Maestría en Economía

## Dollarization and Default Risk

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# Dollarization and Default Risk 

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#### Abstract

I develop a small open economy model with money to study the interaction between dollarization and sovereign debt default. Dollarization requires a large open market operation by which the government buys back the whole stock of national currency using its international reserves or issuing foreign debt. Moreover, money demand is now a demand of US dollars that the economy cannot costlessly supply and has to be financed through trade surplus or foreign debt. I compute conditional default probabilities in the dollarized economy using Arellano (2008) as a benchmark and find that the effects of dollarization on default incentives are twofold. On one hand, holdings of dollars are actively used in the first periods after dollarization as a way to smooth consumption in default states where output is low and due debt is high. This makes the dollarized economy more prone to default. On the other hand, US dollars also act as a reserve asset to hedge against default risk and avoid default output costs. This effect is the sole surviving one in the long run which leads to dollarization eventually decreasing default incentives. I calibrate and solve the model for Ecuador which dollarized in 2000 and perform a counterfactual exercise that shows that dollarization increased Ecuador's default incentives in the last twenty years.


JEL codes: E21, E42, E43, E58, F33, F34, H63.
Keywords: dollarization, sovereign debt default

## 1 Introduction

Emerging countries that cannot commit to a low-inflation regime consider adopting a low-inflation foreign currency instead of their own as way of putting an end to chronic inflation. Dollarization is the adoption of the United States Dollar as a national currency in a country other than the United States of America. Ecuador and El Salvador are two emerging economies which became fully dollarized in the early 2000s. The literature on dollarization stresses the loss of seignorage as one of the costs of this shortcut to monetary stability (Fischer (1982)). However, dollarization requires a large open market operation by which the government buys back the whole stock of national currency using its international reserves or issuing foreign debt. If foreign debt is issued, dollarization causes a large initial increase in the stock of foreign debt of the small open economy. Moreover, money demand is now a demand of US dollars that the economy cannot costlessly supply and thus, the economy's budget constraint calls for real money demand growth to be financed through trade surpluses or foreign debt. Thus, dollarization and foreign debt dynamics appear to be intertwined.

In this paper, I study the effect of dollarization on default incentives in a small-open economy model with limited enforceability of foreign debt contracts and compare default incentives using an Arellano (2008) economy as a benchmark. I find that the effects of dollarization on default incentives are twofold. US dollars serve as a safe asset to smooth consumption in default states (as in Bulow and Rogoff (1989)) but at the same time, these dollars can be used as a reserve asset to hedge against default risk and avoid default output costs. The use of US dollars for consumption smoothing is particularly present in the short run which leads to dollarization increasing default incentives during its early years. The use of US Dollars as a reserve asset (akin to Bianchi et al. (2018)) is present in both the short and long run but is the sole surviving effect in the long run. I calibrate and solve the model using Ecuadorian data and find that dollarization increased Ecuador's default incentives. Ecuador defaulted in 2008 and 2020 and the model actually predicts two defaults (one soon after the dollarization and initial open market operation in 2003 and another one in 2020) while the counterfactual non-dollarized economy only captures the 2020 default.

I now introduce the conditional probabilities used to measure the effect of dollarization on default incentives. Default incentives in both the dollarized and non-dollarized economies can be thought of in terms of default sets which specify the levels of output for which each economy defaults, given levels of debt and real balances. From these default sets, I define two conditional probabilities which measure the extent to which dollarization increases and decreases default incentives with respect to the benchmark economy. The former is the probability that the dollarized economy defaults given that the non-dollarized economy does not default. The latter, on the other hand, is the probability that the dollarized economy does not default given that the non-dollarized economy defaults. These probabilities are a function of the state of the economy. I will show that when output is low and debt is high, these probabilities point to dollarization increasing default incentives. On the other hand, as output increases
default becomes costly and dollarization lowers default incentives.
The Ecuadorian Experience. This paper was motivated by the Ecuadorian dollarization which occurred in 2000. The balance of payments in a dollarized economy such as Ecuador can be written as

$$
\Delta M_{t+1}^{*}=T B_{t}+T R_{t}+\Delta N F L_{t}
$$

where $\Delta M_{t+1}^{*}, T B_{t}, T R_{t}$ and $\Delta N F L_{t}$ represent changes in US dollar cash balances, the trade balance, net transfers (remittances minus debt service) and increase in net foreign liabilities respectively. Figure 1 presents each component of the previous equation between 2000 and 2018 as well as the stock of US dollars in terms of GDP.


Figure 1: Balance of Payments and dollar cash balances for Ecuador in the 2000-2018 period. All variables are expressed in terms of GDP. Trade balance consists of net exports of goods and services. Increase in debt is taken from the capital account net of changes in the cash balances. Remittances minus debt service consists of rents and transfer income composing the current account balance. Cash balances are circulating banknotes among the public and liquid funds at depository institutions. Source: Central Bank of Ecuador.

The data show that cash balances in terms of output have increased in every year since the dollarization. In particular, the stock of US dollars in Ecuador increased approximately 13.5 percentage points of GDP in the 1999-2018 period. During these years, the trade balance was almost always in deficit except for four years (with the trade surpluses after 2000 being below one percentage point of output). Rents and transfers were significant only between 2005 and 2010 and hence only played a minor role in financing the increase in cash balances outside that time frame. Thus, the data supports the premise that the Ecuadorian dollarization was associated to the growth of foreign debt and in turn on default risk.

The paper is organised as follows: Section 2 presents the models and characterises the equilibria, Section 3 presents the calibrations and evaluates the quantitative implications of the model and Section 4 concludes.

## 2 The Model

Preferences. The economy is populated by identical infinitely-lived households with expected utility preferences defined over consumption $c_{t}$ and end-of-period- $t$ real balances $m_{t}$

$$
\begin{equation*}
\mathbb{E}_{0} \sum_{t=0}^{\infty} \beta^{t} u\left(c_{t}, m_{t}\right) \tag{1}
\end{equation*}
$$

where $\beta \in(0,1)$ is the discount factor. The period utility function $u: \mathbb{R}_{+}^{2} \rightarrow \mathbb{R}$ satisfies the Inada conditions, is strictly increasing and strictly concave in both arguments and $u_{12} \in \mathbb{R}$.

Endowment process. Every period the household receives a stochastic stream of a tradable good $y \in Y$ where $Y$ is compact. This endowment follows a first-order Markov process with conditional distribution $F\left(y^{\prime} \mid y\right)$. Let $\mathbb{P}_{y^{\prime} \mid y}$ denote the probability measure associated to it and $\mathbb{E}_{y^{\prime} \mid y}$ the respective conditional expectation.

Government. There exists a benevolent government that seeks to maximise the discounted expected utility of the households while having access to international financial markets. In these markets a one-period discount bond $b$ is traded at price $q$ which is endogenous. A contract is a set $\left\{b^{\prime}, q\right\}$ by which, if $b^{\prime} \geq 0$, the government saves $q b^{\prime}$ units of current period goods today and receives $b^{\prime}$ units of good next period. On the other hand, if $b^{\prime}<0$, the government receives $-q b^{\prime}$ units of current period goods today and must deliver $-b^{\prime}$ units of good the next period. The proceeds from these international credit operations are rebated in a lump-sum fashion to the households. In order to avoid Ponzi schemes on government debt, there is a lower bound on government debt such that $b^{\prime} \geq Z$ for a large negative $Z$.

Budget constraint. In a dollarized regime, the central bank ceases to be the supplier of money. However, the representative household continues to demand real balances which must now come from either trade surplus or foreign debt. Therefore, the economy's budget constraint is given by

$$
\begin{equation*}
c+m^{\prime}+q b^{\prime}=y+b+\frac{m}{\pi} \tag{2}
\end{equation*}
$$

where $\pi$ is the gross inflation rate. I assume that the domestic goods market is open to both foreign and domestic consumers so that arbitrage requires the domestic and US price level to be equal under dollarization. Moreover, a small dollarized economy such as this one takes the US price level as given, as its demand for real balances has no effect on the US money market equilibrium. Hence, inflation is also taken as given and is equal to that in the United States.

Default decision. Debt contracts are not enforceable in the sense that the economy can choose to fully default on its debt. In that case, the economy is excluded from financial markets and is forced to revert to autarky so that the budget constraint becomes

$$
\begin{equation*}
c+m^{\prime}=\ell(y)+\frac{m}{\pi} \tag{3}
\end{equation*}
$$

I assume that defaulting entails two costs: temporary exclusion from financial markets and an output cost. When the government defaults, there is an exogenous probability $\theta$ that
the economy will be allowed to reenter financial markets next period with zero initial debt, i.e. the recovery value of debt is zero. However, with probability $1-\theta$ it remains in autarky. In addition to financial autarky, default entails a one-time output loss of $y-\ell(y) \geq 0$ where $\ell^{\prime}>0$ so that the default cost is increasing in output.

Bond pricing. The international credit market consists of risk neutral foreign lenders who can borrow or lend as much as needed at a constant international risk-free gross interest rate $R>1$. These creditors have perfect information about the state of the economy every period and price defaultable bonds so as to break even in expected value in every contract. Thus, the bond price satisfies

$$
\begin{equation*}
q=R^{-1}(1-\delta) \tag{4}
\end{equation*}
$$

where $\delta$ is the endogenous probability of default. If creditors borrow so that $b^{\prime} \geq 0$ the probability of default is zero. The probability of default is endogenous and depends on the economy's incentives to repay which are captured by the bond price $q$. Also, $\delta \in[0,1]$ implies $q \in\left[0, R^{-1}\right]$. I denote by $R^{c} \equiv q^{-1}$ the country's gross interest rate and the interest rate spread as $R^{c}-R$.

Timing. Given $(b, m)$ and after observing $y$, the economy chooses to default $(d=1)$ or repay $(d=0)$
i) If it defaults, the economy chooses consumption $c$ and real balances $m^{\prime}$ subject to its budget constraint in default.
ii) If it repays, the economy chooses consumption $c$, real balances $m^{\prime}$ and bond holdings $b^{\prime}$ subject to the budget constraint and taking $q$ as given.
(a) Then, foreign lenders choose $b^{\prime}$ taking $q$ as given.
(b) The bond price $q$ adjusts for the bond market to clear

The economy's recursive problem. Every period, the government has the option to default or repay with value

$$
\begin{equation*}
v(b, m, y)=\max _{d \in\{0,1\}}(1-d) v^{r}(b, m, y)+d v^{d}(m, y) \tag{5}
\end{equation*}
$$

where $v^{r}(b, m, y)$ is the value of repayment and $v^{d}(m, y)$ is the value of default.
The value of default is given by

$$
\begin{align*}
v^{d}(m, y)= & \max _{c, m^{\prime}} u\left(c, m^{\prime}\right)+\beta \mathbb{E}_{y^{\prime} \mid y}\left[\theta v\left(0, m^{\prime}, y^{\prime}\right)+(1-\theta) v^{d}\left(m^{\prime}, y^{\prime}\right)\right]  \tag{6}\\
& \text { subject to } c=\ell(y)+\frac{m}{\pi}-m^{\prime}
\end{align*}
$$

The value of repayment is given by

$$
\begin{align*}
v^{r}(b, m, y)= & \max _{c, b^{\prime}, m^{\prime}} u\left(c, m^{\prime}\right)+\beta \mathbb{E}_{y^{\prime} \mid y} v\left(b^{\prime}, m^{\prime}, y^{\prime}\right)  \tag{7}\\
& \text { subject to } c=y+b-q b^{\prime}-m^{\prime}+\frac{m}{\pi}
\end{align*}
$$

The solution to the economy's recursive problem yields decision rules for default $\widehat{d}(b, m, y)$, bond holdings $\widehat{b}^{\prime}(b, m, y)$, consumption under repayment $\widehat{c}^{r}(b, m, y)$, consumption under default $\hat{c}^{d}(m, y)$, real balances under repayment $\widehat{m}^{\prime r}(b, m, y)$ and real balances under default $\widehat{m}^{\prime d}(m, y)$.

Bond pricing function. To be consistent with the lenders' problem, the equilibrium bond price must satisfy

$$
\begin{equation*}
q\left(b^{\prime}, m^{\prime}, y\right)=R^{-1} \mathbb{E}_{y^{\prime} \mid y}\left[1-\widehat{d}\left(b^{\prime}, m^{\prime}, y^{\prime}\right)\right] \tag{8}
\end{equation*}
$$

Definition 1 (Equilibrium). A Markov perfect competitive equilibrium is a set of

1. value functions $\hat{v}, \widehat{v}^{r}$ and $\hat{v}^{d}$
2. policy functions $\widehat{d}, \widehat{b}^{\prime}, \widehat{c}^{r}, \widehat{c}^{d}, \widehat{m}^{r}, \widehat{m}^{d}$, and
3. a bond pricing function $\hat{q}$
such that
4. the value functions solve the economy's recursive problem defined by equations (5)-(7)
5. the policy functions are the economy's optimal choices
6. the bond pricing function satisfies no-arbitrage under risk neutrality and zero recovery value defined by equation (8)

Default and repayment sets. From the value functions (5) - (7) government policy can be characterised by default and repayment sets. Denote $\mathcal{R}(b, m)$ the subset of $y \in Y$ such that repayment is optimal when assets are $b$ and real balances are $m$ while $\mathcal{D}(b, m)$ is the complement of $\mathcal{R}(b, m)$. Then

$$
\begin{align*}
\mathcal{R}(b, m) & =\left\{y \in Y: v^{r}(b, m, y) \geq v^{d}(m, y)\right\}  \tag{9}\\
\mathcal{D}(b, m) & =\left\{y \in Y: v^{r}(b, m, y)<v^{d}(m, y)\right\} \tag{10}
\end{align*}
$$

where I assume that the economy repays upon indifference between repayment and default.
The definition of default and repayment sets allow to express the default probability as

$$
\begin{equation*}
\delta\left(b^{\prime}, m^{\prime}, y\right)=\mathbb{P}_{y^{\prime} \mid y}\left[y^{\prime} \in \mathcal{D}\left(b^{\prime}, m^{\prime}\right)\right]=\int_{\mathcal{D}\left(b^{\prime}, m^{\prime}\right)} d F\left(y^{\prime} \mid y\right) \tag{11}
\end{equation*}
$$

### 2.1 The non-transactional demand for money

From now on, I assume differentiability of the bond price and value functions and refer the reader to Clausen and Strub (2016) for further details on this matter.

The demand for real balances within this model can be thought to have more than a transactional motive. In particular, real balances act as a reserve asset that can be used to smooth consumption in default states. To see this, consider the first order conditions with respect to bond holdings and real balances that can be reduced to

$$
\begin{equation*}
\frac{u_{m^{\prime}}\left(c, m^{\prime}\right)}{\beta \mathbb{E}_{y^{\prime} \mid y} u_{c}\left(c^{\prime}, m^{\prime \prime}\right)}=\frac{1+\partial_{m^{\prime}} q\left(b^{\prime}, m^{\prime}, y\right) b^{\prime}}{q\left(b^{\prime}, m^{\prime}, y\right)+\partial_{b^{\prime}} q\left(b^{\prime}, m^{\prime}, y\right) b^{\prime}} \frac{\mathbb{E}_{y^{\prime} \mid y}\left[u_{c}\left(c^{\prime}, m^{\prime \prime}\right)\left(1-d^{\prime}\right)\right]}{\mathbb{E}_{y^{\prime} \mid y} u_{c}\left(c^{\prime}, m^{\prime \prime}\right)}-\frac{1}{\pi} \tag{12}
\end{equation*}
$$

where $\partial_{x} y(x) \equiv \partial y(x) / \partial x$ and $m^{\prime \prime}$ is the optimal choice of real balances given $\left(b^{\prime}, m^{\prime}, y^{\prime}\right)$. In what follows, I consider the case of $b^{\prime}<0$ so that the economy is a net debtor.

The previous optimality condition can be compared to the following one in a model with full enforceability of debt contracts and exogenous bond price

$$
\begin{equation*}
\frac{u_{m^{\prime}}\left(c, m^{\prime}\right)}{\beta \mathbb{E}_{y^{\prime} \mid y} u_{c}\left(c^{\prime}, m^{\prime \prime}\right)}=\frac{1}{q}-\frac{1}{\pi} \tag{13}
\end{equation*}
$$

The right-hand side of Equation (13) reflects the opportunity cost of holding real balances and depends on the inverse of the bond price and inflation rate which can be written as the associated nominal interest rate. The right-hand of Equation (12) also reflects the opportunity cost of holding real balances but in a model where debt bonds are defaultable and their price is endogenous, it can be seen that there are two additional effects: a general equilibrium one which incorporates changes in prices and another one that depends on how the economy weighs the repayment costs when debt can be defaulted which depends on the expected marginal utility of consumption.

The first factor on the right-hand side of Equation (12) reflects the effective real interest rate after considering endogenous changes in the bond price that arise as the default probability varies. To see this, recall Equation (8) which determines the bond price in equilibrium. This factor can in principle be greater or lesser than $1 / q$ and this depends on the derivatives of the bond pricing function with respect to bonds and real balances. If $q\left(b^{\prime}, m^{\prime}, y\right)>\frac{\partial_{b^{\prime}} q\left(b^{\prime}, m^{\prime}, y\right)}{\partial_{m^{\prime}} q\left(b^{\prime}, m^{\prime}, y\right)}$, then this factor is lesser than $1 / q$ and thus the effective real interest rate falls after considering the changes in default probabilities which translate to changes in the bond price.

However, notice that the effective real interest rate is weighted by the relative marginal utility in repayment states. This factor is lesser than unity and it lowers the marginal cost of holding real balances. The idea is that when the economy borrows to finance an increase in its dollar holdings, it can default on due debt next period and use this safe asset to smooth consumption. This provides an incentive to accumulate dollars that goes beyond transactional motives. Money demand incorporates the fact that the economy's only asset to smooth consumption in default states are dollar holdings. And it uses relative marginal utility as weight. This non-transactional motive grows stronger as default becomes more likely. First, this would directly increase the probability of default but also, it indirectly makes expected marginal utility higher since consumption is low in default states.

Thus, the model generates a money demand that also captures non-transactional motives
which are quantitatively important as I will show in the following section.

## 3 Quantitative Analysis

### 3.1 Computation

The model is solved numerically with discrete-state dynamic programming. I force the vector of states $(b, m, y)$ to lie on finite and discrete grids of points. Then the policy and value functions are obtained via value function iteration. The endowment process is discretized following Tauchen (1986). The curse of dimensionality is particularly present in this model which requires to keep the number of grid points relatively small. Thus, I use linear interpolation to approximate value and bond pricing functions in a finer grid.

### 3.2 Calibration and Functional Forms

The utility function is of the constant elasticity of substitution type

$$
u\left(c, m^{\prime}\right)=\frac{\left[\kappa c^{1-\psi}+(1-\kappa) m^{\prime 1-\psi}\right]^{\frac{1-\sigma}{1-\psi}}}{1-\sigma}
$$

where $\sigma, \psi>0$ and $\kappa \in(0,1)$. The output cost of default is given by $\ell(y)=\min \{\gamma \mathbb{E} y, y\}$ where $\gamma \in(0,1)$. This specification implies that for levels of income $y \leq \gamma \mathbb{E} y$ defaulting entails no output loss but it does for levels of income $y>\gamma \mathbb{E} y$ and this loss increases proportionately with $y$.

The stochastic process of the endowment is assumed to follow a stationary first-order autoregressive process in logs

$$
\log y^{\prime}=\rho \log y+\varepsilon^{\prime}
$$

with $|\rho|<1$ and $\left\{\varepsilon^{\prime}\right\}$ is an independent and identically distributed process with $\mathbb{E}\left[\varepsilon^{\prime}\right]=0$ and $\mathbb{E}\left[\varepsilon^{\prime 2}\right]=\eta_{\varepsilon}^{2}$.

Table 1 presents the values of the parameters and calibration used to obtain the quantitative results. Preference parameters $\psi$ and $\kappa$ are calibrated by fitting the money demand function generated by these CES preferences to the data. In a standard model, these preferences generate the log-log money demand function

$$
\log m^{\prime}=\frac{1}{\psi} \log \left(\frac{1-\kappa}{\kappa}\right)-\frac{1}{\psi} \log \left(\frac{i}{1+i}\right)+\log c
$$

This is the typical form of money demand used in empirical money demand equations. Using data from Ecuador I pinpoint the parameters $\kappa$ and $\psi$ by means of a constrained linear regression model on Ecuadorian M1, nominal interest rate on demand deposits and GDP between 2000Q2-2019Q4. The results are very similar if M2 is used instead of M1 as well as if
household consumption expenditure is used instead of GDP. In particular, they are consistent with the finding of Benati et al. (2021).

| Parameter | Description | Value | Source/target |
| :---: | :--- | :--- | :--- |
| $R$ | Risk free interest rate | 1.01 | Bianchi et al. (2018) |
| $\psi$ | Elasticity of money demand | 2.41 | Money demand estimation |
| $\kappa$ | Consumption relative weight | 0.997 | Money demand estimation |
| $\pi$ | US inflation | 1.0051 | Average US quarterly inflation |
| $\rho$ | Persistence of output | 0.9823 | Ecuador's linearly detrended GDP |
| $\eta_{\varepsilon}$ | Standard deviation of output | 0.0107 | Ecuador's linearly detrended GDP |
| $\theta$ | Probability of re-entry | 0.28 | Arellano (2008) |
| $\sigma$ | IES | 2 | Bianchi et al. (2018) |
| $\beta$ | Discount factor | 0.97 | Bianchi et al. (2018) |
| $\left(n_{b}, n_{m}, n_{y}\right)$ | Grid sizes | $(60,60,60)$ |  |
|  |  |  |  |
|  | $\quad$ Calibrated parameters |  |  |
| $\gamma$ | Output loss | 0.9 | Relative volatility of consumption |

Table 1: Parameters and Calibration

### 3.3 The hedging role of money in a dollarized economy

In order to analyze the hedging role of money, consider a fixed consumption target $\bar{c}$ and denote by $x \equiv\{b, m, y, \bar{c}\}$ the vector of states and target consumption. To sustain $\bar{c}$, real balances $m^{\prime}$ and assets $b^{\prime}$ must be such that

$$
\begin{equation*}
q\left(b^{\prime}, m^{\prime}, y\right) b^{\prime}+m^{\prime}=y-\bar{c}+b+\frac{m}{\pi} \tag{14}
\end{equation*}
$$

Denote by $\widetilde{b}\left(m^{\prime}, x\right)$ the amount of assets that satisfy the budget constraint (14) when the economy demands $m^{\prime}$ and the state is $x$. The debt issuance required to accumulate a unit of $m^{\prime}$ with consumption constant is

$$
\begin{equation*}
\partial_{m^{\prime}} \widetilde{b}\left(m^{\prime}, x\right)=-\frac{1+\partial_{m^{\prime}} q\left(\widetilde{b}\left(m^{\prime}, x\right), m^{\prime}, y\right) \widetilde{b}\left(m^{\prime}, x\right)}{q\left(\widetilde{b}\left(m^{\prime}, x\right), m^{\prime}, y\right)+\partial_{b^{\prime}} q\left(\widetilde{b}\left(m^{\prime}, x\right), m^{\prime}, y\right) \widetilde{b}\left(m^{\prime}, x\right)} \tag{15}
\end{equation*}
$$

Consider the problem of choosing combinations of debt and real balances that deliver the same level of current consumption. This problem is described by

$$
\begin{equation*}
\max _{m^{\prime} \geq 0} u\left(\bar{c}, m^{\prime}\right)+\beta \mathbb{E}_{y^{\prime} \mid y} v\left(\widetilde{b}\left(m^{\prime}, x\right), m^{\prime}, y^{\prime}\right) \tag{16}
\end{equation*}
$$

The first order condition of equation (16) with respect to $m^{\prime}$ together with the BenvenisteScheinkman condition yield

$$
\begin{equation*}
u_{m^{\prime}}\left(\bar{c}, m^{\prime}\right)+\frac{1}{\pi} \beta \mathbb{E}_{y^{\prime} \mid y} d^{\prime} u_{c}\left(c^{\prime}, m^{\prime \prime}\right)=-\left[\partial_{m^{\prime},} \widetilde{b}\left(m^{\prime}, x\right)+\frac{1}{\pi}\right] \beta \mathbb{E}_{y^{\prime} \mid y}\left(1-d^{\prime}\right) u_{c}\left(c^{\prime}, m^{\prime \prime}\right) \tag{17}
\end{equation*}
$$

The left-hand side of equation (17) is the marginal benefit of accumulating an extra unit
of dollars. In particular, the second term captures a Bulow and Rogoff (1989) effect of dollar balances serving as a safe asset to smooth consumption in default states. As for the righthand side, it reflects the marginal cost of this transaction which is that in repayment states, the economy must pay the debt it issued to finance dollar accumulation.

To understand the meaning of $\partial_{m^{\prime}} \widetilde{b}\left(m^{\prime}, x\right)$ I present Figure 2 which shows that derivative as a function of the output for different levels of debt. It illustrates how the bond is pricing the increase in default incentives when issuing debt to finance real money demand when debt is high and output is low. Notice that, for a given level of $b$, low levels of output call for a more than one-to-one increase in debt to accumulate an additional unit of real balances. Moreover, this increase is larger if initial debt is higher. However, as the economy benefits from higher levels of output, default incentives fall and the economy can eventually borrow at the international risk-free rate.


Figure 2: Partial derivative $\partial_{m^{\prime}} \widetilde{b}\left(m^{\prime}, x\right)$ for given levels of debt $b$ and output $y$

### 3.4 Impact of dollarization on default incentives

I will analyze default incentives between a dollarized and a non-dollarized benchmark built on Arellano (2008). To do this, I will define two conditional probabilities of default. But first, let $\mathcal{D}^{D}(b, m)$ be the default set in the dollarized economy just as in Equation (10). And let $\mathcal{D}^{N D}(b)$ be the default set in the non-dollarized economy as defined in Arellano (2008). I define the repayment sets $\mathcal{R}^{D}(b, m)$ and $\mathcal{R}^{N D}(b)$ analogously.

I now define the following conditional probabilities which are a measure of the effects of dollarization on default incentives using the non-dollarized economy as a benchmark

$$
\begin{aligned}
& p_{+}\left(b^{\prime}, m^{\prime}, y\right) \equiv \mathbb{P}_{y^{\prime} \mid y}\left[y^{\prime} \in \mathcal{D}^{D}\left(b^{\prime}, m^{\prime}\right) \mid y^{\prime} \in \mathcal{R}^{N D}\left(b^{\prime}\right)\right]=\frac{\int_{\mathcal{D}^{D}\left(b^{\prime}, m^{\prime}\right) \cap \mathcal{R}^{N D}\left(b^{\prime}\right)} d F\left(y^{\prime} \mid y\right)}{\int_{\mathcal{R}^{N D}\left(b^{\prime}\right)} d F\left(y^{\prime} \mid y\right)} \\
& p_{-}\left(b^{\prime}, m^{\prime}, y\right) \equiv \mathbb{P}_{y^{\prime} \mid y}\left[y^{\prime} \in \mathcal{R}^{D}\left(b^{\prime}, m^{\prime}\right) \mid y^{\prime} \in \mathcal{D}^{N D}\left(b^{\prime}\right)\right]=\frac{\int_{\mathcal{R}^{D}\left(b^{\prime}, m^{\prime}\right) \cap \mathcal{D}^{N D}\left(b^{\prime}\right)} d F\left(y^{\prime} \mid y\right)}{\int_{\mathcal{D}^{N D}\left(b^{\prime}\right)} d F\left(y^{\prime} \mid y\right)}
\end{aligned}
$$

The probability measure defined by $p_{+}\left(b^{\prime}, m^{\prime}, y\right)$ is the probability of default in the dollarized economy given that the non-dollarized economy would repay. This probability measures the effect of dollarization in increasing incentives to default. On the other hand, $p_{-}\left(b^{\prime}, m^{\prime}, y\right)$ is the probability of repayment in the dollarized economy given that the non-dollarized economy would default. This probability measures the effect of dollarization in decreasing incentives to default.

Figure 3 presents these probabilities as a function of assets $b^{\prime}$ and real balances $m^{\prime}$ for two different levels of output: the upper panel corresponds to an output level $5 \%$ below trend while the lower panel to an output level $10 \%$ above trend. When output is low (upper panel) dollarization can have two different effects on default incentives. If debt is low enough, dollarization decreases default incentives and the economy avoids having to bear default output costs. However, when debt goes above around $20 \%$ of mean output dollarization increases default incentives. Dollarization provides a safe asset to the dollarized economy that it uses to smooth consumption in default states which is absent in the non-dollarized economy. On the other hand, when high levels of output are consider (lower panel) the only effect of dollarization is to decrease default incentives relative to the benchmark economy. This is because as output increases, default becomes relatively more costly and dollarization enables the economy to use dollar holdings as a reserve asset to repay debt and avoid these default output costs.


Figure 3: Conditional probabilities of defaulting $p_{+}\left(b^{\prime}, m^{\prime}, y\right)$ and $p_{-}\left(b^{\prime}, m^{\prime}, y\right)$. Plots for $p_{+}=$ $P\left[y^{\prime} \in \mathcal{D}^{D}\left(b^{\prime}, m^{\prime}\right) \mid y^{\prime} \in \mathcal{R}^{N D}\left(b^{\prime}\right)\right]$ and $p_{-}=P\left[y^{\prime} \in \mathcal{R}^{D}\left(b^{\prime}, m^{\prime}\right) \mid y^{\prime} \in \mathcal{D}^{N D}\left(b^{\prime}\right)\right]$ as a function of debt $b^{\prime}$ and real balances $m^{\prime}$ for different levels of output. The upper panel corresponds to an output level $5 \%$ below trend while the lower panel to an output level $10 \%$ above trend.

### 3.5 Simulation

In this section, I simulate the model by feeding it the output process of Ecuador's linearly detrended output and initial asset and real balances positions. Table 2 presents business cycle statistics computed from the data and their model counterparts. The model is capable of reproducing most of the stylized facts of emerging economies presented in Neumeyer and Perri (2005). In particular, consumption is more volatile than income, interest rate spreads are negatively correlated with income and so is the trade balance.

|  | $s d(x)$ |  | $\rho(x, y)$ |  | $\rho\left(x, R^{c}-R\right)$ |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Data | Model | Data | Model | Data | Model |
| Interest rate spread | 5.74 | 5.04 | -0.23 | -0.28 | 1 | 1 |
| Trade balance | 1.18 | 1.10 | -0.08 | -0.04 | -0.40 | 0.09 |
| Consumption | 5.26 | 3.14 | 0.91 | 0.96 | -0.21 | -0.29 |
| Output | 4.92 | 2.94 | 1 | 1 | -0.23 | -0.28 |
| Other statistics | 1.07 | 1.07 |  |  |  |  |
| $s d(c) / s d(y)$ |  |  |  |  |  |  |

Table 2: Business cycle statistics for Ecuador. $\operatorname{sd}(x)$ is standard deviation, $\rho(x, y)$ is correlation with output and $\rho\left(x, R^{c}-R\right)$ is correlation with interest rate spread. All statistics were computed between 2000Q2-2019Q3. The interest rate spread is J.P. Morgan's EMBI spread deflacted by expected GDP deflator, in p.p. Trade balance is in \% GDP. Consumption and output are \% deviations from a linear trend of the log of real private consumption and GDP. The data for consumption, output and trade balance were obtained from the CBE.

### 3.5.1 Short run: role of output and initial open market operation in the first 20 years after dollarization

To understand the role of initial conditions in the short run, I simulate 20.000 draws of 80 quarters each for the output's Markov chain and set initial assets to match Ecuador's net asset position in 2000Q1. By construction, these simulations focus on the first 20 years after dollarization. From these draws I compute sample probabilities of default in the dollarized economy conditional on the non-dollarized not defaulting as well as the probability of repayment in the dollarized economy conditional on the non-dollarized defaulting. These are the sample analogues of $p_{+}\left(b^{\prime}, m^{\prime}, y\right)$ and $p_{-}\left(b^{\prime}, m^{\prime}, y\right)$.

In particular, I find that in the short run dollarization has both positive and negative effects on default incentives. In particular, the sample values for $p_{+}$and $p_{-}$are 0.15 and 0.23 , respectively. This means that, conditional on the non-dollarized economy not defaulting, the dollarized economy defaults, on average, $15 \%$ the time. On the other hand, conditional on the non-dollarized economy defaulting, the dollarized economy repays, on average, $23 \%$ the time. In $33 \%$ of the 20.000 draws did dollarization produce more default events than in the counterfactual economy.

This exercise shows that, in the short run, dollarization has quantitatively significant but opposite effects on default incentives. It increases them since the economy now use dollars
as an instrument to smooth consumption in default states, making default relatively more attractive. But at the same time, it decreases default incentives since the economy can now use dollar holdings to pay debt and avoid default costs.

### 3.5.2 Long run: 50 years after dollarization

In order to understand the long-run effects of dollarization, I compute 1.000 draws of 5.000 quarters each where I drop the first 200 quarters ( 50 years) after dollarization. Thus, these simulations attempt to capture the dynamics well after dollarization has occurred and are clear of any short-run dynamic induced by the initial market operation or subsequent demand for dollars of the first years.

I again compute sample conditional probabilities of default in the dollarized and nondollarized regimes. In the long-run, the average probability of default in the dollarized economy conditional on repayment in the non-dollarized economy (sample long-run $p_{+}$) is 0.006 with a $99 \%$ confidence interval of $(0.002,0.016)$. On the other hand, the average probability of repayment in the dollarized economy conditional on default in the non-dollarized economy (sample long-run $p_{-}$) is 0.43 with a $99 \%$ confidence interval of ( $0.11,0.77$ ). In only $0.1 \%$ of all draws did dollarization produce more default events than in the counterfactual economy.

These sample long run probabilities are consistent with the idea that, well after the first years after dollarization, the economy begins to accumulate dollars which act as a safe asset to hedge against future default risks which are costly in terms of output. For instance, in the longrun, the dollarized has a relative volatility of consumption to output of 1.07 while in the nondollarized economy this relative volatility is 1.17 . The only difference in these simulations that can generate such a difference in the relative volatility of consumption is that the dollarized economy is using dollars to smooth consumption in default states as well as to avoid defaulting if output is not too low.

### 3.5.3 Ecuador: counterfactual exercise

In this quantitative exercise, I simulate the model but with a Markov chain that resembles the output of Ecuador during the 2000Q1-2019Q4 period. The initial values of asset holdings and cash balances are set to match their data counterparts in 2000Q1. This Markov chain for output plus initial conditions on net assets and money are fed to the policy functions of the model. With this, I compute series for the relevant variables and default status.

Figure 4 presents these simulations for both the dollarized and non-dollarized economy. Shaded regions represent quarters in which the economy defaulted. First, both the model and its benchmark capture the Ecuadorian default in early 2020. But the model also predicts a default shortly after dollarization in 2003. Ecuador defaulted in 2008 and 2020. Thus, the model accurately captures the 2020 default and predicts another one in the early 2000s but before it actually occurred. One possible explanation for the model anticipating the 2008 default is the lack of long-term debt in the model.

The lower panel of Figure 4 presents the corresponding ex-ante default probability for each economy. The dollarized economy has, on average, a lower ex-ante default probability than the non-dollarized economy. In particular, notice that slightly before 2003, the default probability for the non-dollarized economy spikes to slightly less than $2.5 \%$ while in the dollarized economy it remains close to 0 . This stands in stark contrast with the default decision in this period: the dollarized economy defaults while the non-dollarized economy does not. This highlights the non-transactional role of money demand in the model. Upon an unexpected negative output shock, the dollarized economy defaults and uses its dollar holdings to smooth consumption during the default period (see that consumption remains above output in this period). On the other hand, since the non-dollarized economy has no alternative but to reduce to autarky if it defaults, it chooses to repay the debt at the cost of lowering present consumption but avoiding default costs in terms of output and financial autarky. Notice that, overall, this has implications on the volatility of consumption relative to output: it is 1.16 in the dollarized economy while 1.27 in the non-dollarized economy.


Figure 4: Ecuadorian economy and non-dollarized counterfactual. Simulation of Ecuador under dollarization and non-dollarization. The Markov chain for output is constructed to match Ecuador's linearly detrended $\log$ GDP in 2000Q2-2019Q4. Initial assets and real balances are taken from World Bank Database.

## 4 Conclusion and future research

In this paper, I have addressed the missing link between dollarization and sovereign default in emerging economies. I developed a small open dollarized economy model which was contrasted with the predictions of a benchmark sovereign default model (Arellano (2008)).

I defined two conditional probabilities which attempt to provide a measure of how dollarization can increase or decrease default incentives. In particular, the model predicts that dollarization can have diverse effects on default incentives. On one hand, it can increase default incentives in the sense that it provides a safe asset that the economy can now use to smooth consumption in default states. But on the other hand, it also provides an asset that can be accumulated to hedge against future default risk and avoid bearing default output costs. In this sense, dollarized real balances are akin to international reserves (see Bianchi et al. (2018)).

The quantitative exercise consisted of calibrating the model for Ecuador and performing simulation exercises that capture the importance of the effects of dollarization on default incentives in the short and long run. The effect of dollarization on increasing default incentives is active in the short run, especially in the first 20 years after dollarization. In the years following the dollarization, the economy uses the stock of US dollars to smooth consumption in default states. Meanwhile, in the long run, the surviving effect of dollarization is that of decreasing default incentives as dollarization serves as a hedge against default risk. In a final counterfactual exercise, I show that if the model is fed with Ecuador's GDP and initial asset position, it predicts Ecuador's two defaults while the benchmark model does not. One of them occurs shortly after dollarization and the other one in 2020. However, the benchmark model of Arellano (2008) fails to capture the first default that occurs shortly after 2000.

### 4.1 Future research

The current version of the paper uses Arellano (2008) as a benchmark for the non-dollarized economy. However, this counterfactual could be misleading since it does not provide the economy with an asset that could be used to smooth consumption in default states. I am currently working on a model with a national currency and international reserves which serves as a more natural benchmark since in the dollarized economy money plays the role of reserves. Moreover, the benefit of having a national currency is that I will be able to make a meaningful welfare analysis of dollarization that takes into account the welfare cost of high inflation.

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