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A polyhedral study of the berth allocation problem with tides

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Abstract

The *berth allocation problem* models the spatial and temporal allocation of ships into berth space in container terminals. In this work we are interested in the discrete version of this problem, in which the container terminal is viewed as a set of atomic berths. We are particularly interested in modeling the existence of tides, in such a way that ships can be moved in/out of the container terminal only in high-tide periods. We introduce a natural extension of the standard formulation for the discrete berth allocation problem that considers this feature, and we perform a polyhedral exploration of this formulation. We present valid inequalities involving the variables that model high-tide periods, we explore conditions ensuring that these inequalities induce facets of the associated polytopes, and we present computational experiments showing that the reinforcement of the formulation with some of these inequalities has a better performance with a general integer programming solver.

Keywords: berth allocation problem; tides; integer programming; polyhedral combinatorics

1. Introduction

The *berth allocation problem* (BAP) is a combinatorial optimization problem arising in maritime logistics, where the goal is to efficiently assign berthing spaces to incoming ships at a port [3, 4]. Given the constraints of limited quay space, varying ship arrival times, and different service priorities, port operators must determine an optimal berth schedule that minimizes delays, reduces operational costs, and maximizes port efficiency. The problem is further complicated by factors such as tidal constraints, cargo handling requirements, and ship size variations. All realistic variants of the berth allocation problem give rise to NP-hard problems. In this work we are interested in the *static version* of the berth allocation problem, in which all ships are known in advance. We furthermore assume that ships are restricted to predefined slots along the quay (called *berths* in this context), in what is known as the *discrete version* of this problem.

Tides play a crucial role in the berth allocation problem, particularly in ports where water depth fluctuates significantly throughout the day [1, 5, 8]. Some ships, especially those with deep drafts, can only berth or depart during specific tidal windows when the water depth is sufficient to accommodate them safely. Ignoring tidal constraints can

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lead to inefficient berth assignments, where ships may be scheduled to dock or leave at times when it is physically impossible, causing delays and disruptions in port operations.

In this work we are interested in the modeling of constraints imposed by high tides within integer programming formulations. In particular, we are interested in exploring how such constraints affect the modeling and whether such a model can be reinforced with valid inequalties. We propose in Section 2 a combinatorial optimization problem capturing the main aspects of high-tide consideration within the berth allocation problem, and a natural mixed integer programming formulation for this problem. In Section 3 we explore the facetness properties of the model constraints and we present families of valid inequalities for this formulation. Finally, in Section 4 we report preliminary computational experiments with these inequalities.

2. The berth allocation problem with tides

We now introduce the *berth allocation problem with tides* (BAPT). Let $N = \{1, \ldots, n\}$ be the set of ships, let $M = \{1, \ldots, m\}$ be the set of berths, and let $T = \{1, \ldots, p\}$ be the set of high-tide periods. For each $i \in N$, we define $a_i \in \mathbb{R}_+$ to be the attention time of the ship i. For each $t \in T$, we define $s_t, e_t \in \mathbb{R}_+$ to be the start and end time of the high-tide period t, respectively. We assume $s_t < e_t$ for $t \in T$, and $e_t < s_{t+1}$ for $t \in T$, $t < p$. An instance of BAPT is given by the tuple $\mathcal{I} = (N, M, T, a, s, e)$.

Given an instance $\mathcal{I}$ of BAPT, the objective is to assign a berth, a starting time, and an ending time to each ship, so that (a) the length of the interval assigned to each ship is greater than or equal to its attention time, (b) the intervals of no two ships assigned to the same berth overlap, (c) the starting and ending time of each ship belongs to some high-tide period, and (d) the makespan (i.e., the maximum ending time) is minimized. BAPT is clearly NP-hard, since it includes the classical BAP as a particular problem.

The consideration of high-tide periods adds an interesting constraint to BAP, by incorporating the requirement that all ship intervals start (respectively, end) within one of the pre-specified high-tide periods. Once an assignment of ships to berths has been determined, BAP reduces to the classical interval coloring problem [7] on a *conflicts graph* given by the union of disjoint cliques, each clique corresponding to ships assigned to the same berth. To the best of our knowledge, the additional constraint imposed by BAPT has not been addressed previously in the interval coloring literature.

In order to present a mixed integer programming formulation for BAPT, we introduce the following variables. For $i \in N$ and $k \in M$, the binary variable x_{ik} takes value 1 if the ship i is assigned to the berth k. For $i \in N$, the continuous variables l_i and r_i represent the start and the end of the allocated interval for the ship i, respectively. For $i, j \in N$, $i \neq j$, the binary variable y_{ij} takes value 1 if the ships i and j are assigned to the same berth and furthermore $r_i < l_j$, i.e., the ship i is assigned to the berth before the ship j. For $i \in N$ and $t \in T$, the binary variables wl_{it} and wr_{it} take value 1 if $l_i \in [s_t, e_t]$ and $r_i \in [s_t, e_t]$, respectively. Finally, the continuous variable z represents an upper bound on the makespan. Define $\mu := e_p - s_1$ to be the available scheduling interval. In this setting, the following mixed integer formulation models BAPT.

$$\min \ z \tag{1}$$

$$\sum_{k \in M} x_{ik} = 1 \qquad \forall i \in N \tag{2}$$

$$\sum_{t \in T} wl_{it} = 1 \qquad \forall i \in N \tag{3}$$

$$\sum_{t \in T} wr_{it} = 1 \qquad \forall i \in N \tag{4}$$

$$r_i \geq l_i + a_i \qquad \forall i \in N \tag{5}$$

$$r_i \leq l_j + \mu(1 - y_{ij}) \qquad \forall i, j \in N,\ i \neq j \tag{6}$$

$$r_i \leq z \qquad \forall i \in N \tag{7}$$

$$x_{ik} + x_{jk} - 1 \leq y_{ij} + y_{ji} \qquad \forall i, j \in N,\ i \neq j,\ \forall k \in M \tag{8}$$

$$l_i \geq s_t - \mu(1 - wl_{it}) \qquad \forall i \in N,\ \forall t \in T \tag{9}$$

$$l_i \le e_t + \mu(1 - wl_{it}) \qquad \forall i \in N,\ \forall t \in T \tag{10}$$
$$r_i \ge s_t - \mu(1 - wr_{it}) \qquad \forall i \in N,\ \forall t \in T \tag{11}$$
$$r_i \le e_t + \mu(1 - wr_{it}) \qquad \forall i \in N,\ \forall t \in T \tag{12}$$
$$x_{ik} \in \{0,1\} \qquad \forall i \in N,\ \forall k \in M \tag{13}$$
$$wl_{it},\ wr_{it} \in \{0,1\} \qquad \forall i \in N,\ \forall t \in T \tag{14}$$
$$y_{ij} \in \{0,1\} \qquad \forall i,j \in N,\ i \ne j \tag{15}$$

The objective function (1) asks to minimize the total makespan. Constraints (2) require every ship to be assigned to exactly one berth, whereas constraints (3) and (4) ensure that the starting and ending time of each ship belongs to exactly one high-tide period. Constraints (5) guarantee that the interval allocated to each ship is greater than or equal to its attention time. Constraints (6) ensure that $r_i \le l_j$ whenever $y_{ij} = 1$, and constraints (7) bind the r-variables with the variable z. Constraints (8) states that if any two ships i and j are assigned to the same berth, then either $y_{ij} = 1$ or $y_{ji} = 1$. Constraints (9)-(12) guarantee that the starting and ending time of each ship belongs to the high-tide period assigned by the wl- and wr-variables, respectively. Finally, constraints (13)-(15) impose the domains for the variables.

3. Polyhedral exploration

We define $P_{\text{BAPT}}(\mathcal{I})$ to be the convex hull of the vectors (x, y, wl, wr, l, r, z) satisfying constraints (2)-(15). For each $i \in N$, define $\sigma_i := \max\{t \in T : wl_{it} = 1$ for some $(x, y, wl, wr, l, r, z) \in P_{\text{BAPT}}(\mathcal{I})\}$ and $\tau_i := \min\{t \in T : wr_{it} = 1$ for some $(x, y, wl, wr, l, r, z) \in P_{\text{BAPT}}(\mathcal{I})\}$. Calculating these parameters may be nontrivial. However, if there exists a solution using at most $m - 1$ berths, then $\sigma_i = \max\{t \in T : e_t \le e_p - a_i\}$ and $\tau_i = \min\{t \in T : s_t \ge s_1 + a_i\}$.

Theorem 1. *If there exists a solution using at most $m - 1$ berths and such that any two ships can be allocated to the remaining berth in any order, then constraints (2)-(4) together with the equations*

$$wl_{it} = 0 \qquad \text{for } i \in N \text{ and } t \in T,\ t > \sigma_i,$$
$$wr_{it} = 0 \qquad \text{for } i \in N \text{ and } t \in T,\ t < \tau_i$$

are a minimal equation system for $P_{BAPT}(\mathcal{I})$.

We omit the proofs in this extended abstract due to space limits. We refer to the hypothesis in Theorem 1 as the *dimensionality hypothesis*. This hypothesis ensures that the only equations satisfied by every point in $P_{\text{BAPT}}(\mathcal{I})$ are the trivial equations. It is interesting to note that the dimensionality hypothesis is trivially satisfied if there is a large enough number of high-tide periods, which is usually the case in a practical setting.

We now explore the facetness properties of the model constraints. Constraints (5) do not induce facets of $P_{\text{BAPT}}(\mathcal{I})$ in general, since solutions in the faces induced by these constraints have an exact separation between the high-tide periods containing the interval bounds for the associated ship, and this may imply additional equalities for the wl- and wr-variables. As an example, assume $s_t = 10t$ and $e_t = 10t + 2$ for every $t \in T$. If $a_i = 31$ for some $i \in N$, then $wl_{it} = wr_{i,t+3}$ for every $t \in T$, $t \le p - 3$, in any feasible solution, thus making it impossible for (5) to induce a facet of $P_{\text{BAPT}}(\mathcal{I})$ if the dimensionality hypothesis holds.

A similar argument shows that constraints (6) do not induce facets in general. In this case, any solution satisfying the constraint (6) associated with $i, j \in N$ will have $wr_{it} = 0$ for any $t \in T$ with $s_t > e_p - a_j$, and $wl_{it} = 0$ for any $t \in T$ with $e_t < a_i$. Again, if the dimensionality hypothesis holds then these equations do not allow the face induced by (6) to be a facet of $P_{\text{BAPT}}(\mathcal{I})$. Also, constraints (7) do not induce facets if the dimensionality hypothesis holds, since $y_{ij} = 0$ for every $j \in N$, $j \ne i$, in every solution satisfying (7) with equality.

Proposition 1. *If there exists a solution using at most $m - 2$ berths and such that any two ships can be allocated to one of the remaining berths in any order, then constraints (8) induce facets of $P_{BAPT}(\mathcal{I})$.*

The model constraints (9)-(12) do not induce facets in general, but can be reinforced in such a way that the resulting inequalities are indeed facet-inducing, as the following results show.

Proposition 2. *The inequalities*

$$l_i \geq s_1 + \sum_{t=2}^{p} (s_t - s_1) wl_{it}, \tag{16}$$

$$l_i \leq e_1 + \sum_{t=2}^{p} (e_t - e_1) wl_{it}, \tag{17}$$

$$r_i \geq s_1 + \sum_{t=2}^{p} (s_t - s_1) wr_{it}, \tag{18}$$

$$r_i \leq e_1 + \sum_{t=2}^{p} (e_t - e_1) wr_{it} \tag{19}$$

are valid for $P_{BAPT}(\mathcal{I})$, for any $i \in N$.

Theorem 2. *If there exists a solution using at most $m-2$ berths and such that any two ships can be allocated to one of the remaining berths in any order, then the inequalities (16)-(19) induce facets of $P_{BAPT}(\mathcal{I})$.*

Theorem 3. *Fix $i \in N$. If the dimensionality hypothesis holds, then the projection of $P_{BAPT}(\mathcal{I})$ onto the variables $\{l_i\} \cup \{wl_{it}\}_{t=1}^{p}$ is given by (16)-(17) and the constraints*

$$\sum_{t=1}^{p} wl_{it} = 1$$
$$wl_{it} \geq 0, \quad t = 1, \ldots, p$$

We now explore families of valid inequalities not necessarily arising from reinforcements of the model constraints. For $i \in N$ and $t \in T$, define $\delta_{it} \in T$ to be the smallest $t' \in T$ such that there exists a solution with $wl_{it} = 1$ and $wr_{it'} = 1$. If the dimensionality hypothesis holds, then $\delta_{it} := \min\{r \in T : s_r \geq s_t + a_i\}$.

Proposition 3. *Let $i \in N$ and $t \in T$. The inequality*

$$\sum_{t'=t}^{\sigma_i} wl_{it'} \leq \sum_{t'=\delta_{it}}^{p} wr_{it'} \tag{20}$$

is valid for $P_{BAPT}(\mathcal{I})$.

Theorem 4. *Let $i \in N$ and $t \in T$ such that $t \leq \sigma_i$. If there exists a solution using at most $m-2$ berths and such that any two ships can be allocated to one of the remaining berths in any order, then the inequality (20) associated with i and t induces a facet of $P_{BAPT}(\mathcal{I})$.*

Theorem 5. *Fix $i \in N$. If the dimensionality hypothesis holds, then the projection of $P_{BAPT}(\mathcal{I})$ onto the variables $\{wl_{it}, wr_{it}\}_{t=1}^{p}$ is given by the inequalities (20) and the constraints*

$$\begin{aligned} wl_{it} &= 0 && \text{for } t \in T,\ t > \sigma_i, \\ wr_{it} &= 0 && \text{for } t \in T,\ t < \tau_i, \\ wl_{it} &\geq 0 && \text{for } t \in T,\ t \leq \sigma_i, \\ wr_{it} &\geq 0 && \text{for } t \in T,\ t \geq \tau_i, \\ \sum_{t \in T} wl_{it} = \sum_{t \in T} wr_{it} &= 1. \end{aligned}$$

The inequalities presented in this work are not sufficient in order to characterize the projection of $P_{\text{BAPT}}(\mathcal{I})$ onto the variables $\{l_i, r_i\} \cup \{wl_{it}, wr_{it}\}_{t=2}^{p}$ in general.

Proposition 4. *Let $t \in T$ and suppose that $e_t - s_t < a_i$ for every $i \in I$. The inequalities*

$$\sum_{i \in N} wl_{it} \leq m, \tag{21}$$

$$\sum_{i \in N} wr_{it} \leq m \tag{22}$$

are valid for $P_{BAPT}(\mathcal{I})$.

Although these inequalities might be effective in order to reinforce the formulation, they do not induce facets in general. For example, if the attention time of every ship spans more than two high-tide periods, then any solution satisfying (21) with equality will have $wl_{i,t+1} = 0$ for every $i \in N$, thus precluding the facetness of (21) if the dimensionality hypothesis holds. A symmetric observation applies to (22).

Proposition 5. *Let $i, j \in N$, $i \neq j$, and let $t \in T$ with $\tau_i \leq t \leq \sigma_j$. The inequality*

$$wr_{it} + y_{ij} + \sum_{t'=\sigma_j+1}^{p} wr_{it'} \ \leq \ 1 + \sum_{t'=t}^{\sigma_j} wl_{jt'} \tag{23}$$

is valid for $P_{BAPT}(\mathcal{I})$.

Theorem 6. *Let $i, j \in N$, $i \neq j$, and let $t \in T$ with $\tau_i \leq t \leq \sigma_j$. If there exists a solution using at most $m - 2$ berths and such that any two ships can be allocated to one of the remaining berths in any order, then the inequality (23) associated with i, j, and t induces a facet of $P_{BAPT}(\mathcal{I})$.*

A similar result is obtained by considering the symmetric inequality to (23). In this case, let $i, j \in N$, $i \neq j$, and let $t \in T$ with $\tau_j \leq t \leq \sigma_i$. The inequality

$$wl_{it} + y_{ji} + \sum_{t'=1}^{\tau_j - 1} wl_{it'} \ \leq \ 1 + \sum_{t'=\tau_j}^{t} wr_{jt'} \tag{24}$$

is valid for $P_{\text{BAPT}}(\mathcal{I})$, and is facet-inducing if the hypotheses of Theorem 6 hold.

4. Computational experiments

We report in this section preliminary computational experiments in order to evaluate the contribution of the reinforced constraints and valid inequalities presented in Section 3 to the solution of BAPT with an integer programming solver. To this end, we implemented the formulation (1)-(15) in the ZIMPL modeling language [6] and used SCIP 8.0.4 [2] as an integer programming solver. All the experiments reported in this section were run in a computer with and Intel Core i7-10700 CPU with 8 cores running at 2.90 GHz, and 32 GB of RAM memory.

Given the number n of ships, the number m of berths, the number p of high-tide periods, and a parameter $f \in \mathbb{R}_{>0}$, we generate instances with the following procedure. Each ship has an attention time of $5 + r$ hours, where $r \in \mathbb{Z}_+$ is randomly generated with uniform distribution in the interval $[0, 20]$. Each high-tide period has a duration randomly generated as a real number in the interval $[0, f]$ with uniform distribution, and is separated from the previous high-tide period by an amount of hours randomly generated in the interval $[8, 12]$. The parameter f controls the "tightness" of the constraints imposed by high tides, and we expect the instances to become harder as the value of f gets closer to zero.

The first set of experiments intends to evaluate the contribution of each of the families of reinforced constraints and valid inequalities presented in Section 3. Figure 1 presents the running time of the formulation reinforced with several subsets of these families of valid inequalities divided by the running time of the formulation (1)-(15), i.e., the formulation with no additional reinforcements. This coefficient has a value strictly less than 1 if and only if the addition of the corresponding family of valid inequalities decreases the running time to optimality. The figure presents a boxplot showing these coefficients for 80 instances with $n \in \{10, \ldots, 16\}$, $m = 6$, $p = 7$, and generated with

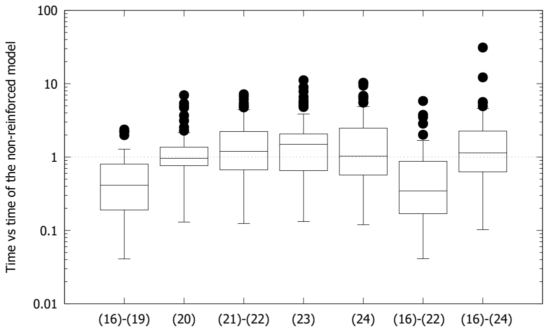

Fig. 1. Boxplot of the running time to optimality of the formulation reinforced with the families of valid inequalities specified in each boxplot label divided by the time to optimality of the formulation (1)-(15), for 80 randomly-generated instances with $n \in \{10, \ldots, 16\}$, $m = 6$, $p = 7$, and generated with $f \in \{0.1, 0.2, \ldots, 1.5\}$. Vertical axis in logarithmic scale and with no associated units, since it measures the relative time with respect to the non-reinforced model.

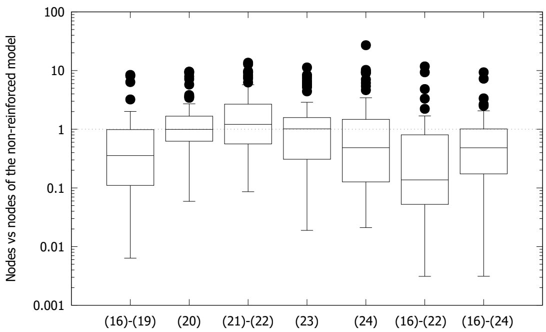

Fig. 2. Boxplot of the number of nodes in the enumeration tree when solving the formulation reinforced with the families of valid inequalities specified in each boxplot label divided by the number of nodes when solving the formulation (1)-(15), for 80 randomly-generated instances with $n \in \{10, \ldots, 16\}$, $m = 6$, $p = 7$, and generated with $f \in \{0.1, 0.2, \ldots, 1.5\}$. Vertical axis in logarithmic scale and with no associated units, since it measures the relative number of nodes with respect to the non-reinforced model.

$f \in \{0.1, 0.2, \ldots, 1.5\}$. The labels associated with each boxplot correspond to the equation numbers of the families of valid inequalities, as introduced in Section 3. As this figure shows, the addition of the inequalities (16)-(19) reinforcing constraints (9)-(10) provides the single most important improvement to the overall running times. These experiments suggest that the valid inequalities (23) and (24) are not effective in this context, as the addition of all the families of valid inequalities but these two families provides the best results.

Similarly, Figure 2 reports the number of nodes in the enumeration tree when solving to optimality the same instances as in the previous analysis, relative to the number of required nodes to solve the initial formulation (1)-(15). Again, the addition of all the families of valid inequalities with the exception of (23) and (24) provides the best results, the average number of nodes being only 14% of the average number of nodes for the formulation with no reinforcements. It is interesting to note that the number of nodes when all the inequalities are added to the formulation is substantially decreased, although the running time is larger. This suggests that solving each linear relaxation is harder, and in this sense the inequalities (23) and (24) may be the most problematic inequalities.

We now turn our attention to evaluating the running time to optimality as a function of the number n of ships, the number m of berths, and the number p of tides. To this end, Figure 3 presents measurements over 600 instances varying these parameters. In each subfigure, the running time is divided by the running time corresponding to the execution for the value immediately before the first value in the horizontal axis, in order to normalize across different instances. Figure 3(a) shows that the time to optimality mildly increases as n increases, although for $n = 16$ most of the instances could not be solved within the time limit of 5 minutes (note that the vertical axis in this figure does not

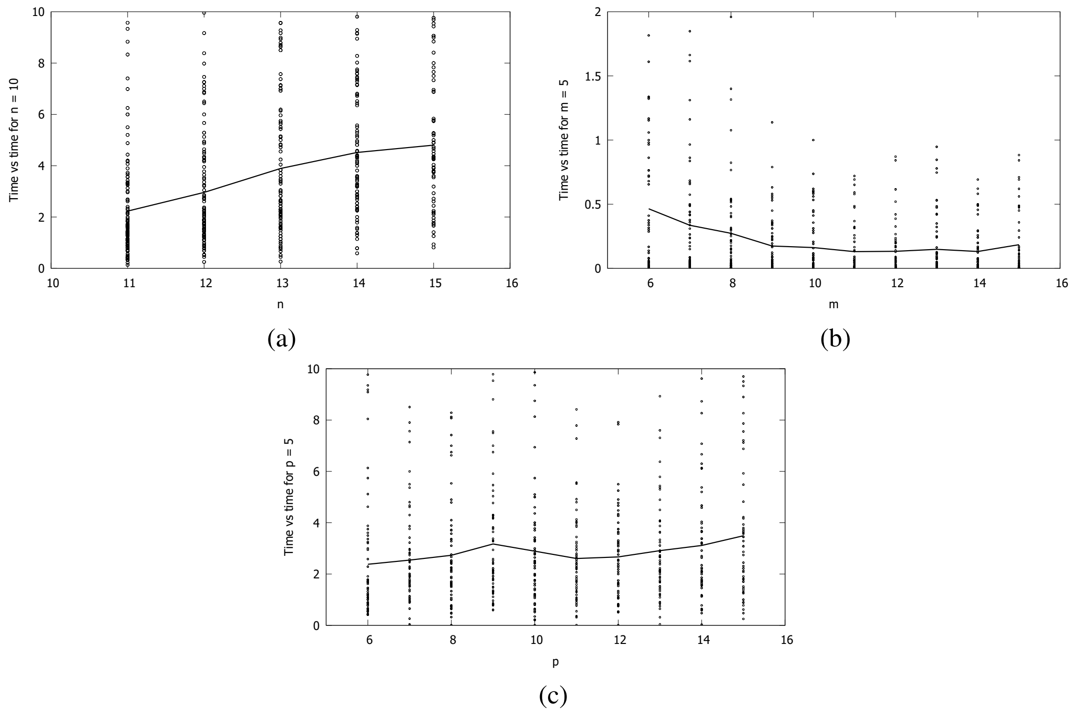

Fig. 3. Normalized running times to optimality as a function of (a) the number n of ships, (b) the number m of berths, and (c) the number p of high-tide periods. All subfigures report relative running times for 600 instances generated by taking $n \in \{10, \ldots, 15\}$, $m \in \{5, \ldots, 15\}$, $p \in \{5, \ldots, 15\}$, and $f = 0.5$. The normalization of running times is performed by dividing each measurement by the measurement obtained for the instance corresponding to the value immediately before the first value in the horizontal axis, while keeping the remaining parameters unchanged. Solid lines report the average running times, and each dot represents an individual measurement.

provide absolute time measurements, but the relation of the execution time with the execution time for $n = 10$ instead). Figure 3(b) shows that the running time to optimality decreases as the number of berths incresases, although at some point this measurement stabilizes and tends upwards due to the size of the resulting formulations. Finally, Figure 3(c) shows that the number p of high-tide periods has a mild negative impact on the running times to optimality.

As mentioned before, the smallest instances that cannot be solved with optimality have $n = 16$ ships. In this case, it is interesting to explore the optimality gap of the different reinforcements of the initial formulation. Figure 4 reports the optimality gap of the formulation (1)-(15) (solid line at the top), and the optimality gap of the reinforcements considered in Figure 1 (circles). Solid circles correspond to the addition of the families (16)-(22) to the initial formulation, which generate an average optimality gap of 16.82%, the best among all reinforcements when compared to the average optimality gap of 34.29% for the initial formulation. For these instances, the addition of the valid inequalities reinforcing the initial formulation has an interesting impact on the optimality gap, although it does not allow to solve most of these instances with optimality.

5. Concluding remarks

In this work we have started a polyhedral study of a natural formulation of the berth allocation problem with tides, focusing on valid inequalities coming from the variables representing the compliance of the constraints imposed by high-tide periods. Our computational results show that these inequalities may be helpful in order to reinforce the formulation, although further explorations should be conducted in order to deepen the knowledge on the associated polytope and to understand the best way to incorporate these theoretical results to a cutting plane environment.

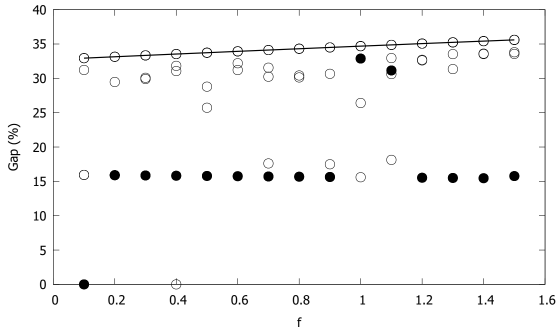

Fig. 4. Optimality gap of the formulation (1)-(15) with (circles) and without (solid line) the valid inequalities identified in Section 3. Solid circles correspond to the addition of the families (16)-(22) to the initial formulation. These experiments were conducted with 100 instances having $n = 16$, $m = 7$, $p = 6$, and $f \in \{0.1, 0.2, \ldots, 1.5\}$.

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