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A polyhedral study of a relaxation of the routing and spectrum allocation problem

(Brief Announcement)

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Abstract

The *routing and spectrum allocation (RSA) problem* arises in the context of flexible grid optical networks, and consists in routing a set of demands through a network while simultaneously assigning a bandwidth to each demand, subject to non-overlapping constraints. One of the most effective integer programming formulations for RSA is the DR-AOV formulation, presented in a previous work. In this work we explore a relaxation of this formulation with a subset of variables from the original formulation, in order to identify valid inequalities that could be useful within a cutting-plane environment for tackling RSA. We present basic properties of this relaxed formulation, we identify several families of facet-inducing inequalities, and we show that they can be separated in polynomial time.

Keywords: routing and spectrum allocation; integer programming; facets; separation

Given (i) a directed graph $G = (V, E)$ representing an optical fiber network, (ii) a set of demands $D = \{(s_i, t_i, v_i)\}_{i=1}^k$, sorted by v_i in descending order, such that each demand $(s_i, t_i, v_i) \in D$ has a source $s_i \in V$, a target $t_i \in V$, and a volume $v_i \in \mathbb{Z}_+$, and (iii) a fixed number $\bar{s} \in \mathbb{Z}_+$ of available slots, the *routing and spectrum allocation (RSA) problem* consists in determining a *lightpath* for each demand $(s_i, t_i, v_i) \in D$ (i.e., a path in G from the source node s_i to the target node t_i together with an interval of v_i consecutive slots in $\{1, \ldots, \bar{s}\}$ that forms a channel) in such a way that the channels of any two demands that share an arc do not overlap. For any $i = (s_i, t_i, v_i) \in D$, we define $s(i) = s_i$, $t(i) = t_i$, and $v(i) = v_i$.

Various integer programming approaches have been studied in the literature in order to solve RSA (see, e.g., [1, 2, 3, 4, 5, 6]). In [7], several integer programming models for RSA have been presented and assessed by computational experiments. It turned out that the integer programming model called the *DR-AOV formulation* showed the best

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performance among all evaluated models. In this formulation, for each demand $i \in D$ and each arc $e \in E$, the binary variable y_{ie} represents whether the demand i is routed along the arc e or not, thus defining a path in G between $s(i)$ and $t(i)$. For each pair of demands $i, j \in D$, $i \neq j$, the binary variable x_{ij} takes value 1 if i and j share an edge and if the channel assigned to i is located before the channel assigned to j. These x-variables define a partial linear ordering on the channels. Finally, for each demand $i \in D$, the integer variable ℓ_i represents the slot number of the first slot assigned to i, so i uses the channel $[\ell_i, \ell_i + v(i)]$. Since a polyhedral analysis of the DR-AOV formulation is not straightforward, in this work we propose to study the following relaxed formulation focused on the spectrum assignment:

$$\ell_i + v(i) \leq \ell_j + (\bar{s} - v(i))(1 - x_{ij}) \qquad \forall i, j \in D, i \neq j, \tag{1}$$

$$x_{ij} + x_{ji} \leq 1 \qquad \forall i, j \in D, i \neq j, \tag{2}$$

$$0 \leq \ell_i \leq \bar{s} - v(i) \qquad \forall i \in D. \tag{3}$$

We define the polyhedron $RP(D, \bar{s})$ as the convex hull of all solutions $(\ell, \mathbf{x}) \in \mathbb{Z}^{|D|} \times \{0, 1\}^{|D|^2 - |D|}$ satisfying constraints (1)-(3).

Theorem 1. *The polyhedron $RP(D, \bar{s})$ has full dimension $|D|^2$ if and only if $\bar{s} \geq v(i) + v(j)$ for every $i, j \in D$, $i \neq j$.*

Theorem 2. *For every valid (resp. facet-defining) inequality $\mathbf{a}^T(\ell, \mathbf{x}) \leq b$ of $RP(D, \bar{s})$, the inequality $\mathbf{a}^T(\bar{\ell}, \bar{\mathbf{x}}) \leq b$, obtained by replacing ℓ_i by $\bar{s} - v(i) - \ell_i$ and x_{ij} by x_{ji}, is also valid (resp. facet-defining) for $RP(D, \bar{s})$.*

Theorem 3. *If $\bar{s} \geq v(i) + v(j)$ for every $i, j \in D$, $i \neq j$, then*

1. *the anti-parallelity constraints (1) define facets of $RP(D, \bar{s})$,*
2. *the 2-cycle constraints (2) define facets of $RP(D, \bar{s})$, and*
3. *the non-negativity constraint $x_{ij} \geq 0$ defines a facet of $RP(D, \bar{s})$, for $i, j \in D$, $i \neq j$.*

We are interested in finding families of valid inequalities for $RP(D, \bar{s})$, motivated by the fact that any valid inequality for this polytope will also be valid for the DR-AOV formulation for RSA. Facetness results do not translate directly from one polytope to the other one, but still may provide hints on the strength of the identified inequalities, so in this work we also tackle this issue.

Let $i \in D$ and $A = \{j_1, \ldots, j_t\} \subseteq D \backslash \{i\}$ with $j_{l-1} < j_l$ for $l = 2, \ldots, t$. We define

$$\ell_i \geq \sum_{j \in A} \alpha_j x_{ji}, \tag{4}$$

where $\alpha_{j_l} = v(j_l)$ if $l = 1$ and $\alpha_{j_l} = v(j_l) - v(j_{l-1})$ otherwise for $l = 2, \ldots, t$ (recall that the demands are sorted in decreasing order of their volumes, so $\alpha_{j_l} \geq 0$), to be the *left telescopic inequality* associated with i and A. The *right telescopic inequality* is defined similarly by Theorem 2.

Theorem 4. *The left/right telescopic inequalities are valid for $RP(D, \bar{s})$, and can be separated in $O(|D|^3)$ time. For $i \in D$ and $A \subseteq D \backslash \{i\}$, the inequality (4) defines a facet of of $RP(D, \bar{s})$ if*

- *$\bar{s} > v(i) + v(A)$,*
- *$A \neq \emptyset$,*
- *the volumes of all the demands $j \in A$ are different, and*
- *either i is a demand with maximum volume (over all demands in D) and A contains a demand with maximum volume over all demands in $D \setminus \{i\}$, or A contains a demand with maximum volume (over all demands in D).*

Let $i, j \in D$, $i \neq j$, and $A = \{j_1, \ldots, j_t\} \subseteq D \backslash \{i, j\}$ with $j_{l-1} < j_l$ for $l = 2, \ldots, t$. We define

$$\ell_j - (\ell_i + v(i)) \geq \sum_{k \in A} \alpha_k (x_{ik} - x_{kj} - 1) - (\bar{s} - v(i) - v(k))(1 - x_{ij}) \tag{5}$$

where $\alpha_{j_l} = v(j_l)$ if $l = 1$ and $\alpha_{j_l} = v(j_l) - v(j_{l-1})$ otherwise for $l = 2, \ldots, t$, to be the *middle telescopic inequality* associated with i, j, and A. This inequality captures the fact that if the demand i is allocated before the demand j (so the last term is null), then the difference between ℓ_i and ℓ_j can be bounded by the volumes of the demands in A located between i and j.

Theorem 5. *The middle telescopic inequalities are valid for $RP(D, \bar{s})$, and can be separated in $O(|D|^4)$ time. For $i, j \in D$, $i \neq j$, and $A \subseteq D \backslash \{i, j\}$, the inequality (5) defines a facet of $RD(D, \bar{s})$ if*

- $\bar{s} > v(i) + v(j) + v(A)$,
- $A \neq \emptyset$,
- *the volumes of all the demands $j \in A$ are different, and*
- *A contains a demand with maximum volume.*

Let $i, j \in D$, $i \neq j$ and $A = \{j_1, \ldots, j_t\} \subseteq D \backslash \{i, j\}$ with $j_{l-1} < j_l$ for $l = 2, \ldots, t$. We define

$$\ell_i \geq (v(j) + v(j_1))x_{ji} + \sum_{k \in A} \alpha_k x_{kj} - v(j_1) \tag{6}$$

where $\alpha_{j_l} = v(j_l)$ if $l = 1$ and $\alpha_{j_l} = v(j_l) - v(j_{l-1})$ otherwise for $l = 2, \ldots, t$, to be the *left reinforced telescopic inequality* associated with i, j, and A. The *right reinforced telescopic inequality (RRTI)* is defined similarly by Theorem 2.

Theorem 6. *The left/right reinforced telescopic inequalities are valid for $RP(D, \bar{s})$, and can be separated in $O(|D|^4)$ time. For $i, j \in D$, $i \neq j$, and $A \subseteq D \backslash \{i, j\}$, the inequality (6) defines a facet of $RD(D, \bar{s})$ if*

- $\bar{s} > v(i) + v(j) + v(A)$,
- $A \neq \emptyset$,
- *the volumes of all the demands $j \in A$ are different, and*
- *A contains a demand with maximum volume.*

Let $A = \{j_1, \ldots, j_t\} \subseteq D$. We define

$$x_{j_t j_1} + \sum_{l=1}^{t-1} x_{j_l j_{l+1}} \leq t - 1 \tag{7}$$

to be the *k-cycle inequality* associated with A.

Theorem 7. *The k-cycle inequalities are valid for $RP(D, \bar{s})$, and can be separated in polynomial time. Furthermore, the inequality (7) defines a facet of $RD(D, \bar{s})$ if $\bar{s} \geq \sum_{i \in D} v(i)$.*

As a future work, it would be interesting to identify further families of facet-inducing inequalities and to explore under which hypotheses the facetness results translate to the original formulation. On the practical side, it would be relevant to assess the practical contribution of these inequalities within a cutting-plane environment for solving RSA.

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