partial \mu}|_\tau = -\frac{1}{2}\left[(1-\upsilon_{ss}) + (1-\mu)\frac{\partial \upsilon_{ss}}{\partial \mu}|_\tau\right]$$

$$\frac{\partial \upsilon_{ss}}{\partial \mu}|_\tau = D_{\tau+1}(x^*)\frac{\partial x^*}{\partial \mu} - \sum_{n=1}^{+\infty} n(1-\mu)^{n-1}\int_{x_{n-1}}^{x_n} D_{\tau+1}dx + \\ + \sum_{n=1}^{+\infty}(1-\mu)^n\left[D_{\tau+1}(x_n)\frac{\partial x_n}{\partial \mu} - D_{\tau+1}(x_{n-1})\frac{\partial x_{n-1}}{\partial \mu}\right]$$

$$\frac{\partial \upsilon_{ss}}{\partial \mu}|_\tau = D_{\tau+1}(x^*)\frac{\partial x^*}{\partial \mu} - \sum_{n=1}^{+\infty} n(1-\mu)^{n-1}\int_{x_{n-1}}^{x_n} D_{\tau+1}dx + \\ + \sum_{n=1}^{+\infty}(1-\mu)^n a^{n-1}\frac{\partial x^*}{\partial \mu}\left[D_{\tau+1}(x_n)a - D_{\tau+1}(x_{n-1})\right]$$

As it can be seen, the derivate of unemployment against the educative rate of success has two parts. On the one hand, those who would have been educated workers in the first place, now have a lower rate of unemployment. On the other hand, less individuals become uneducated; this can be seen in the derivative of the uneducated sector of the economy against μ. In this derivative, it shows the effect that the change in the success rate have in the economy. It changes the bequests that a parent must have so it leaves their child a sufficient bequest so he educates. This is captured by $\frac{\partial x_n}{\partial \mu}$. Not only the intervals are changed, but the probability that a medium worker's offspring finishes as uneducated is lowered.

Regarding economy's output, there are two different forces that hit aggregate production. In the first place, there is more product that comes from the educated sector of the economy, more educated individuals are skill. On the other hand, more individuals are part of the most productive sector of the economy.

$$\frac{\partial \mathcal{Y}_{ss}}{\partial \mu}|_{\tau} = \frac{1}{2}\frac{\theta}{\alpha}\left(\left(2\eta_s - \eta_m\right)\left(1 - \upsilon_{ss}\right) - \left(\frac{\partial \upsilon_{ss}}{\partial \mu}|_{\tau}\right)\left[2\eta_s \mu + \left(1 - \mu\right)\eta_m - 2\eta_{ne}\right]\right)$$

The net input on output per capita depends on the gap between productivities. A sufficient condition for aggregate output to increase is $\eta_s \mu > \eta_{ne}$.

Regarding economic inequality, it does not change among educated individuals but increases between the educated and uneducated sector of the economy. No ones income change, but the bequest that an educated individual receives increases.

$$\frac{\partial I_2^{ne}|_{ss}}{\partial \mu}|_{\tau} = 0$$

$$\frac{\partial I_2^{m}|_{ss}}{\partial \mu}|_{\tau} = \frac{\partial I_2^{s}|_{ss}}{\partial \mu}|_{\tau} = \frac{\partial \widehat{x}_e}{\partial \mu}(1 + r_w) > 0$$

$$\frac{\partial R}{\partial \mu} > 0$$

R increases not only because educated individuals receive a larger bequest but also because skill individual's income weights more in the average.

4 Conclusions

In this work, different shocks were examined in the theoretical framework developed in "Caramp, Espinosa, Melero and Szenig" (2009). The model is built on basically three major assumptions: capital market imperfections, indivisibilities in investment in human capital and uncertainty about the results of education. It has already been shown that the model acts as expected when a skill biased technological change occurs. It has been succesfull explaining qualitatively Argentinean experience during the 1990's. This work complements those findings. A set of comparative statistics - productivity boosts, changes in interest rate and educative reforms - were analyzed to see how the model reacts.

An increase in the uneducated worker's productivity leads to a decrease in the proportion of educated workers of the society by an increase in the threshold training cost. It reduces unemployment and inequality. A change in the medium worker's productivity increases the proportion of the educated individuals in the society as the threshold training cost decreases. Inequality between the educated and uneducated sector of the society increases.

Changes in interest rates also affect the economy's dynamics and output. An increase in the active interest rate that may be interpreted as a reduction in subsidized rates for students leads to a reduction of the educated sector of the economy as the threshold training cost increases and medium worker's income falls. Unemployment decreases and the effect on output depend on the gap between skill and uneducated individuals' productivities. An increase in the passive interest rate may decrease the threshold training cost as studying is less expensive. Literacy rate and unemployment increases.

Finally, a reduction in education cost to individuals leads to a reduction of the educated proportion of individuals. This reduces unemployment and the net effect on output depends on the gap of productivities. If the success rate of the educative process increases, the threshold training cost decreases. The proportion of educated individuals increases. Unemployment faces two different effects. On the one hand, unemployment rate among educated individuals decreases as the success rate increases. On the other hand, the amount of educated individuals in the society increases what may lead to an increase in the unemployment rate of educated individuals in the whole society.

To conclude, it is possible to study different governments´ policies may influence the economy's dynamics, unemployment rate and output through different policies. Getting involved in the financial market offering subsidized rates to students or doing a educative reform that may lead to a reduction in its cost or an increase in the success rate may affect the whole economy. This occurs as these policies change the individuals' optimal decisions and affect the whole dynamic.

The main effects that the different shocks have on the economy are the following (in the following chart, a big gap between the productivity of the skill and uneducated sector is assumed, $\eta_s\mu > \eta_{ne}$)[16]:

[16] For the case of the passive interest rate a larger gap in productivity has been assumed.

Shock	Uneducated Sector's Productivity (η_{ne})	Medium Sector's Productivity (η_m)	Active Interest Rate (r_b)	Passive Interest Rate (r_w)	Educative Cost (ε, ε_2)	Educative Success Rate (μ)
Educated Sector of the Economy	Decreases	Increases	Decreases	Increases	Decreases	Increases
Unemployment	Decreases	Increases	Decreases	Increases	Decreases	Depends on the Net Effect (more educated, higher success rate)
Aggregate Product	Depends on the Gap of Productivities	Increases	Decreases	Increases	Decreases	Increases
Society's Inequality	Decreases	Increases	Unchanged	Increases	Decreases	Increases

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