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The Thirlwall's Model of Economic Growth using State Space representations and time-variying parameter estimation through the Kalman filter. Theory and empirical application for El Salvador, 1965-2010.

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Index

Abstract[1]

The Thirlwall Law is a series of Post Keynesian postulations that allow to explain the long-term economic growth between different nations as a result of the ratio of the income elasticity of demand for exports and imports. The case of El Salvador is explored in this research and the methodological approach consists of casting the basic Thirlwall Law equation into state-space representations to estimate a Time-varying Parameter Model that is estimated through the implementation of an algorithm known as the Kalman filter.

[1] **Key words:** State-Space representations, Kalman filter, Thirlwall Model

Introduction

This thesis work centers its attention in understanding, from a Post Keynesian point of view, which are the main factors that have influenced the economic growth rate of El Salvador during the period of 1965 to 2010. The theoretical model that is developed in this research is commonly known as the Thirlwall Law. Its relevance in this scenario resides in the fact that it places demand as the main force that configures the dynamic of the economies and its link to the external sector.

With this framework of analysis, the estimation is performed through the specification of time-variying coefficients in the form of a State-Space representation and the application of the Kalman filter algorithm to obtain a series of the price and demand elasticities of imports and exports.

The first chapter covers the theoretical aspects of the Thirlwall Law; the second section explores the theoretical aspects of the State-Space representations and the Kalman filter algorithm. The third section examines the empirical evidence using data of El Salvador in the period of 1965-2010. The reader will find in the fourth section the conclusions. As a final section the implementation codes for the estimation are posted.

Finally, it is important to mention that a diversity of papers such as: Pugno (1998), Krishna (2002), Setterfield (2011), Garate et al. (2011), among others, explore theorethical and empirical applications of the Thirlwall Law. However it is necessary to mention that the only study that offers empirical evidence casting these models in State-Space representations and applying the Kalman filter for this Real Business Cycle model, was found in Pavel (2008).

Chapter I.

Understanding divergent growth between nations with the Thirlwall's Model of Economic Growth

The relationship can be found between international trade policies and economic growth in the long term can be studied from different points of view, so it results necessary to define the theory that assesses how the opening trade may or may not be a strategically desired policy for the performance of economies.

This chapter discusses the Thirlwall Law, which explains the long-term economic growth between different nations as a result of the ratio of the income elasticity of demand for exports and imports. This subject is presented with a first section that incorporates demand as the driving force of the economies, using as a starting point the ideas of Keynes seizable in Thirlwall’s postulations.

1.1.Demand as the driving force of the economy

The Keynesian theory establishes that demand, as the unique empowering force of economic growth, should be encouraged in order to achieve full employment. In that sense, variables such as investment, government spending and exports are assumed to rule the government aggregate activity.

As part of the Postkeynesian breakthroughs, Thirlwall contributes to the study of economic growth putting its attention on the external sector, by developing what is currently known as the Thirlwall’s Law. The author explains about this matter: “My particular contribution is to try to put demand back as a driving force in the growth theory” (Thirlwall, 2003).

1.2. Economic growth and trade imbalances between nations

Trade between nations is perceived as an essential practice to their economic growth and dynamization. This is logically due to the need of expansion that markets eventually reach for their products and the diversity of needs that usually cannot be met by the local production.

1.2.1. Free trade and the growing divergence between nations

From the conventional economic perspective[2], opening borders to foreign trade policies through the elimination of tariff and non-tariff practices, enlisting the country into trade agreements, whether these are unilateral, bilateral and/or multilateral, constitutes a requirement for economic growth. These policies have been promoted by the main international organizations, such as the Inter-american Development Bank (IADB), the International Monetary Fund (IMF) and the World Bank (WB), among others:

> [The aim is to] "increase international trade, particularly exports, and increase the efficiency of the allocation of domestic resources as a mean to support sustainable growth faster in the long run, and thus raise living standards and employment "(World Bank, 1989, cited in Abrego, 1991).

Nevertheless, it is well known that some countries grow faster than others, so in terms of per capita income levels they rapidly take distance from each other, generating diverging growth and leaving as a result some economies better positioned than the rest.

[2] Absolute Advantage (Adam Smith), Comparative Advantage (David Ricardo) and other derivatives from both theories (authors).

1.3. The Harrod-Thirlwall's Law: new perspectives to study divergent growth

More than three decades have gone by since the first release of a Post Keynesian model that linked the long term economic growth to the external sector. This model is due to A. Thirlwall, whose formulations explore the problem of divergent growth between nations.

In Thirlwall (1979), it is demand the factor that plays the key role in the study of growth differentials between countries. Furthermore, the author states that the balance of payments and, in particular the current account, is a fundamental constraint to the rate of long-term economic expansion of developing countries. So, for the purposes of this paper it is noted that a country cannot rely on foreign capital to finance its trade deficit incessantly.

It is important to keep in mind that the performance of the nation's trade balance plays a key role in the model due to the existence of a proportional relationship between the long-term rate of income elasticities of demand for exports and imports of a country, and the ratio of economic growth in relation to the rest of the world.

To explore these concepts, the equation that exhibits the trade balance equilibrium is taken as a starting point for the development of the Thirlwall's model:

(1.1) $$P_d\, X = P_f\, M\, E$$

The notation is as it follows:

P_d=price of exports expressed in the national currency.

X=amount of exports in real terms made by the national country.

P_f =price of imports expressed in foreign currency.

M=amount of imports in real terms for the home country.

E=exchange rate measured as the domestic currency price of one unit of foreign currency.

If logarithms and differentiation are taken, expression (1.1) can be linearized in the following form:

(1.2) $$\frac{\partial p_d}{p_d}+\frac{\partial x}{x}=\frac{\partial p_f}{p_f}+\frac{\partial m}{m}+\frac{\partial e}{e}$$

Equation 1.2 expresses the growth rates of the following variables: the price of exports in local currency $\left(\frac{\partial p_d}{p_d}\right)$, the volume of exports $\left(\frac{\partial x}{x}\right)$, the price of imports in foreign currency $\left(\frac{\partial p_f}{p_f}\right)$, the volume of imports $\left(\frac{\partial m}{m}\right)$ and finally the rate of growth rate of the nominal exchange rate $\left(\frac{\partial e}{e}\right)$. The standard equations for exports and imports raised by Thirlwall and Hussain are the following:

(1.3) $$X=\alpha\left(\frac{P_d}{P_f\,e}\right)^{\eta}W^{\pi}$$

(1.4) $$M=\beta\left(\frac{P_f}{P_d\,e}\right)^{\phi}\,Y^{\xi}$$

Let W represents some income measurement[3] of the foreign economies (rest of the world) while Y represents the income of the local economy. The interpretation of η, ϕ, π, ξ will be held for a moment to be discussed more adequately below[4]. Using the standard linearization procedures, 1.3 and 1.4 can be expressed in terms of their rates of growth, as it is shown below:

(1.5) $$\frac{\partial x}{x}=\eta\left(\frac{\partial p_d}{p_d}-\frac{\partial p_f}{p_f}-\frac{\partial e}{e}\right)+\pi\,\frac{\partial w}{w}$$

(1.6) $$\frac{\partial m}{m}=\phi\left(\frac{\partial p_f}{p_f}-\frac{\partial p_d}{p_d}+\frac{\partial e}{e}\right)+\xi\,\frac{\partial y}{y}$$

[3] Such as real GDP.

[4] Also notice that the parameters α and β will be dismissed during the linearization procedure.

Equations 1.5 and 1.6 allow the model to be interpreted in a more intuitive manner, for instance: let η represent the price elasticity of demand for exports: in that sense it is expected that $\eta < 0$ holds. The parameter ϕ can be interpreted as the price elasticity of demand for imports, thus $\phi < 0$; π symbolizes the foreign income elasticity demand for exports and ε is the national income elasticity of imports; $\frac{\partial w}{w}$ represents the rate of income growth in other economies of the world, and finally $\frac{\partial y}{y}$ symbolizes the growth rate of income of the local economy.

Based on the remarks discussed above, it is possible to replace 1.5 and 1.6 into 1.7, which will lead to obtain, with some algebraic operations, an expression for the growth rate of the local economy in the long run. This can be observed in the following expression:

(1.7)

$$\frac{\partial y}{y} = \frac{\pi\frac{\partial w}{w} + (\phi + \eta + 1)\left(\frac{\partial p_d}{p_d} - \frac{\partial p_f}{p_f} - \frac{\partial e}{e}\right)}{\varepsilon}$$

The expression described in 1.7 summarizes a series of known economic assumptions. Such formulations can be found in Thirlwall (2003):

a) An improvement in the terms of real exchange, $\left[\left(\frac{\partial p_d}{p_d} - \frac{\partial p_f}{p_f} - \frac{\partial e}{e} > 0\right)\right]$, contributes to increase the rate of growth of the local economy. This is the pure effect of terms of trade on the growth of real income.

b) The fastest growth rates of a country relative to another, measured in a common currency decreases the growth rate of the balance of payments of that country if the sum of price elasticities is larger than unit in absolute value, which is $|\phi + \eta| > 1$, then $(\phi + \eta + 1) < 0$ holds.

c) The currency depreciation $\left(\frac{\partial e}{e} > 0\right)$ will drive to an enlargement in the growth rate of the economy in equilibrium with the balance of payments if both price elasticities add up to more than one in absolute value, as shown in literal b).

d) The interdependence of countries is highlighted by 1.7 because the local economy's performance $\left(\frac{\partial y}{y}\right)$ is linked to other economies $\left(\frac{\partial w}{w}\right)$. Based on the values of the exports and imports elasticities (π and ε, respectively), the local economy grows faster with respect to others as long as π is larger than ε. When the ratio of these two elasticities increases over time it is expected to perceive an acceleration of the local economy.

e) Finally, there exists an inverse relationship between the growth rate $\left(\frac{\partial y}{y}\right)$ with the income elasticity of imports (ε) while, on the other hand, this growth rate holds a direct relationship with the income elasticity of exports (π).

With that said, the equations to be estimated can be expressed as it shown ahead:

(1.8)
$$\frac{\partial y}{y} = \frac{\pi}{\varepsilon}.\frac{\partial w}{w} + \frac{(\phi + \eta + 1)}{\varepsilon}.\left(\frac{\partial p_d}{p_d} - \frac{\partial p_f}{p_f} - \frac{\partial e}{e}\right)$$

For simplicity, it is also possible to follow Thirlwall (2003) and assume that the real exchange rates remain stable through time, so $\left(\frac{\partial p_d}{p_d} - \frac{\partial p_f}{p_f} - \frac{\partial e}{e}\right) = 0$. Which leads to the following equation:

$$\frac{\partial y}{y} = \frac{\pi}{\varepsilon}.\frac{\partial w}{w}$$

(1.9)
$$\frac{\frac{\partial y}{y}}{\frac{\partial w}{w}} = \frac{\pi}{\varepsilon}$$

Expression 1.9 allows us to see that he ratio of growth between the rest of the world and the local economy can be explained by the ratio of the foreign income elasticity demand for exports π and the national income elasticity of import, ε.

As it can be seen, the idea behind this model is that the local economy is able to produce an impact that enlarges the ratio of elasticies in order to increase the national growth ratio. It is logical to see that these ratios can be modified through the implementation of economic policies, such as free trade agreements, etc.

In that sense, it is reasonable to consider that these ratios are not fixed constants to be estimated, in lieu they can be thought as parameters that are allowed to modify its composition throughout time. If this is so, it is convenient to model the parameter evolution in a way that allows these parameter fluctuations. These specifications can be found in State-Space representations, particularly of time-varying parameters models, that are estimated using the Kalman filter as it will be exposed in the following chapter.

Chapter II.

The theoretical approach for modeling and estimation

2. State-Space representations and the Kalman filter

State space representations are a convenient form of expressing a wide variety of time series models. Some of their specific uses include: i) modeling non-observable components; ii) representations of autoregressive integrated moving average models (ARIMA), and iii) the estimation of time-varying parameters (TVP) models, which is the application that is put into practice in this research.

Let y_t to be a $n\ x$ 1 vector of variables that are observed at time t, where $t = 1,2, \dots, T$. From this point it is possible to represent a wide variety of dynamic models by another vector of unobserved elements, called β_t with dimensions $r\ x$ 1. In the literature this vector frequently receives the name of State Vector. In that sense the State-Space representation can be noted by the two equations shown below:

(2.1) $$y_t = A'x_t + H'\beta_t + w_t$$ ***[Observation equation]***

(2.2) $$\beta_t = \tilde{\mu} + F\beta_{t-1} + v_t$$ ***[State equation or transition equation]***

Where the parameter matrices are the following: On the one hand, F is of dimension r x r; A and H are both defined of dimension k x n. On the other hand, x_t is a k x 1 vector of exogenous/predetermined variables defined in date t. Keep in mind that in the TVP model, the $A'x_t$ part of the Observation equation is used to include variables that are not varying parameters to be estimated, such as dummy variables.

Furthermore, v_t is a r x 1 vector, while w_t is a vector of dimensions n x 1. In this case, v_t and w_t are both defined as Gaussian independent and identically distributed multivariate vectors:

(2.3) $$E(v_t v_\tau') = \begin{cases} Q & \forall\ t = \tau \\ 0 & \text{otherwise} \end{cases}$$

(2.4) $$E(w_t w_\tau') = \begin{cases} R & \forall\ t = \tau \\ 0 & \text{otherwise} \end{cases}$$

Where Q and R represent the variance and covariance matrices of dimensions r x r and n x n, respectively. Furthermore, v_t and w_t are assumed to display a null correlation between them, which is:

(2.5) $$Cov(v_t w_\tau') = E(v_t w_\tau') = 0\ \forall\ t\ and\ \tau$$

2.1. Additional assumptions

The equations shown before about the State-Space representations are used to describe a finite series of observations, for which it is necessary to stablish some additional assumptions:

(2.6) $$E\left(v_t \beta_\tau'\right) = 0\ \ \forall\ \ t = 1,2, \dots, T$$

(2.7) $$E\left(w_t \beta_\tau'\right) = 0\ \ \forall\ \ t = 1,2, \dots, T$$

The two equations described above allow to see that v_t is uncorrelated with lagged values of β:

(2.8) $$E\left(v_t \beta_\tau'\right) = 0\ \ \forall\ \ \tau = t-1, t-2, \dots, 1$$

However, in terms of w_t it occurs that:

(2.9) $$E\left(v_t\beta_\tau'\right) = \theta \text{ for } \tau = 1,2,\dots,T$$

(2.10) $$E(w_t y_\tau') = E\left(w_t\left[A'x_t + H'\beta_t + w_t\right]'\right) = 0 \ \forall \ \tau = t-1, t-2, \dots, 1$$

(2.11) $$E(v_t y_\tau') = E\left(v_t\left[A'x_t + H'\beta_t + w_t\right]'\right) = 0 \ \forall \ \tau = t-1, t-2, \dots, 1$$

2.2. Implementation of the algorithm

The basic tool used to deal with the standard state-space models is the Kalman filter, a recursive procedure for computing the estimator of the unobserved component or the State Vector at time *t*, base on available information at time *t*.

Notice that for the State-Space representation it is necessary to observe all the elements contained in the y_t and x_t vectors. If for now it is assumed that the matrices F, A, H, Q and R are known[5], it is only necessary to count with an algorithm that produces forecasts of the State Vector, taking into account the available information through date *t*:

(2.12) $$\hat{\beta}_{t+1|t} = \hat{E}\left(\beta_{t+1}|\mathcal{Y}_t\right),$$

Where: $\mathcal{Y}_t = (y_t', y_{t-1}', \dots, y_1';\ x_t', x_{t-1}', \dots, x_1')'$

Assuming that the information in the x_t vectors is available and the information in the y_t vector can be predicted, the Kalman filter consists in the following two steps: 1) Prediction and 2) Updating.

[5] In a further section the details to estimate these parameter matrices from the data will be discussed.

2.2.1. Prediction

For this step initial step, we are aiming to derive an optimal predictor of y_t, based on all the available information up to time *t-1*. Basically, the procedure of the prediction process goes through four stages: i) estimating $\beta_{t|t-1}$, then ii) estimating and computing the covariance matrix of β_t conditional on information up to t-1, which will be referred as $P_{t|t-1}$. The next stage consists in iii) computing de prediction error: $\eta_{t|t-1} = y_t - y_{t-1}$. Finally, iv) the conditional variance of the prediction error is required for the filtering process, $f_{t|t-1} = E\left[\eta^2_{t|t-1}\right]$.

For stage I, it is necessary to produce estimates of $\beta_{t|t-1}$, as it can be concluded from expressions 2.1 and 2.2. Since the recursion needs to start with a forecasted State Vector that is based on no prior observations of β_t, the expression $\beta_{0|0}$ makes reference just to the unconditional mean. Following Kim & Nelson (1999), the unconditional mean of stationary β_t is derived as:

$$E\left[\beta_t\right] = E\left[\tilde{\mu} + F\beta_{t-1} + v_t\right]$$

(2.13)
$$E\left[\beta_t\right] = \tilde{\mu} + FE\left[\beta_{t-1}\right] + E[v_t]$$

Because $E[v_t] = 0$ and the State Vector is supposed to be covariance stationary, which implies $E\left(\beta_{t+1}\right) = E\left(\beta_t\right)$, the expression 2.14 can be written as $E\left(\beta_t\right) - FE\left(\beta_t\right) = \tilde{\mu}$. Which is $(I_r - F).E\left(\beta_t\right) = \tilde{\mu}$. Take into consideration that by definition unity cannot be an eigenvalue of the F matrix, therefore $(I_r - F)$ is invertible. This property simplifies the problem with a unique solution of $E(\xi_t) = 0$. Once again, this is possible when the dynamic system is stationary:

(2.14)
$$\beta_{0|0} = \tilde{\mu}(I_r - F)^{-1}$$

For stage II, notice that in the State equation, if all the eigenvalues of F lie inside the unit circle, then the State Vector at any time t is said to be covariance-stationary. Furthermore, the unconditional variance of the State Vector at time t can be obtained as:

$$E(\beta_t\, \beta_t') = E\left\{(\tilde{\mu} + F\beta_{t-1} + v_t)(\tilde{\mu} + F\beta_{t-1} + v_t)'\right\}$$

Notice that since $(v_t) = 0$, the former expression is reduced to the following:

(2.15) $$E(\beta_t\, \beta_t') = FE(\beta_{t-1}\, \beta_{t-1}')F' + E(v_t v_\tau')$$

Let $P_{t|t-1}$ denote the variance and covariance matrix of the β_t vector based on information up to *t-1* and let Q denote the one for the v_t vector, as mentioned in 2.3. For the stationary case[6] $Cov(\beta_t) = cov(\beta_{t-1})$, therefore:

(2.16) $$P_{0|0} = FP_{0|0}F' + Q$$

The solution of the former expression can be obtained by expressing it in the shape of column vectors of. This is given by the following:

$$Vec(P_{0|0}) = Vec(FP_{0|0}F') + Vec(Q)$$

$$Vec(P_{0|0}) = (F \otimes F).Vec(P_{0|0}) + Vec(Q)$$

(2.17) $$Vec(P_{0|0}) = (I_r - F \otimes F)^{-1}.Vec(Q)$$

[6] The nonstationary case will be addressed in one of the following sections.

With that said, notice that if all the eigenvalues of the F matrix lie inside the unit circle, the Kalman filter iterations can be started with $\beta_{1|0} = 0$. In the same fashion, $P_{1|0}$ can be expressed as[7]:

For stage III, based on the estimated information of $\beta_{t|t-1}$ it is possible to obtain the prediction error:

$$\eta_{t|t-1} = y_t - y_{t-1}$$

(2.18)
$$\eta_{t|t-1} = y_t - x_t\beta_{t|t-1}$$

For Stage IV of the Prediction step, computing the conditional variance of the prediction error, $f_{t|t-1}$, one can proceed in the following fashion:

$$E\left[\eta^2_{t|t-1}\right] \equiv f_{t|t-1} = E\left[\left(y_t - x_t\beta_{t|t-1}\right)\left(y_t - x_t\beta_{t|t-1}\right)'\right]$$

(2.19)
$$f_{t|t-1} = E\left[(y_t - x_t\beta_t)(y_t - x_t\beta_t)' + x_t\beta_{t|t-1}\beta'_{t|t-1}x'_t\right]$$

Provided that $(y_t - x_t\beta_t)(y_t - x_t\beta_t)' = w_t w'_t$, then:

(2.20)
$$f_{t|t-1} = R + x_t P_{t|t-1} x'_t$$

2.2.2. Updating

This step comprises two stages. Once the prediction of y_t is realized at the end of time *t* the prediction error, $\eta_{t|t-1} = y_t - x_t\beta_{t|t-1}$, is calculated. This prediction contains new information about β_t, beyond that contained in $\beta_{t|t-1}$. In other words, after observing

[7] According to Hamilton, 1994, if the eigenvalues of F do not lie inside the unit circle it is also possible to define the initial state with the researchers best guess, which will be addressed later.

y_t, a more accurate inference of β_t can be made, let's say $\beta_{t|t}$. This update can be obtained with the following expression:

(2.21) $$\beta_{t|t} = \beta_{t|t-1} + K_t \eta_{t|t-1}$$

As it can be seen, the updating process allows to achieve a more accurate estimate of the $\beta_{t|t}$ vector, which is provided by this stage. The update for the coefficients vector is computed as a linear combination of the latest estimate of β_t, which is $\beta_{t|t-1}$ and the prediction error, $\eta_{t|t-1}$. The K_t element is commonly referred as the Kalman gain, which determines the weight assigned to new information about β_t, that is contained in the prediction error, $\eta_{t|t-1}$.

The Kalman gain is an inverse function of R, the variance of w_t in the State Equation, and it can be written as follows:

(2.22) $$K_t = P_{t|t-1} x_t' f_{t|t-1}^{-1}$$

For some intuition, using 2.20, the equation above can be expressed in the following manner:

(2.23) $$K_t = P_{t|t-1} x'_t x_t \left(P_{t|t-1} x'_t + R\right)^{-1}$$

It can be seen that the Kalman gain is a positive function of the uncertainty associated with $\beta_{t|t-1}$, $P_{t|t-1}$. Which means that K_t retrieves a higher weight from the prediction error if the uncertainty is higher. Similarly, it receives a lower weight if R, the variance in the State equation due to shock in w_t, is high.

The second stage of the updating process involves estimating a more accurate value of the P_t matrix, which is $P_{t|t}$ and contains more information than $P_{t|t-1}$. This is achieved with the following expression:

(2.24) $$P_{t|t} = P_{t|t-1} - K_t x_t P_{t|t-1}$$

And so the process continues until the algorithm has reached the totality of times data points. The following diagram provides a more intuitive logic about the implementation of the basic Kalman filter. It is only pending to explore the theoretical approach of how to produce the estimates of the hyperparameter matrices, which is described in the following section:

Flowchart 1: Steps of the Kalman filter estimation

Initial values at time 0

$$\beta_{0|0} \; P_{0|0}$$

Prediction of the State Vector at time 1

$$\beta_{t|t-1} = F\beta_{t-1|t-1}$$

$$P_{t|t-1} = FP_{t-1|t-1}F' + Q$$

Calculation of the prediction error

$$\eta_{t|t-1} = y_t - y_{t-1} = y_t - x_t\beta_{t|t-1}$$

$$f_{t|t-1} = x_tP_{t|t-1}x_t' + R$$

Updating

$$K_t = P_{t|t-1}x_t' f_{t|t-1}^{-1}$$

$$K_t = P_{t|t-1}x'_t x_t\left(P_{t|t-1}x'_t + R\right)^{-1}$$

$$\beta_t = \beta_{t-1|t-1} + K_t\eta_{t|t-1}$$

$$P_t = P_{t-1|t-1} - K_t\eta_{t|t-1}x_tP_{t|t-1}$$

Source: Own elaboration.

2.3. Maximum Likelihood estimation of the parameter matrices

Up to this point it has been assumed that all the elements of the matrices: F, A and H are known. However, in reality these elements are required to be estimated. Following Hamilton (1994), since the Kalman filter is used to provide the optimal forecasts of $\beta_{t|t-1}$ and y_{t-1} within the family of forecasts that are linear in $(x_t, \mathcal{Y}_{t-1})$. If it is assumed that the initial State Vector (β_1) and the error terms that figure in the Observation Equation and the State Equation, w_t and v_t follow a multivariate Gaussian distribution, then it is possible to think about the distribution of $y_t|x_t, \mathcal{Y}_{t-1}$ as Gaussian too:

$$y_t|x_t, \mathcal{Y}_{t-1} \sim N(A'x_t + H'\beta_{t|t-1}, R + x_t P_{t|t-1} x'_t)$$

(2.25) $$y_t|x_t, \mathcal{Y}_{t-1} \sim N(y_{t|t-1}, f_{t|t-1})$$

With this, the log likelihood function is presented as:

(2.26) $$\ln L = -1/2 \sum_{t=1}^{T} \ln\left(2\pi^n \det\left(f_{t|t-1}\right)\right) - 1/2 \sum_{t=1}^{T} \eta'_{t|t-1} f_{t|t-1}^{-1} \eta_{t|t-1}$$

Which according to Kim & Nelson (1999) can be maximized with respect to the unknown parameters of the model. For details about the estimation of the Hyperparameters with the Maximum Likelihood approach, refer to Tomasso and Alessandra (2012). The following flowchart summarizes the filtering process:

Flowchart 2: Steps of the Kalman filter estimation including the MLE

$$\beta_{0|0}, P_{0|0}, \mu_0, Q_{0|0}, F_0, R_0$$

Prediction of the State Vector at time 1

$$\beta_{t|t-1} = F\beta_{t-1|t-1}$$

$$P_{t|t-1} = FP_{t-1|t-1}F' + Q$$

Calculation of the prediction error

$$\eta_{t|t-1} = y_t - y_{t-1} = y_t - x_t\beta_{t|t-1}$$

$$f_{t|t-1} = x_t P_{t|t-1} x'_t + R$$

Updating

$$K_t = P_{t|t-1} x'_t f_{t|t-1}^{-1}$$

$$\beta_t = \beta_{t-1|t-1} + K_t \eta_{t|t-1}$$

$$P_t = P_{t-1|t-1} - K_t \eta_{t|t-1} x_t P_{t|t-1}$$

Estimation of the Maximum Likelihood for each time t

$$1/2 \sum_{t=1}^{T} \ln\left(2\pi^n \det\left(f_{t|t-1}\right)\right) - 1/2 \sum_{t=1}^{T} \eta'_{t|t-1} f_{t|t-1}^{-1} \eta_{t|t-1}$$

Source: Own elaboration.

2.4. Implementation of the Kalman filter with nonstationary time-series models

As it has been presented, state-space models and state-space representation of data are important tools for econometric modeling and computation. However, when applied to observed data, many such models have a mixture of stationary and non-stationary roots.

Particularly, the exact nature of the parametric variation in TVP models is difficult to predict: while changes in the behavioral characteristics of dynamic systems are often relatively slow and smooth, more rapid and violent changes do occur from time-to-time and lead to similarly rapid changes, or even discontinuities, in the nature of the related time series (Young & Runkle, 1989: 17).

To be specific, for stationary time-series models the usual initialization uses the unconditional mean and variance of series models, described by equations 2.15 and 2.16. However, this approach is not available if their means and covariances change over time, because 2.15 and 2.15 do not exist (see Bell & Hillmer, 1991).

Nevertheless, some authors have proposed different methodologies for dealing with this problem. For instance, (1) augmenting the state vector for the differenced data with d observations (when differencing is of order d) and initializing at time d since the augmented part of the state vector is then known exactly (Harvey and Pierse 1984). On the other hand, (2) Kohn and Ansley (1986) present a "modified Kalman filter" that allows the variance of the part of the initial state due to the nonstationary starting values to be infinite, so the transformation eliminates the effect of the nonstationary starting values; (3) De Jong (1988) proposes to run the Kalman filter with initial state estimate and variance 0, and then computing an adjustment to yield results invariant to the mean and variance of the initial state.

These 3 approaches are appealing, but they have a common drawback: they require complex computations and none of the existing Kalman filter softwares and their programming codes can be edited for dealing with this matter.

Even so, Kim & Nelson (1999) propose a methodology that can be easily implemented and that is feasible for the purposes of this research. The approach recognizes that when facing with a time-series dynamic model with potential nonstationarity, the unconditional mean and the covariance matrix of β_t do not exist. However, it is possible to set the initial state vector $\beta_{0|0}$, as a an arbitrary *kx1* vector, but in order to assign the "deserved" uncertainty to this wild guess, it is necessary to allocate "very large" values to the diagonal elements of $P_{0|0}$, .

In simple words, with the Kim & Nelson (1999) approach, granted that $\beta_t = \beta_{t-1|t-1} + K_t \eta_{t|t-1}$ in the Kalman filter algorithm, most of the weight in the updating process is provided by new information contained in the forecast error, $\eta_{t|t-1}$, projected by the Kalman gain, K_t. This procedure strongly discards the information coming from the initial wild guess. Therefore, with this maneuver it is possible to initialize the Kalman filter, preventing for the potential nonstationarity in the system and gaining precision in the estimation as the algorithm iterates.

Chapter III.

Empirical approach of the Kalman filter to estimate a time-varying coefficients for the Harrod-Thirlwall's Law

During recent years, El Salvador has participated from a new perspective in the world trade. This view has been expressed in some measures of liberalization and external opening that were launched at the end of the decade of the 80's. However, the following question arises: What effect do these measures have in the performance of El Salvador's economy? The purpose of this chapter is to answer this question with the implementation of econometric methods.

3.1. Brief review of the international commerce in El Salvador

Historically in El Salvador there has existed a tendency for large volumes of commercial transactions with other countries of the world, even for the years between 1965 and 1979, a period in which the Salvadoran economy was characterized by having a high degree of protectionism, as part of the Import Substitution Model (ISI in Spanish) adopted by the time.

According to Acevedo (2003), the period mentioned above initially recorded an index of trade liberalization of 0.45, with an upward trend for the following years. The openness index reached at the end of the same period a value of 0.74. This means that the volume of commercial transactions that the country had with the rest of the world represented 45% and 74% of GDP, respectively.

However, by the beginning of 80's a civil war between the military-led government of El Salvador and the Farabundo Martí National Liberation Front (FMLN) took place, this period is now commonly referred as the "lost decade". Due to this internal conflict or civil war, a notorious decay of trade the openness took place. This conflict officially ended in January of 1992, when a treaty was negotiated by representatives of the Salvadoran government, the rebel movement FMLN, and political parties. This conflict fostered that throughout almost the “lost decade” the international openness resulted significantly lowered.

Nevertheless, in 1989, with the arrival of a new government democratically elected, a model of open economy to international trade was adopted. These measures were conducted as part of the requirements of the International Organisms, which promoted this strategy as a mean to foster and reestablish economic growth in the long run.

Complying with the recommendations, El Salvador embarked on the project of trade liberalization in 1989, which certainly increased the volume of business transactions with the rest of the world. Particularly, the relationships with some commercial partners, such as Chile, Dominican Republic, Mexico and the United States were strengthened by the signing of different Free Trading Agreement (FTA).

3.2. Data

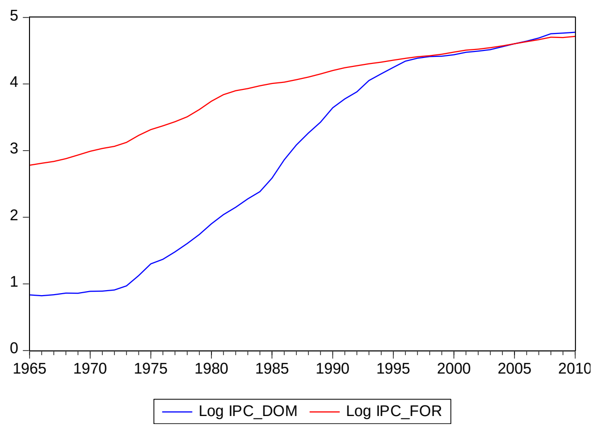
The empirical evidence worked in this chapter has been elaborated based on information issued by the World Bank’s, World Development Indicators data base (WDI) and the Central Bank of El Salvador. The annual series have a length of 45 data points (1965-2010).

All the series are expressed in local currency units (LCU) constant terms of year 2000. Figure 1 presents the evolution of the series in logarithmic scale: a) consumer price indexes (CPI), b) exports and imports of goods and service, c) GDP and d) exchange rate (domestic/foreign - units).

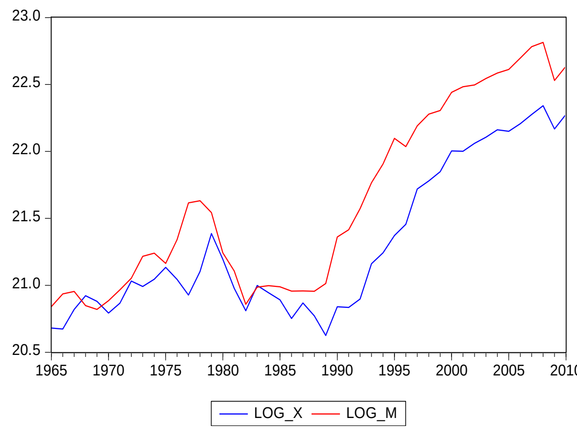
It is important to mention that for simplicity purposes, data from the United States of America will be used as a proxy of the world, since according to the Central Bank of El Salvador it represents around 55% of the international trade volumes.

Figure 1: Evolution of the variables involved 1965-2010

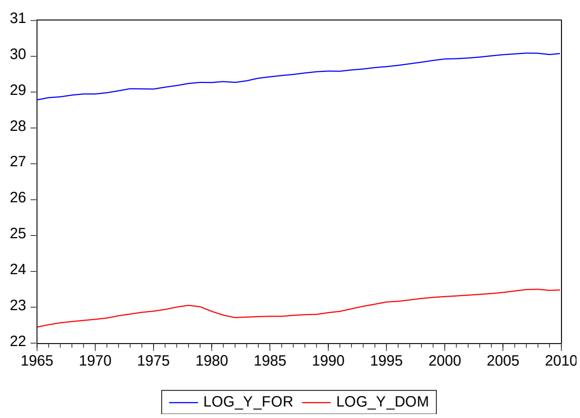
a)



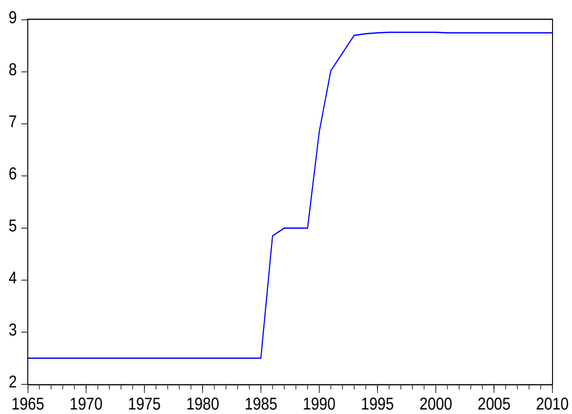
b)



c)



d)



Source: Own elaboration.

3.1.1. Unit root tests

In the following table the results of the Augmented Dickey-Fuller and Perron Unit root test are shown in order to assess the stationary of the individual time series. Nevertheless, according to Kohn and Ansley (1986) and Kim & Nelson (1999) among others, the real restriction for the implementation of the Kalman filter lies on the stationary of the State Vector, which means the stationarity of the complete dynamic system. As it will be exposed in the following sections.

Table 1: Augmented Dickey-Fuller and Perron Unit Root Tests

Variable endógena	*DFA*[(a)]	*Perron* [(b)]	*I(d)*
LN IPC DOM	-2.2814 (1)	-0.3310 (0)	I(1)
Δ*LN IPC DOM*	-3.0820* (3)	-1.9641* (0)	I(0)
LN IPC FOR	-3.4368 (1)	-0.01213 (2)	I(1)
Δ*LN IPC FOR*	-3.6784* (0)	-3.3877* (2)	I(0)
LN X	-1.3105 (0)	-0.4522 (0)	I(1)
Δ*LN X*	-6.1505* (0)	-2.3583* (0)	I(0)
LN M	-2.9330 (1)	-0.9023 (0)	I(1)
Δ*LN M*	-6.2595* (1)	-2.3583* (0)	I(0)
LN Y FOR	-2.9212 (1)	-1.4828 (0)	I(1)
Δ*LN Y FOR*	-5.3723* (0)	4.1519* (0)	I(0)
LN Y DOM	-2.3125 (7)	-1.8601 (0)	I(1)
Δ*LN Y DOM*	-3.5388* (1)	4.0093* (0)	I(0)
E	-2.3467 (4)	-0.8750 (0)	I(0)
Δ*E*	-1.8249* (3)	-1.3238 (0)	I(0)

(a) **Note:** Number of lags in parentheses. Order of integration in fourth column.
(b) For the DFA test the following sequence of hypotheses have been used: a) trend and intercept, b) trend, c) none. Result that the three tests fail to reject the null hypothesis of a unit root in levels. For more details about this sequence Enders (2004:213) can be consulted.
(c) For the Perron tests of the variables in levels, the statistics for deterministic trend and dummy level or "step" are reported; for the variables expressed in differences the no deterministic trend and dummy pulse are reported.

Source: Own elaboration.

3.2. State-Space and filtering specification

The realization of the empirical analysis of the approach required some fundamental assumptions, such as the existence of an unlimited capacity to supply, so that the volume of exports and imports is determined by the demand side also alluded to the assumption that goods produced in El Salvador and the United States does not have characteristics that become perfect substitutes.

The calculation of the estimated elasticities of demand for exports and imports are performed through the determination of the export and import functions previously mentioned in Chapter I, which are taken up to be detailed below:

(3.1) $$X = \delta \left(\frac{P_d}{P_f e}\right)^{\eta} W^{\pi}$$

(3.2) $$M = \lambda \left(\frac{P_f e}{P_d}\right)^{\emptyset} Y^{\xi}$$

Following standard procedures, natural logarithm is applied to expressions 3.1 and 3.2, leading to obtain the following equations:

(3.3) $$lnX_t = \gamma_0 + \eta_t ln\left(P_{d_t} - P_{f_t} - e_t\right) + \pi_t lnW_t + v_t$$

(3.4) $$lnM_t = \gamma_1 + \emptyset_t ln\left(P_{f_t} - P_{d_t} + e_t\right) + \varepsilon_t lnY_t + u_t$$

Where $\gamma_0 = ln\, \delta$ and $\gamma_1 = ln\, \lambda$

Equation 3.3 contains as endogenous variable the natural logarithm of X_t. This, represents the volume of exports of the home country (El Salvador) to the world. The indicator is measured in units of local currency at constant prices. Additionally, the W_t variable is used to represent the income of the rest of the world, which similarly measured at constant prices.

Equation 3.4 contains as endogenous variable the natural logarithm of M_t, which represents the volume of local country imports from the rest of the world, measured in units of local currency at constant prices. The Y_t variable is used to represent the income of the rest of the local country, which similarly is measured at constant prices.

With the linearized representations in 3.3 and 3.4, the parameter η_t represent the price elasticity of demand for exports: in that sense it is expected that $\eta_t < 0$ holds. The parameter $\emptyset_t$ can be interpreted as the price elasticity of demand for imports, thus $\emptyset_t < 0$. Finally, π_t symbolizes the foreign income elasticity demand for exports and ε_t is the national income elasticity of imports.

Notice that the parameters have been indexed by the time indicator, *t*. This indexations means that it is expected that they change as time moves forward. The two State-Space observation equations to be estimated are displayed in equations 3.5 to 3.6.

[Observation equations]

(3.5) $$lnX_t = \begin{bmatrix} 1 & ln\left(P_{d_t} - P_{f_t} - e_t\right) & lnW_t \end{bmatrix} \begin{bmatrix} \gamma_0 \\ \eta_t \\ \pi_t \end{bmatrix} + v_t$$

(3.6) $$lnM_t = \begin{bmatrix} 1 & ln\left(P_{f_t} - P_{d_t} + e_t\right) & lnY_t \end{bmatrix} \begin{bmatrix} \gamma_1 \\ \emptyset_t \\ \varepsilon_t \end{bmatrix} + u_t$$

The two State equations to be estimated are displayed in equations 3.7 to 3.8.

[State equations]

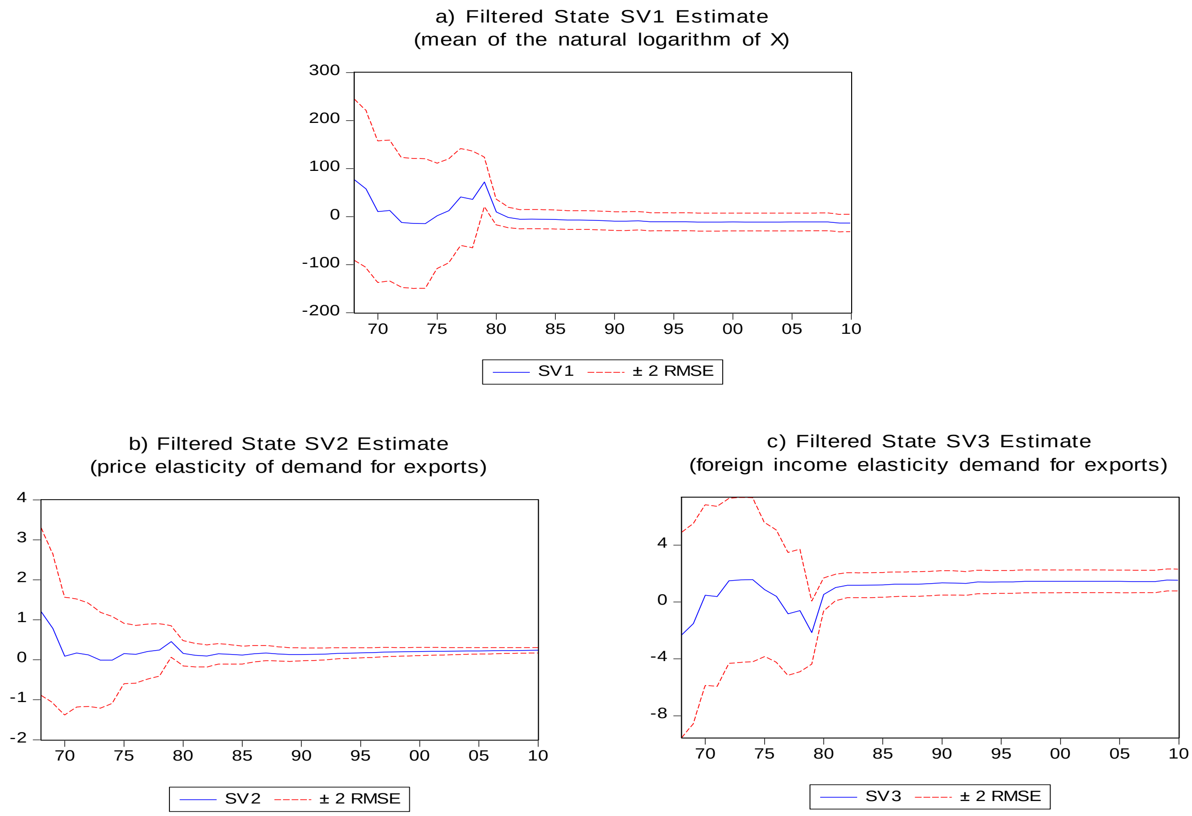
(3.7) $$\begin{bmatrix} \gamma_{0t} \\ \eta_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} \theta_{x0} \\ \theta_{x1} \\ \theta_{x2} \end{bmatrix} + \begin{bmatrix} f_{x11} & 0 & 0 \\ 0 & f_{x22} & 0 \\ 0 & 0 & f_{x33} \end{bmatrix} \begin{bmatrix} \gamma_{0t-1} \\ \eta_{t-1} \\ \pi_{t-1} \end{bmatrix} + \begin{bmatrix} 0 \\ \nu_{1t} \\ \nu_{2t} \end{bmatrix}$$

(3.8) $$\begin{bmatrix} \gamma_{1t} \\ \emptyset_t \\ \varepsilon_t \end{bmatrix} = \begin{bmatrix} \theta_{m0} \\ \theta_{m1} \\ \theta_{m2} \end{bmatrix} + \begin{bmatrix} f_{m11} & 0 & 0 \\ 0 & f_{m22} & 0 \\ 0 & 0 & f_{m33} \end{bmatrix} \begin{bmatrix} \gamma_{1t-1} \\ \emptyset_{t-1} \\ \varepsilon_{t-1} \end{bmatrix} + \begin{bmatrix} 0 \\ \upsilon_{1t} \\ \upsilon_{2t} \end{bmatrix}$$

The State-space systems are estimated using the basic Kalman filter. In the following figures it is possible to appreciate the time-varying estimated parameters. Figure

2 contains the estimation of the intercept of the natural logarithm of exports, the price of exports elasticity if exports (η_t) and the foreign income elasticity of exports (π_t).

Figure 2: State Space Kalman filtered Series of Exports



Source: Own elaboration.

Notice that for the initial vector, we can pre-define the parameters of the F matrix and allow the Kalman algorithm to produce sequential updates. As it has been described the correction will be performed by the Kalman gain, which will take into consideration the "large" variances arbitrarily assigned. The initial State-vectors for natural logarithm of X_t are shown in the following table:

Table 2. Initial State Vector of the exports function

SV1 (γ_{0t})	SV2 (η_t)	SV3 (π_t)
0.000	0.181	0.952

Source: Own elaboration.

As it has been mentioned, in order to initialize the filter with the "wild guess" that Kim & Nelson (1999) suggest, to prevent for potential nonstationarity, the initial variances where set to 100 each, which are large figure if one considers that we are estimating price and income elasticities. With that said, the initial variance and covariance matrix are shown in the following table:

Table 3. Initial Covariance Matrix of State Vector of the exports function

	SV1(γ_{0t})	SV2 (η_t)	SV3 (π_t)
SV1(γ_{0t})	100	0.000	0.000
SV2 (η_t)	0.000	100	0.000
SV3 (π_t)	0.000	0.000	100

Source: Own elaboration.

The final State-vectors for natural logarithm of X_t along with the root of the MSE and the statistical significance are reported in the following table:

Table 4. Final State Vector of the exports function

	Final State	Root MSE	z-Statistic	Prob.
SV1 (γ_{0t})	-12.534	9.071	-1.381	0.167
SV2 (η_t)	0.375	0.122	3.056	0.002
SV3 (π_t)	1.5164	0.386	3.924	0.000

Source: Own elaboration.

Similarly, the final covariance matrix of the algorithms iterations are shown in the following table:

Table 3. Final Covariance Matrix of State Vector of the exports function

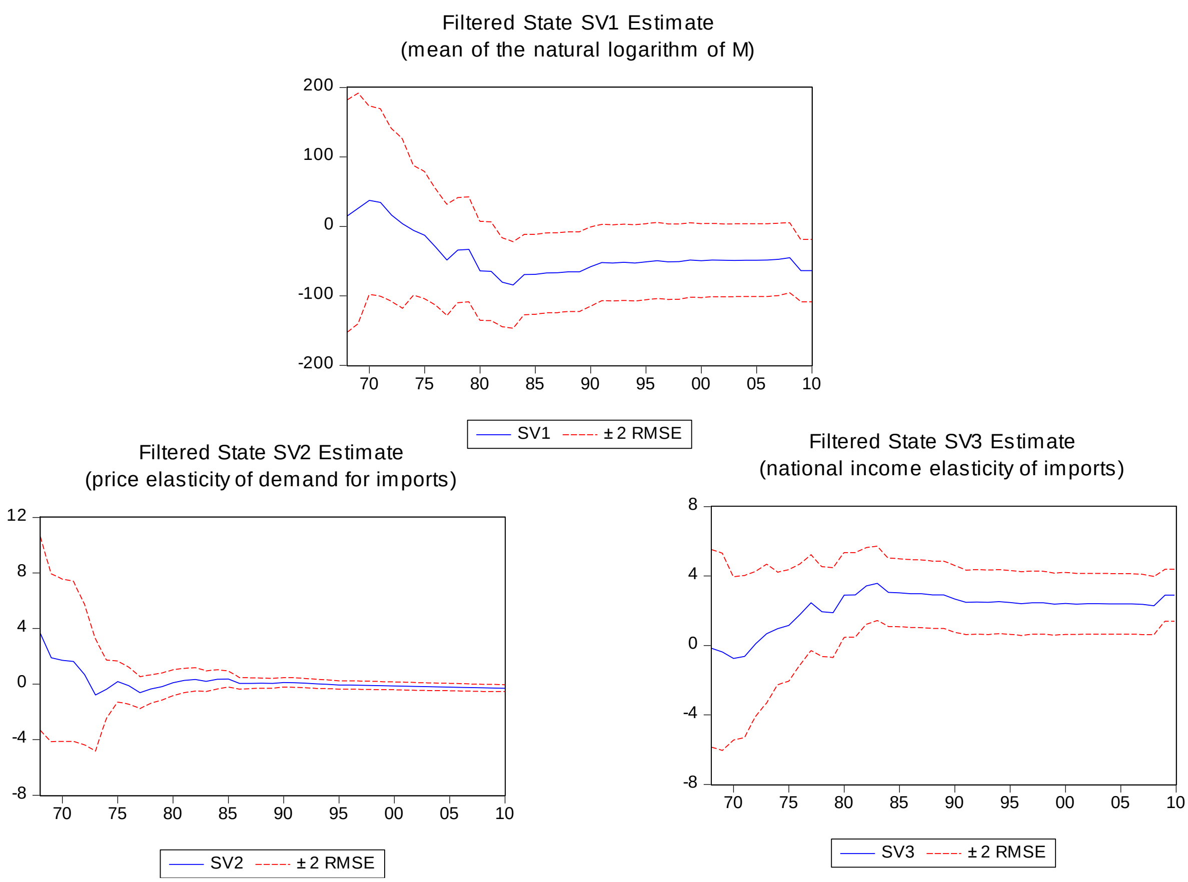
	SV1(γ_{0t})	SV2 (η_t)	SV3 (π_t)
SV1(γ_{0t})	2.85E-13	0.000	0.000
SV2 (η_t)	0.000	3.39E-16	0.000
SV3 (π_t)	0.000	0.000	4.65E-16

Source: Own elaboration.

In figure 3 it is possible to observe the estimated price of imports elasticity ($\emptyset_t$) and the national income elasticity of imports (ε_t), using the basic Kalman filter. Notice that in the beginning of the filtering process the confidence intervals are much larger than by the middle of the period or at the end of the recursive estimation. Mainly, this is due to two reasons: first, the arbitrarily large variances that were assigned to the initial state space vectors. The second reason is due to the correction provided by the Kalman gain.

On the other hand, notice that price elasticity of demand of imports tends to stabilize around the value of 1, while the national income elasticities of imports tend to stabilize in values close to 2.

Figure 3: State Space Kalman filtered Series of Imports



Source: Own elaboration.

The initial State-vectors for the natural logarithm of M_t is shown in the following table:

Table 5. Initial State Vector of the imports function

SV1 (γ_{0t})	SV2 (η_t)	SV3 (π_t)
0.000	0.088	0.224

Source: Own elaboration.

For the case of the imports functions, its State Space representation and the recursive estimation through the Kalman Filter, the initial variance and covariance matrix are shown in the following table:

Table 6. Initial Covariance Matrix of State Vector of the imports function

	SV1 (γ_{1t})	SV2 ($\emptyset_t$)	SV3 (ε_t)
SV1 (γ_{1t})	100	0.000	0.000
SV2 ($\emptyset_t$)	0.000	100	0.000
SV3 (ε_t)	0.000	0.000	100

Source: Own elaboration.

The final State-vectors for natural logarithm of M_t along with the root of the MSE and the statistical significance are reported in the following table:

Table 7. Final State Vector of the imports function

	Final State	Root MSE	z-Statistic	Prob.
SV1 (γ_{1t})	-64.761	15.668	-4.133	0.000
SV2 ($\emptyset_t$)	-0.338	0.128	-2.641	0.008
SV3 (ε_t)	2.929	0.523	5.599	0.000

Source: Own elaboration.

Finally, the covariance matrix of the State Vector is displayed in the following table:

Table 8. Final Covariance Matrix of State Vector of the imports function

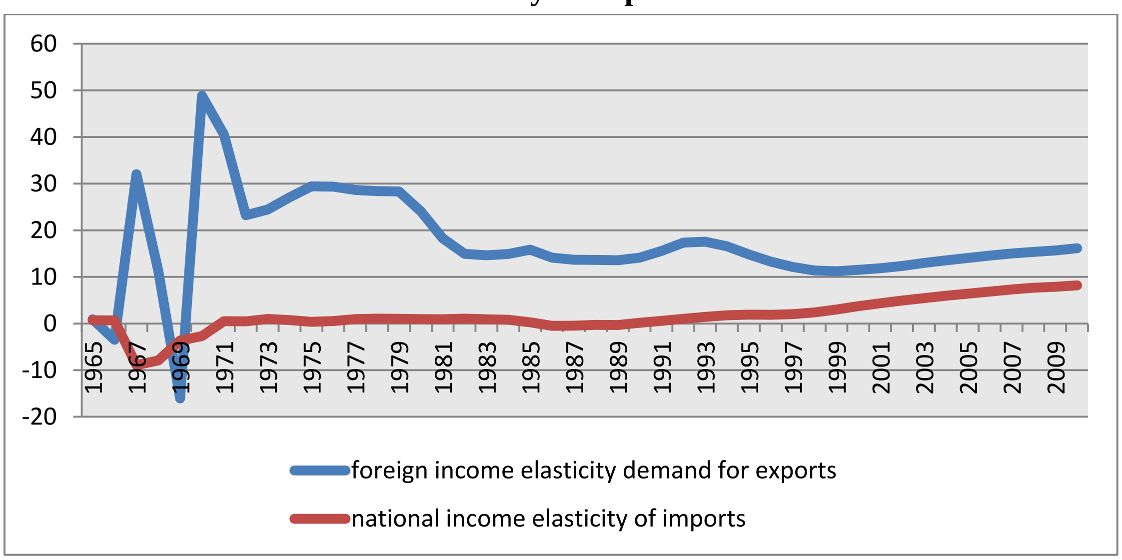
	SV1 ($\boldsymbol{\gamma_{1t}}$)	SV2 ($\emptyset_t$)	SV3 ($\boldsymbol{\varepsilon_t}$)
SV1 ($\boldsymbol{\gamma_{1t}}$)	0.026	0.000	0.000
SV2 ($\emptyset_t$)	0.000	0.000	0.000
SV3 ($\boldsymbol{\varepsilon_t}$)	0.000	0.000	0.154

Source: Own elaboration.

Thus, the trajectory of the ratio between the foreign income elasticity demand for exports (π_t) and the national income elasticity of imports ($\boldsymbol{\varepsilon_t}$) can be obtained with the filtered State Vectors ($\pi_t/\boldsymbol{\varepsilon_t}$), as it is shown in the following figure.

It can be seen that both elasticities remain relatively stable throughout all the period examined. Aditionally, the gap elasticities is favorable in the sense that in most of the periods π_t exceeds $\boldsymbol{\varepsilon_t}$, which according to Thirlwall's Law it allows that El Salvador grows faster than the rest of the world. However, it can also be seen that the gap between this two elasticities has been slowly reduced through time.

Figure 5. Foreign income elasticity demand for exports and the national income elasticity of imports

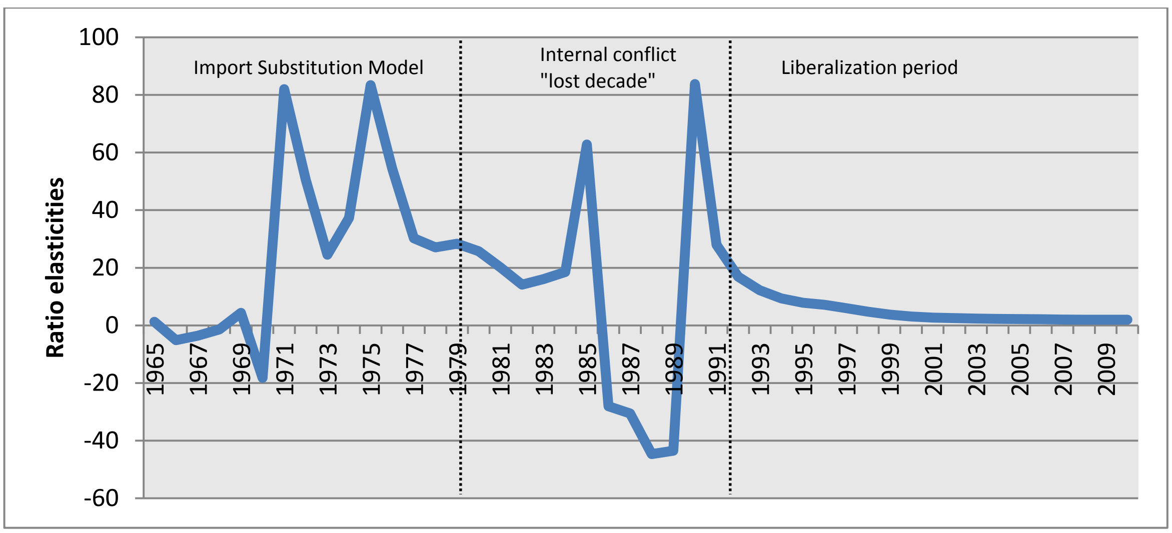


Source: Own elaboration.

When examining the ratio of elasticities it can be seen that, contrary to what it could be assumed, the period in which the Import Substitution Model (ISI) dominated, the ratio of elasticities was larger, which means that the relative growth of El Salvador was larger than the rest of the world.

Additionally, it can be observed that the during the internal conflict, the ratio of elasticities estimated through the Kalman filter reflects the low rates of economic growth and commercial transactions with the rest of the world, producing as a consequence a delay with the rest of the world, in the Thirlwall sense. Finally, it can also be seen that in the 1995-2001 period, the gap elasticities was diminished to the point of becoming almost null.

Figure 5. Ratio of the estimated income elasticity of demand for imports and the national income elasticity of imports (π_t/ε_t),



Source: Own elaboration.

A final remark is that income elasticities of demand for exports and imports are directly influenced by the type of goods traded, this is why trade policies should be focused on modifying these elasticities favorably, implying expand the gap between them and thus find El Salvador in a better position, compared to the rest of the world in terms of economic growth.

3. Conclusions

As it was exposed international trade, although is not the only factor, directly influences the long-term economic growth of countries, and El Salvador is not the exception.

The Thirlwall's Law allows to study the long-term economic growth of the countries, in this particular case El Salvador. This theory can be used to explore the relative rates of growth through the analysis of the price and demand elasticities of exports and imports. According to the Thirlwall's Law these variables govern an important fraction of the growth rate and therefore, its application provides guidance to the public policy makers when it comes to the implementation of trade practices.

The theory suggests that almost all time series models can be casted into State-Space representations. In this particular case, it has been very useful to represent the model as a Time-Varying-Parameter model (TVP), because this representation allows us understand that the elasticities can be modified through time and by the effects of economic and trade policies.

As it has been discussed, once the system is represented in the form of a State-Space model, the basic tool to perform the estimation is the Kalman filter algorithm. This tool is very powerful, in the sense that it performs the estimation providing a optimum mean squared error at a low modest computational cost.

Finally, the estimation of the Thirlwall's model suggests that the future growth of the salvadorean economy will depend on the ability to maintain export growth and to reduce income elasticity of demand for imports. Given that new trade agreements keep accumulating, exports are expected to continue growing in the coming years. However, prospects could be better and bring benefits to a larger proportion of the economy if import substitution were made more efficient and if other export sectors with a greater multiplier effect were explored and expanded.

4. Anex

Anex 1: Implementation codes

Exports State Space

```
@signal log_x = SV1 + SV2*log_tcr_x + SV3*log_y_dom + [ename = e1 , var=exp(C(1))]

@state SV1 =   SV1(-1) + [ename = e2, var =exp(C(2))]

@state SV2 = c(7) + c(8)*SV2(-1) + [ename = e3, var = exp(C(3))]

@state SV3 = c(9) + c(10)*SV3(-1) +  [ename = e4, var = exp(C(4))]

@param  C(2) 100 c(3) 100 c(4) 100

@param  C(7) 1 C(9) 1

@param  c(8) 1 c(10) 1

@evar cov(e1, e2)=0

@evar cov(e1, e3)=0

@evar cov(e1, e4)=0
```

Imports State Space

```
@signal log_m = SV1 + SV2*log_tcr_m + SV3*log_y_for+ [ename = e1 , var=exp(C(1))]

@state SV1 =   SV1(-1) + [ename = e2, var =exp(C(2)) ]

@state SV2 =  c(7) + c(8)*SV2(-1) + [ename = e3, var = exp(C(3))]

@state SV3 = c(9) + c(10)*SV3(-1) +  [ename = e4, var = exp(C(4))]

@param  C(2) 100 c(3) 100 c(4) 100

@param  C(7) 1 C(9) 1

@param  c(8) 1 c(10) 1
```

```
@evar cov(e1, e2)=0

@evar cov(e1, e3)=0

@evar cov(e1, e4)=0
```

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