TORCUATO DI TELLA UNIVERSITY

REGULATION IN THE CABLE TELEVISION INDUSTRY

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Part I

Introduction

During the last decade, many countries of Latin America have promoted laws regarding the regulation of media, a presumable priority in the continent's political agenda. Arguments that justify such interventions point at widening the variety of content offered to consumers by helping new actors become part of the media industry. Another frequently quoted reason is the desintegration of monopolies which are typical of this industry.

However, reasons behind regulation may well go beyond the economic arguments alluded to fight against big media corporations. The annual survey of Freedom House, published since 1980, assesses the political, legal and economic climate for the press in all countries of the world on a scale of 100 points (where fewer points mean more freedom). According to this year's report, out of the 35 countries that constitute the Americas, 15 are considered to have a Free Press, in 14 press is Partly Free and in 6 it is Not Free (Cuba, Ecuador, Honduras, Mexico, Paraguay and Venezuela). A lot of legislative projects presented by these countries' governments arguably look for the democratization and plurality of the media, whereas their true motivation might be concealed behind the open political battle set by the executives against big media corporations. The following examples reveal a tendency of Latin American countries to increase the power and control exerted by the State over the media industry:

- Uruguay: the Uruguayan Consititution (1967) protects the freedom of communications media. Recently, however, President José Mujica sent to Congress the draft of a law that aims at regulating the audio-visual telecommunication services. One of its articles sets a limit to the number of subscribers a cable or internet company may have to be no more than 25% of the total number of households with access to mass media. This law also establishes the creation of a Council of Audiovisual Communication, composed of five members elected by the executive, that will act "according to the general interest". The project foresees possible sanctions, such as the loss of licensing permissions, for those who might violate precepts of this legislation.
- Paraguay: the Paraguayan Constitution (1992) protects the freedom of expression and of the press, and guarantees the dissemination of thought and opinion without censorship. In 2004, a reform of the Telecommunications Law No. 642 was passed in an attempt to regulate the emission and propagation of

electromagnetic signals. The National Telecommunications Commission was created to promote, control and regulate telecommunications. In June 2012, after the coup that removed President Fernando Lugo from power, the government intervened the media.

- Bolivia: the Bolivian Constitution guarantees freedom of expression and protects freedom of communication and access to individual and collective information. In 2011, the General Law on Telecommunications, Information and Communication Technologies was passed, allowing for much discretion on the side of the government regarding the regulation of media. This law has been said to represent the “democratization of communication in Bolivia”. Its most controversial article, number 11, limits the participation of private firms to 33% of the electromagnetic spectrum (today it is 90%): 33% will pass into the hands of the State, 17% to social community and 17% to native people and peasant farmers. What is debated is the fact that Evo Morales’ government would end up controlling 67% of the spectrum given that the last two groups depend on the executive.

- Brazil: the Brazilian Constitution protects freedom of expression and prohibits any kind of censorship. It also states that social media may not be subject to monopolistic or oligopolistic control. Ex president Lula promoted a new law in order to strengthen social control over the media, a measure very much opposed by private companies of the industry.

- Ecuador: the Ecuadorian Constitution enshrines the right to communication and access to information technologies. What is more, provides for the State to encourage plurality and diversity in communication, and prevent the creation of monopolies and oligopolies. In October 2011, the National Assembly of Ecuador began debating a new law of telecommunications. President Correa threatened to place financial pressure on the private media that were critical of the government.

- Venezuela: the Venezuelan Constitution protects freedom of expression and the right of people’s access to document of any kind. However, Venezuela has been referred to as a country were freedom of expression has sufered increasing restraints, particularly for those critical of ex-president Hugo Chávez. Since 2004, the law on Social Responsibility in Radio and Television establishes high penalties for media that promote public incitement against the government. The application of punishments is in charge of the National Commission in Telecommunications which depends on the executive branch.

- Panama: law No. 31 dictates standards for the regulation of telecommunications. The aim of this law (1996) is to modernize and develop the sector by promoting private investment and encourage fair competition in the provision of telecommunication services. According to the Freedom Press ranking, Panama fell 2 points as a consequence of the block in the distribution of the newspaper “La Prensa”.

- Nicaragua: TELCOR stands for Nicaraguan Telecommunications and Postal Institute, which is responsible for the regulation and control of telecommunication and postal services. This country's position in the Freedom Press ranking has deteriorated as a result of the government regulating media content and interfering with freedom of press.

- Honduras: in August 2012, the National Commission of Telecommunications issued a resolution forbidding the extension of new licenses to broadcasting stations of low power.

- Guatemala: In December 2011, Congress approved a reform of the General Telecommunications Act, allowing for radio and television stations that already have a license of 15 years to expand these almost automatically for another 25 years.

- Mexico: the Federal Telecommunications Law and the Federal Law on Radio and Television regulate telecommunications and media activity. In 2006, reforms were passed, some of which were later declared unconstitutional by the Supreme Court.

- Cuba: Cuba's legislation regarding freedom of expression and press is highly restrictive and violates some fundamental rights.

In Argentina, President Cristina Fernández de Kirchner presented in August 2009 the controversial law of Audiovisual Communication Services N°26.522, approved by Congress in October of the same year, and whose constitutionality is now being debated by the Supreme Court. The aim of such law, according to the Argentinean government, is to reduce the level of concentration of media and guarantee a greater variety of content, with special emphasis on national production. The executive has argued that this new regulation stands for the democratization of communication and it will help slow down the process of concentration that is said to be taking place in the continent.

Among its numerous articles, this legislation envisages the creation of a new regulatory organism, the Federal Authority of Audiovisual Communication Services (AFCSA). It limits the number of licenses (of radio, air and pay television) a company may have to a maximum of 10 (as opposed to the 24 permitted by the previous legislation); these are not transferable and will be granted for a 10-year period (before this new law it was 15 years) and an extension of a further 10 years is possible. However, controls will take place every two years to avoid that, with the introduction of new technologies such as digitalization, the owner of a license might multiply the amount of signals it provides. If the standards of quality, investment and content were in any case violated, the law envisages sanctions that could lead to the loss of the license. Furthermore, no operator will be allowed to provide its service to more than 35% of the country's total population or of the total amount of subscriptions; the owner of a channel of air TV cannot be a distributor of pay television in the same district (or viceversa) and companies that provide telephone services are not allowed to enter the cable television business.

This law would force some media conglomerates, notably Grupo Clarín, to divest most of their assets. In view of this, Clarín secured an injunction suspending the law's implementation for three years while it sought judicial annulment of four clauses it claims are unconstitutional. In December 2012 a federal court ruled against Clarín, but in April this year an appeals court overturned that decision and judged the media law as arbitrary and disproportionate. The government therefore appealed to the Supreme Court whose final verdict is to be announced in a near future.

Over the last six years, Argentina has lost 34 positions on the ICI (International Index of Institutional Quality). An index annually produced by the Foundation of Freedom and Progress, the ICI determines the degree of respect for the political and economic rules of game through eight indicators. Ours is the Latin American country that has most recoiled and one of the five countries that has most receded in the world, going down from postion 127 to 191.

Beyond any political implications surrounding the regulation of media, the purpose of this work is to study the mecanism through which prices of channels are set in the television industry and to analyse the impact that measures which attempt to disolute big media corporations may have on total producer surplus. Following much of the literature that exists on this subject, we model the TV industry by means of a chain of production in which two program providers, the upstream firms, sell their channels to one monopolistic cable operator, the downstream firm. We do not allow for competition between upstream firms to avoid technicalities regarding product differentiation (for more on oligopolistic competition between upstream firms, see Bourreau, Hombert et al. 2010); instead, we think of each channel as unique and each program provider as a monopoly of the content it sells. We do allow for heterogeneity in the consumers' valuation for each channel. Under this setting, we study how the interaction between the members of the chain and the prices they set is affected when firms that are initially integrated are obliged to desintegrate. As we shall see, one of the consequences that we will encounter is the well-known vertical externality of double marginalization, first introduced and studied by Cournot (1838)-Spengler (1950).

Furthermore, bundling is characteristic of markets where sellers have market power. In the TV industry in particular, subscription to cable television is typically to a package of channels together, rather than to each channel separately. We therefore allow for the cable operator to choose between bundling and separate selling (usually known as providing programs "à la carte"), and we study how this decision depends on the chain being integrated or not. Traditionally, bundling has been regarded as a practice that facilitates consumer price discrimination (Adams and Yellen 1976, Bakos and Brynjolsson 1999) and possibly deters entry (Nalebuff 2002). According to the former, when consumers have heterogeneous tastes for several products a firm will choose bundling over separate selling for it allows to derive greater benefits when the price set for the bundle is lower than the sum of the prices of each of its components. The setting under which such conclusion is reached parallels that of our chain being totally integrated. But as argued in a recent paper by Adilov et al. (2012), when there is no such integration, the interaction within the chain of production affects the decision of the downstream firm. Based on this work, we explore the role of negotiations and the division of surplus between upstream and downstream firms in conditioning the cable operator's decision to bundle. The cable operator might strategically use bundling or separate selling in order to enhance its bargaining position in negotiations with program suppliers. We shall see that in the extreme case in which the chain is completely desintegrated and no negotiation takes place in the process through which prices are set, provision of channels à la carte (i.e., separate selling) is preferred over bundling.

Finally, we introduce a further extension of our work. The pay-offs of the upstream and downstream firms obtained under different degrees of integration of the chain are used to construct a signalling game in which a government that would like to reduce the level of concentration in the TV industry, can threat to impose a regulation that will oblige to disolute the chain. Depending on the type of government, this regulation may or may not be effectively passed, but firms must make a decision ex-ante (due to transaction, legal costs) if they wish to avoid such regulation in the future. In particular, we shall see that they have the possibility to coordinate a negotiation game and replicate the prices and total producer surplus of the integrated chain.

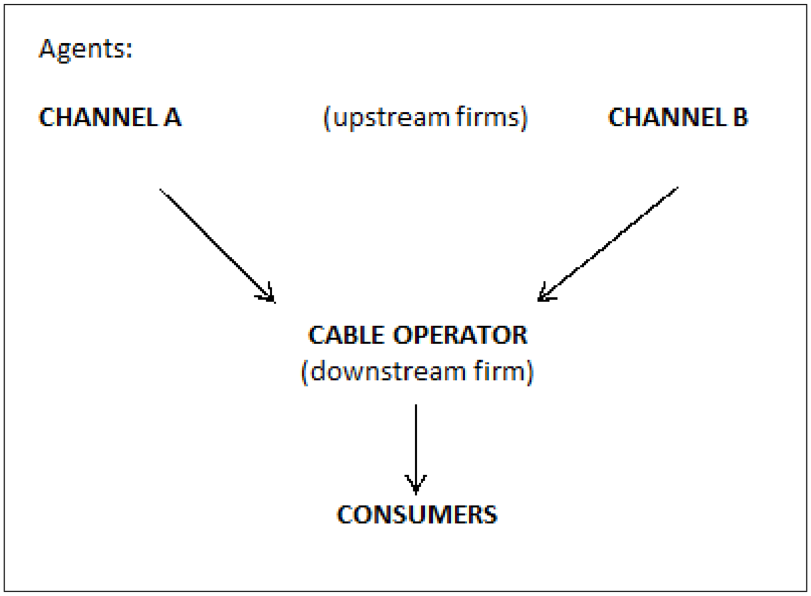
This work is organised as follows: in part two, we present the model and obtain prices, profits and total producer surplus in four different scenarios that differ in the level of integration of the chain and the dynamics of the interaction between the firms; in part three, we conclude on the different scenarios analysed; in part four, we describe the signaling game where both government and firms are players.

Part II

Model

In most industries, production and sale do not take place in a single stage but, on the contrary, there is a whole chain of production and commercialization that involves transactions between upstream and downstream firms. In particular, we wish to study the dynamics of the chain of production in the cable industry. In our simplified model, there are two upstream firms (the program providers or channels, indexed $i = A, B$) and a single downstream firm (the cable operator, indexed o) that purchases programs from suppliers and provides programs to consumers in its franchise area. All firms maximize profits. Each channel is unique and cannot be substituted by the other, therefore a program provider behaves as a monopolist when selling its channel to the cable operator. Cost of production of each channel is fixed (c_A, c_B). To distribute the channels to consumers, the cable operator can either choose to practice separate selling or pure bundling. In the former case, each channel i is sold separately to consumers at price p_i; in the latter case, both channels are sold in a bundle at price p_{AB} (for simplicity, we omit the possibility of mixed bundling, where the channels are sold both in a bundle and separately) and consumers are restricted to purchasing either the entire bundle or nothing at all. Consumers will be buying at most one unit of each good.

The degree of integration of the chain of production determines the way in which the prices are set. In particular, we distinguish and analyse the dynamics of four possible scenarios: when the chain is desintegrated ("desintegration"), when one channel agrees to negotiate a particular transfer payment with the cable operator ("semi-bargaining"), when both channels bargain a transfer payment ("bargaining"), and when the whole chain of production is integrated ("integration"). In each case, we provide a brief description of the stages through which prices are set. We compare prices, profits and total producer surplus, and study the incentives that a cable operator may have to practice bundling or separate selling.



We begin by obtaining the demand for the channels and the bundle of channels. A unit mass of consumers have preferences over the different channels. Each consumer is identified by a vector (θ_A, θ_B) where θ_i is the consumer´s valuation for channel i. Consumers are distributed with density $f(\theta_A, \theta_B)$. For simplicity, we assume that valuation for each channel is independent and uniform over [0,1]. Therefore, (θ_A, θ_B) is uniformly distributed over the unit square. Furthermore, the consumer's valuation for the bundle is equal to the sum of her separate valuations for the component goods. As noted by Belleflamme and Peitz (2010), this assumption of strict additivity is justified for independently valued goods. (If valuations were not independent, i.e., they were interrelated: when goods are complements (substitutes), the valuation for the bundle is larger (lower) than the sum of the separate valuations for the component goods).

When the channels are sold separately, they are priced independently. Demand for each channel is (for more detail, *see Appendix, part A*):

$$D_A(p_A) = \int_{p_A}^{1} 1 d\theta_A = 1 - p_A$$

$$D_B(p_B) = \int_{p_B}^{1} 1 d\theta_B = 1 - p_B$$

(If $p_i > 1$, demand is zero).

When the channels are offered in a bundle, demand for this package is:

$$D_{AB}(p_{AB}) = \iint_{\theta_A+\theta_B>p_{AB}} f(\theta_A, \theta_B) d\theta_B d\theta_A = 1 - \int_0^{p_{AB}} \int_0^{p_{AB}-\theta_A} 1 d\theta_B d\theta_A = 1 - \int_0^{p_{AB}} (p_{AB} - \theta_A) d\theta_A =$$
$$= 1 - \tfrac{1}{2}(p_{AB})^2$$

Notice that, since θ_A and θ_B are independent variables, $f(\theta_A, \theta_B) = f(\theta_A).f(\theta_B) = 1.1 = 1$

(If $p_{AB} > \sqrt{2}$, demand is zero).

1) Integration

In this scenario, the uptream firms (program providers) and the downstream firm (cable operator) are all integrated.

In stage one, the firm decides whether to bundle the programs or to provide programs to consumers à la carte (separate selling).

In stage two, the firm sets a price for each channel (p_A, p_B), if it has decided to sell separately, or for the bundle of channels (p_{AB}).

In stage three, consumers decide whether to subscribe or not to the cable operator. If provision of channels is à la carte, they decide which channels to buy.

We use backward induction to retrieve the price and profits that result from this set of stages. We begin by analysing these under separate selling and we then repeat our procedure under bundling. Finally, the firm chooses separate selling or bundling by comparing profits.

Under separate selling:

Demands (stage three) were obtained previously. The firm chooses at what price to sell each good (stage two) by maximizing its profit and we can state this problem as follows:

$$\max_{p_B, p_A} \; (1 - p_A)p_A + (1 - p_B)p_B - c_A - c_B$$

$$(p_i) \quad 1 - 2p_i = 0 \quad \text{for } i = A, B$$

$$p_A^* = p_B^* = \frac{1}{2}$$

The firm's profit under separate selling is: $\Pi^s = (1 - p_A^*)p_A^* + (1 - p_B^*)p_B^* - c_A - c_B$

$$\Pi^s = \frac{1}{2} - c_A - c_B$$

What would the firm's benefits be if it only sold one of both channels? Selling only A would give a profit of $\Pi^s = \frac{1}{4} - c_A$; selling only B, $\Pi^s = \frac{1}{4} - c_B$.

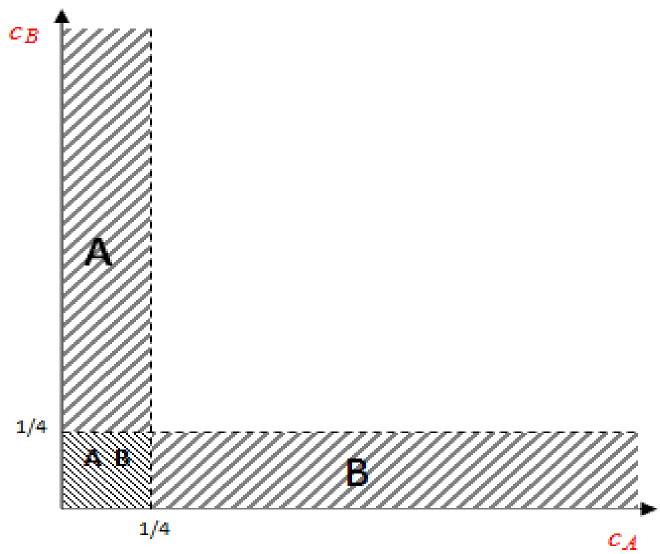
The firm will sell both channels when:

$\frac{1}{2} - c_A - c_B \geq \frac{1}{4} - c_A \quad \wedge \quad \frac{1}{2} - c_A - c_B \geq \frac{1}{4} - c_B \quad \wedge \quad \frac{1}{2} - c_A - c_B \geq 0$

$\Rightarrow \frac{1}{4} \geq c_B \quad \wedge \quad \frac{1}{4} \geq c_A.$

Then, if $\frac{1}{4} \geq c_B \wedge \frac{1}{4} \geq c_A$, it sells both channels. This case is represented in Figure 1 by the area AB. If $\frac{1}{4} < c_B \wedge \frac{1}{4} \leq c_A$, it only sells channel A (area A); if $\frac{1}{4} \geq c_B \wedge \frac{1}{4} < c_A$, it only sells channel B (area B); if $\frac{1}{4} < c_B \wedge \frac{1}{4} < c_A$, it doesn't sell.

Figure 1



Under bundling:

$\max\limits_{p_{AB}} \; (1 - \frac{1}{2}(p_{AB})^2)p_{AB} - c_A - c_B$

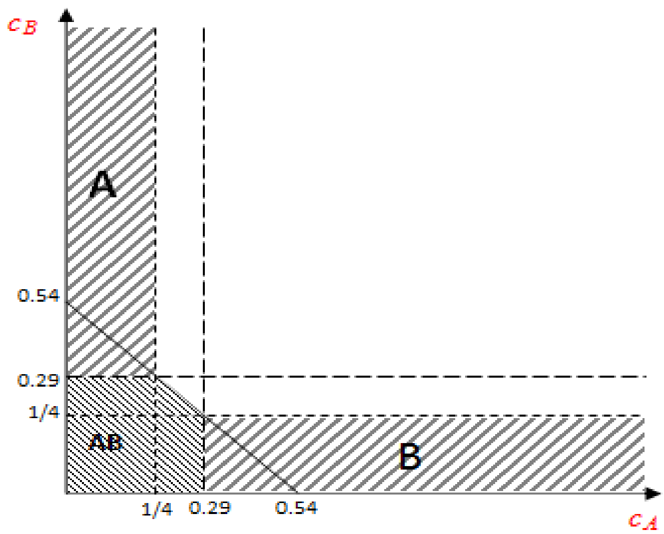
$(p_{AB}) \quad 1 - \frac{3}{2}p_{AB}^2 = 0$

$$p_{AB}^* = \sqrt{\frac{2}{3}}$$

The firm's profit under bundling is: $\Pi^{b*} = (1 - \frac{1}{2}(p_{AB}^*)^2)p_{AB}^* - c_A - c_B$

$$\Pi^{b*} \;= \sqrt{\frac{2}{3}}\frac{2}{3} - c_A - c_B$$

Figure 2



The firm will sell both channels when:

$$\frac{2}{3}\sqrt{\frac{2}{3}} - c_A - c_B \geq \frac{1}{4} - c_A \quad \wedge \quad \frac{2}{3}\sqrt{\frac{2}{3}} - c_A - c_B \geq \frac{1}{4} - c_B \quad \wedge \quad \frac{2}{3}\sqrt{\frac{2}{3}} - c_A - c_B \geq 0$$

$$\Rightarrow \frac{2}{3}\sqrt{\frac{2}{3}} - \frac{1}{4} \geq c_B \quad \wedge \quad \frac{2}{3}\sqrt{\frac{2}{3}} - \frac{1}{4} \geq c_A \quad \wedge \quad \frac{2}{3}\sqrt{\frac{2}{3}} \geq c_A + c_B$$

This case is represented in Figure 2 by the area AB. The channels' costs must be smaller than 0.29 and the sum of them should not be larger than 0.54.

If $\frac{2}{3}\sqrt{\frac{2}{3}} - \frac{1}{4} < c_j$ and $\frac{1}{4} \geq c_i$, the firm will offer only channel i. The area A (B) in Figure 2 contains costs' values such that only channel A (B) is offered to the consumers. In any other case, it doesn't sell.

Comparing bundling vs. separate selling:

In stage one, the firm decides whether to bundle the programs or to provide them à la carte. Its decisions will depend on the costs of the channels.

If $\frac{1}{4} \geq c_A$ and $\frac{1}{4} \geq c_B$, the firm chooses to bundle since:

$$\Pi^b > \Pi^s$$

$$\frac{2}{3}\sqrt{\frac{2}{3}} - c_A - c_B > \frac{1}{2} - c_A - c_B$$

$$\frac{2}{3}\sqrt{\frac{2}{3}} > \frac{1}{2}$$

Producer surplus is maximized under bundling, therefore the firm chooses to sell the channels in a package. This result is related to the use of bundling as a strategy for sorting consumers and perfom some kind of price discrimination of second degree but in the opposite direction: instead of increasing the menu of prices to exploit the heterogeneity of consumers, the set of a unique price for several goods is a way of reducing this heterogeneity and gain higher profits. In particular, in this monopoly setting with fixed costs and heterogeneous but uncorrelated valuations for two products, the firm could set a price for the bundle equal to the sum of the prices obtained under separate selling and thus earn the same profits (what wouldn't be same would be the identity of the consumers who buy the channels). Instead of doing this, selling the bundle of channels at a lower price than the sum of the prices of its components allows the firm to increase demand and derive greater benefits. Why? There are now more marginal consumers, meaning that lowering the price a bit would raise demand of the bundle (i.e., of both channels) in an amount larger than what an equivalent increase in the price of both channels would do under separate selling. Therefore, the firm has an incentive to lower the price of the bundle. Such gains from bundling would work even better if the values for the products were negatively correlated, as noted by Belleflamme and Peitz.

If $\frac{2}{3}\sqrt{\frac{2}{3}} - \frac{1}{4} < c_j$ and $\frac{1}{4} \geq c_i$, the firms sells channel i since it is the only alternative that brings nonnegative profits. In Figure 3, these cases are represented by the areas A and B.

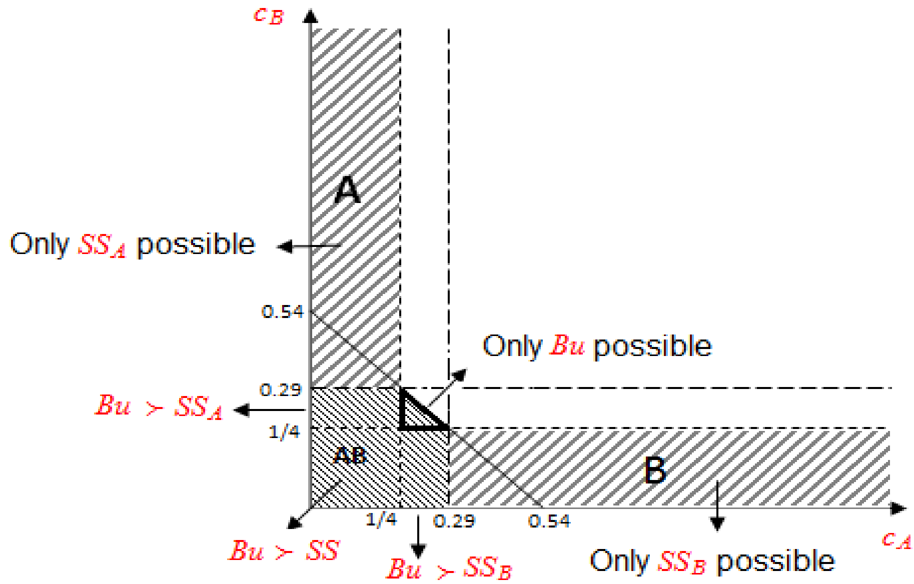
If $\frac{1}{4} \geq c_i$ and $\frac{1}{4} < c_j \leq \frac{2}{3}\sqrt{\frac{2}{3}} - \frac{1}{4}$, the firm prefers bundling rather than selling only channel i.

$$\Pi^b > \Pi^{s,i}$$

$$\frac{2}{3}\sqrt{\frac{2}{3}} - c_A - c_B \geq \frac{1}{4} - c_A$$
$$\frac{2}{3}\sqrt{\frac{2}{3}} - \frac{1}{4} \geq c_B$$

If $\frac{1}{4} < c_A \leq \frac{2}{3}\sqrt{\frac{2}{3}} - \frac{1}{4}$, $\frac{1}{4} < c_B \leq \frac{2}{3}\sqrt{\frac{2}{3}} - \frac{1}{4}$ and $\frac{2}{3}\sqrt{\frac{2}{3}} \geq c_A + c_B$, the firm chooses to bundle.

Then, the area AB in Figure 3 sums up three different scenarios under which the cable operator prefers to bundle. When the costs of both channels are smaller than 1/4, bundling is preferred over separate selling. The rectangle delimited by $c_A \in [1/4, 0.29]$ and $c_B = 1/4$ refers to the case when the firm prefers bundling rather than selling only channel B. The other rectangle is analogous. When the channels' costs belong to the triangle, bundling is the only alternative that brings nonnegative profits.



Notice how, whenever costs are such that both bundling and separate selling bring positive profits to the firm, the former is preferred over the latter. Also, there is a region of costs under which, if only separate selling were allowed, no channels would be sold, whereas under bundling they both are. Finally, for certain costs only one channel would be sold under separate selling, whereas both channels are when bundling is allowed.

2) Desintegration

In this scenario, cable operator, channel A and channel B are all individual agents taking decisions as follows:

In stage one, the cable operator decides whether to practice separate selling or bundling.

In stage two, each program provider decides at which price to sell its channel to the cable operator (w_A, w_B). The cable operator will pay this price for each subscriber it has. We assume that the cost of production for the program provider is independent of demand (it is a fixed cost: c_A, c_B).

In stage three, taking the price of each channel as given, the cable operator sets the final price of the channels, depending on whether it is doing separate selling or bundling. Its only cost is what it pays to the program provider.

In stage four, consumers decide whether to buy or not.

Once again, we solve this problem by backward induction: given the price that the program provider charges for its channel, the cable operator sets the final price. Then, taking this final price into account, each program provider sets the price for the channel. Finally, the cable operator compares the different results and chooses bundling or separate selling.

Under separate selling:

The cable operator chooses at what price to sell each channel by maximizing its profit. If both channels are sold:

$\max_{p_B, p_A} (1-p_A)(p_A - w_A) + (1-p_B)(p_B - w_B)$

$(p_i): \quad 1 - 2p_i + w_i = 0 \quad$ for $i = A, B$

Solving for $p_i : p_i = \frac{1+w_i}{2}$

Second order conditions are met to guarantee this is a maximum.

Number of subscribers to each channel will be: $q_i = 1 - \frac{1+w_i}{2} = \frac{1-w_i}{2}$. With this in mind, each program provider solves:

$\max_{w_i} w_i \frac{1-w_i}{2} - c_i$

$(w_i) : \frac{1}{2} - w_i = 0$

$$w_i^* = \frac{1}{2}$$

Second order conditions that guarantee this is a maximum are met. The program provider will sell the channel so long as $\frac{1}{2}\frac{1-\frac{1}{2}}{2} > c_i \Rightarrow \frac{1}{8} > c_i$.

So the final price of each channel is:

$$p_i^* = \frac{3}{4}$$

Profits are:

For each program provider: $\Pi_i^s = \frac{1}{8} - c_i \quad \forall i = A, B$

For the cable operator: $\Pi_o^s = \frac{1}{8}$

Total surplus is: $\Pi_A^s + \Pi_B^s + \Pi_o^s = \frac{3}{8} - c_A - c_B$

On the other hand, if only one channel is sold, the cable operator solves:

$$\max_{p_i}(1-p_i)(p_i-w_i)$$
$$(p_i):\ (1-2p_i+w_i)=0 \rightarrow\ p_i=\frac{1+w_i}{2}$$

Then channel i solves:

$$\max_{w_B}\ w_i\frac{1-w_i}{2}-c_i$$
$$(w_i):\ \frac{1}{2}-w_i=0 \rightarrow\ w_i^*=\frac{1}{2}\quad;\quad p_i^*=\frac{3}{4}$$

Profits are:

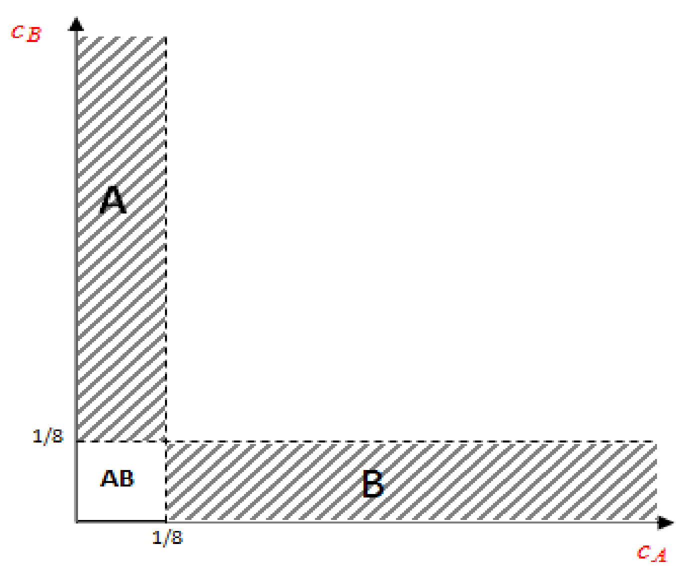
For the program provider: $\Pi_i^{s,i}=\frac{1}{8}-c_i$

For the cable operator: $\Pi_o^{s,i}=\frac{1}{16}$

If $\frac{1}{8}\geq c_A \wedge \frac{1}{8}\geq c_B$, both channels will be offered. In Figure 4, this case is represented by the square AB. Compared with the total integration scenario, in order to sell both channels costs must be smaller ($\frac{1}{4}$ vs $\frac{1}{8}$).

If $\frac{1}{8}\geq c_i\ \wedge \frac{1}{8}<c_j$, only channel i will be offered. Areas A and B refer to those cases.

Figure 4



It is interesting to compare the results, under separate selling, of desintegration vs integration (we concentrate on the region of costs for which both channels are sold). Notice how, when firms are integrated, the final price of the channels ($p_A^* = p_B^* = \frac{1}{2}$) is lower than when they are desintegrated ($p_A^* = p_B^* = \frac{3}{4}$) and total producer surplus is higher ($\frac{1}{2} - c_A - c_B$ vs $\frac{3}{8} - c_A - c_B$ when both channels are sold). This is caused by what is commonly referred to as "double marginalization": when firms are desintegrated and the cable operator chooses the prices that maximize its own benefits, it considers its own marginal costs (w_A, w_B) and disregards the chain's marginal cost (which in our particular case is 0); in other words, there is a vertical externality that results in lower total producer surplus. We should take into account that when integrating the whole chain of production we are not only eliminating the vertical externality that exists between an upstream and a downstream firm but also the potencial interaction between program providers. We can hypothetically consider a scenario in which there are two chains of production composed by a program provider and a cable operator that sells its products separately. Prices and total producer surplus do not differ from the total integration scenario. We can argue that under separate selling there is no such interaction between program suppliers since each channel is a monopoly of the content it sells. We should be more careful when analysing bundling since that interaction is likely to appear.

In addition, under desintegration the conditions on channels' costs for both of them being sold are more restrictive than under integration.

Under bundling:

The cable operator chooses at what price to sell each good by maximizing its profit:

$$\max_{p_{AB}} \ (1 - \tfrac{1}{2}p_{AB}^2)(p_{AB} - w_A - w_B)$$

$$(p_{AB}): \ \tfrac{-3}{2}\ p_{AB}^2 + p_{AB}(w_A + w_B) + 1 = 0$$

Solving for $p_{AB}: p_{AB} = \frac{w_A+w_B}{3} + \frac{\sqrt{(w_A+w_B)^2+6}}{3}$; $p_{AB} = \frac{w_A+w_B}{3} - \frac{\sqrt{(w_A+w_B)^2+6}}{3}$

Second order conditions are met to guarantee this is a maximum: $-3p_{AB} + w_A + w_B < 0 \Rightarrow p_{AB} > \frac{w_A+w_B}{3}$, which is satisfied by the first price obtained.

Number of subscribers will be:

$$q = 1 - \frac{1}{2}\left(\frac{w_A+w_B}{3} + \frac{\sqrt{(w_A+w_B)^2+6}}{3}\right)^2 = \frac{2}{3} - \frac{1}{9}(w_B^2 + w_A^2) - \frac{2}{9}w_A w_B - \frac{1}{9}\sqrt{(w_A + w_B)^2 + 6}\,(w_A + w_B)$$

Therefore, each program provider solves:

$$\max_{w_i} w_i q - c_i$$
$$\max_{w_i} w_i \left[\tfrac{2}{3} - \tfrac{1}{9}(w_B^2 + w_A^2) - \tfrac{2}{9}w_A w_B - \tfrac{1}{9}(w_A + w_B)\sqrt{(w_A + w_B)^2 + 6}\right] - c_i$$

In particular, program provider A solves:

$$\max_{w_A} w_A \left[\tfrac{2}{3} - \tfrac{1}{9}(w_B^2 + w_A^2) - \tfrac{2}{9}w_A w_B - \tfrac{1}{9}(w_A + w_B)\sqrt{(w_A + w_B)^2 + 6}\right] - c_A$$

$$(w_A) : \tfrac{2}{3} - \tfrac{1}{9}(w_B^2 + w_A^2) - \tfrac{2}{9}w_A w_B - \tfrac{1}{9}(w_A + w_B)\sqrt{(w_A + w_B)^2 + 6}+$$
$$w_A \left[-\tfrac{2}{9}(w_A + w_B) - \tfrac{1}{9}\left(\sqrt{(w_A + w_B)^2 + 6} + \tfrac{1}{\sqrt{(w_A+w_B)^2+6}}(w_A + w_B)^2\right)\right] = 0$$

Solving for w_A we obtain the reaction function $w_A(w_B)$.

Second order conditions are met to guarantee this is a maximum:

$$\frac{-2\sqrt{(w_A+w_B)^2+6}}{9} - \frac{2(w_A+w_B)(2w_A^3+5w_A^2 w_B+w_A(4w_B^2+15)+w_B(w_B^2+6))}{9((w_A+w_B)^2+6)^{\frac{3}{2}}} - \frac{2(3w_A+2w_B)}{9} < 0$$

Program provider B solves:

$$\max_{w_B} w_B \left[\tfrac{2}{3} - \tfrac{1}{9}(w_B^2 + w_A^2) - \tfrac{2}{9}w_A w_B - \tfrac{1}{9}(w_A + w_B)\sqrt{(w_A + w_B)^2 + 6}\right] - c_B$$

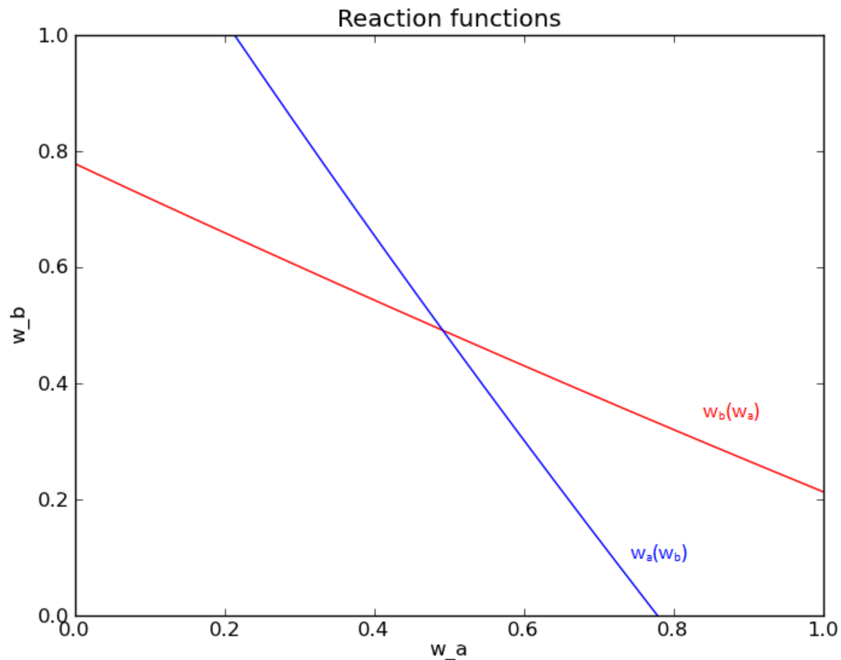
$$(w_B) : \tfrac{2}{3} - \tfrac{1}{9}(w_B^2 + w_A^2) - \tfrac{2}{9}w_A w_B - \tfrac{1}{9}(w_A + w_B)\sqrt{(w_A + w_B)^2 + 6}+$$
$$w_B \left[-\tfrac{2}{9}(w_A + w_B) - \tfrac{1}{9}\left(\sqrt{(w_A + w_B)^2 + 6} + \tfrac{1}{\sqrt{(w_A+w_B)^2+6}}(w_A + w_B)^2\right)\right] = 0$$

Solving for w_B we obtain the reaction function $w_B(w_A)$.

By intersecting both reaction functions, we obtain the optimal prices of the channels. But retrieving explicit reaction functions is mathematically complicated, instead we find a numerical solution to this system of equations.

Nevertheless, in order to get some intuition, we use the Implicit Funtion Theorem to see the way w_A depends on w_B (and viceversa). Since $\frac{\partial w_A}{\partial w_B} < 0$ and $\frac{\partial w_B}{\partial w_A} < 0$, prices are substitutes. We also graph an approximation of both reaction functions (for more details, see *Appendix part B)*.

Intuitively, if a program provider sets a higher price, it makes the cable operator set a higher final price and hence subscribers are less, therefore it reduces the other program provider's profit, who will respond by setting a lower price to encourage more subscriptions. As a consequence of this dynamics, the price of each channel set by the program providers under bundling is less than the prices set under separate selling.



The numerical solution that solves the system of equations given by both first order conditions is:

$$w_A^* = w_B^* = 0.49126$$

Equilibrium is stable since:

$$\frac{\partial w_A}{\partial w_B}\Big|_{w_A^*=w_B^*=0.49126} = \frac{-0.7195}{-1.2717} = -0.5657$$
$$\frac{\partial w_B}{\partial w_A}\Big|_{w_A^*=w_B^*=0.49126} = \frac{-0.7195}{-1.2717} = -0.5657$$

$$\Longrightarrow \left| \frac{\partial w_B}{\partial w_A}\Big|_{w_A^*=w_B^*=0.49126} \right| < \frac{1}{\left| \frac{\partial w_A}{\partial w_B}\Big|_{w_A^*=w_B^*=0.49126} \right|}$$

Program providers sell their channels so long as $w_i q - c_i > 0 \Rightarrow 0.133\,27 > c_i$

So the final price is:

$$p_{AB}^* = 1.207\,2$$

Profits are:

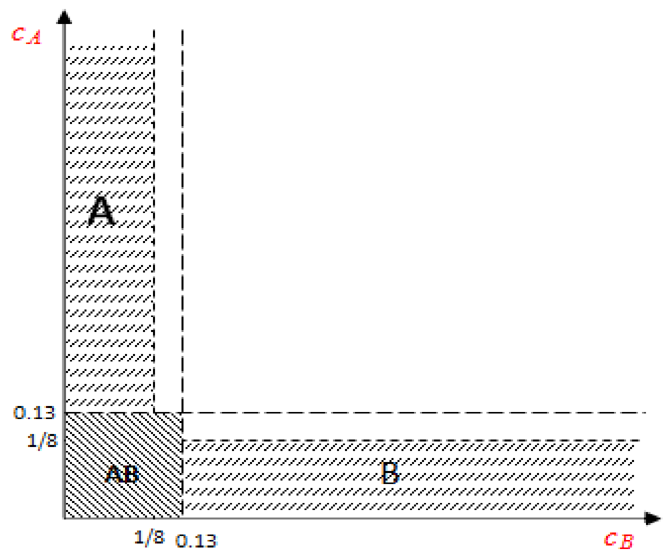
For each program provider: $\Pi_i^b = 0.133\,27 - c_i$

For the cable operator: $\Pi_o^b = 0.06\,096\,3$

And total surplus: $\Pi_A^b + \Pi_B^b + \Pi_o^b = 0.327\,5 - c_B - c_A$

If $0.133\,27 \geq c_A \wedge\ 0.133\,27 \geq c_B$, both channels will be offered. In Figure 5, the square AB represents this case. If $0.133\,27 < c_j$, channel j will not be offered but channel i will be offered as long as $c_i \leq \frac{1}{8}$ since $\Pi_i^{s,i} = \frac{1}{8} - c_i$. Only channel A will be sold if the channels' costs belong to the area A in the figure. The area B is analogous.

Figure 5

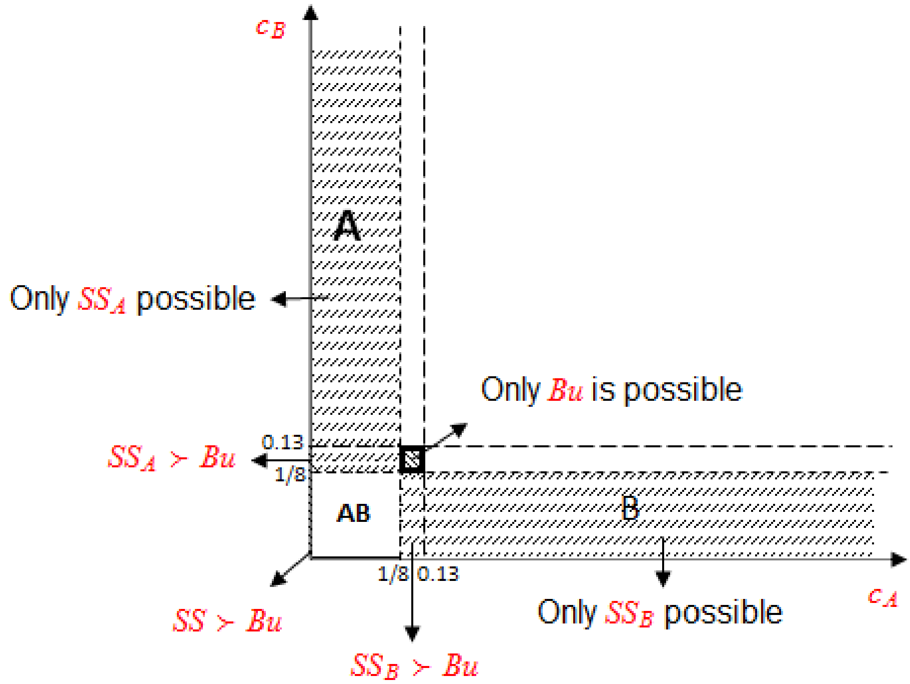


As already noticed in the case of separate selling, when we compare (now, under bundling) desintegration vs integration (concentrating on the case in which both channels are sold), we see that the final price of the bundle under integration ($p_{AB}^* = \sqrt{\frac{2}{3}} \approx 0.8165$) is lower than when they are desintegrated ($p_{AB}^* = 1.207\,2$) and total producer surplus is higher ($0.327\,5 - c_B - c_A$ vs $\sqrt{\frac{2}{3}}\frac{2}{3} - c_A - c_B$ when both channels are sold). Once again, this result is related to the vertical externality implied by double marginalization.

Comparing bundling vs separate selling:

When deciding whether to practice bundling or separate selling, the cable operator compares its profits under each scenario.

Figure 6



If $\frac{1}{8} \geq c_A$ and $\frac{1}{8} \geq c_B$, the cable operator chooses to sell the channels separately over selling them as a bundle since $\Pi_o^s = 0.125 > \Pi_o^b = 0.060963$. In Figure 6, the square AB represents this case.

If $c_i \leq \frac{1}{8}$ and $\frac{1}{8} < c_j \leq 0.13327$, the cable operator prefers selling channel i rather than practicing bundling since $\Pi_o^{s,i} = 0.0625 > \Pi_o^b = 0.060963$. If $c_i \leq \frac{1}{8}$ and $c_j \geq 0.13327$, the cable operator offers only channel i. Then, the area A in Figure 6 (and area B analogously) sums up two different scenarios under which the cable operator prefers to sell channel A. The rectangle delimited by $c_B \in [1/8, 0.13]$ and $c_A = 1/8$ refers to the case when the firm prefers selling only channel A rather than bundling. In the other scenario, the cable operator sells channel A since it is the only alternative that brings nonnegative profits.

If $\frac{1}{8} < c_i \leq 0.13327$ and $\frac{1}{8} < c_j \leq 0.13327$, the cable operator offers both channels as a bundle. This case is represented by the little square in Figure 6.

To gain some intuition, let us concentrate on the first region of costs, for which selling both channels gives positive profits either under bundling or separate selling. We see that, contrary to what happened under integration, the cable operator chooses to practice separate selling. What is the intuition behind this? When we analysed bundling in the traditional monopoly setting (ie, under integration of the chain, case 1) we saw that the firm sets a final price for the bundle that is lower than the sum of the final prices of the channels under separate selling, therefore captures a greater demand and derives greater benefits. But in this case, where we introduce a chain of production, profits decrease. Notice that the market structure we have been analysing is very simple (there is literally no competition). Yet, when bundling is imposed, we observe some interaction between the upstream firms, who, when setting their prices, take into consideration the other firm's decision. The price set by a program supplier depends negatively on the price set by the other channel. However, in spite of this competition between program providers, it is not enough to lower the price of the channels for the cable operator in an amount big enough to justify the choice of bundling. Under integration of the chain costs taken into account when setting the final price of the bundle are fixed, therefore incrementing the number of subscribers by setting a lower price means no additional costs. Here, if the cable operator lowers the price, he gets more subscribers but at the same time, this means paying more to the program provider. So if the channels are too expensive, the cable operator profits will decrease when offering the bundle. As regards total producer surplus, separate selling is preferable to bundling under this scenario; then, the cable operator's decision is in line with a greater total producer surplus. On the contrary, program suppliers would rather choose bundling than separate selling.

3) Bargaining

Many times, special contracts are involved in the transactions between members of a chain. Through such contracts, commonly refered to as "vertical restrictions", they can establish conditions to coordinate their policies, avoid costs of transation, guarantee a long term relationship and more. Their impact on competition and wellfare can either be positive or negative; when they allow for the internalization of externalities, eficiency is gained and both producer and consumer surplus can increase. In particular, we aim to replicate the price and total producer surplus obtained in the extreme scenario in which firms are all integrated. As explained, desintegration of the chain results in higher prices and lower producer surplus due to double marginalization. But is there a way to replicate the results of integration without necessarily having to integrate the firms? As we shall see, there is. A vertical restriction that would allow for this can be summarized as follows: the downstream firm (i.e., the cable operator) sells the product (the channels) at a certain price and then agrees on a sharing of profits with the upstream firms (i.e., the program providers). We choose to model such negotiations through an asymmetric Nash bargaining problem. Bilateral negotiations have been studied extensively building on Nash (1950) and Rubinstein (1982), as detailed in Muthoo (1999). We shall see that the price that the cable operator sets (of the channels if it is selling them separately, or of the package of channels if it chooses to do bundle) is the same price set by the integrated firm in the first case we analysed. Therefore, the chain's surplus is maximized.

In stage one, the cable operator decides whether to bundle the programs or to provide them à la carte.

In stage two, the cable operator sets a price for each channel, if it has decided to sell separately, or for the bundle of channels.

In stage three, the cable operator enters into simultaneous negotiations with each program supplier separately. Each negotiation determines the transfer payment, T_i, the cable operator pays to program provider i. According to this result, the cable operator decides which channels to include in its package keeping its decision of whether to bundle or not. If we alter the order of stages two and three, the results won't change. Without loss of generality, we suposse that the cable operator chooses the channels' prices before negotiations.

In stage four, consumers decide whether to subscribe or not to the cable operator. If provision of channels is à la carte, it decides which channels to buy.

The cable operator decides whether to sell i depending on the transfer it will have to pay to the program supplier. This transfer payment, T_i, is a solution to the asymmetric Nash bargaining maximization problem:

$$\max_{T_i} (\Pi - d)^{\alpha_i} (\Pi_i - d_i)^{1-\alpha_i}$$

Where d is the disagreement profit of the cable operator (profit must be greater than this if the cable operator is to include channel i); in particular, it is the profit the cable operator would have if it didn't include channel i: $d = p_j D(p_j) - T_j$ (if it sells the other channel) or $d = 0$ (if no channel is sold). $d_i = 0$ is the disagreement profit of the program provider. It is assumed that the cable operator keeps α_i share of marginal surplus created from selling channel i, while supplier i keeps the remaining $1 - \alpha_i$. So, $\alpha_i \in [0, 1]$ is the cable operator's bargaining power when negotiating with supplier i. Correspondingly, supplier's bargaining power is $1 - \alpha_i$. These powers are exogenous. Π is the cable operator's profit when it includes channel i and Π_i is the program provider's profit for selling the channel.

Under separate selling:

The result of negotiations with other channels is the same, regardless of our initial assumption of how many other channels are being sold (it doesn't matter if the cable operator is including channel j or not because $\Pi - d$, the marginal profit of including this new channel, is always the same).

$$\max_{T_i}(p_iD_i(p_i) - T_i + p_jD_j(p_j) - T_j - p_jD_j(p_j) + T_j)^{\alpha_i}(T_i - c_i)^{1-\alpha_i} \quad \text{with } i \neq j$$
$$\max_{Ti}\ (p_iD_i(p_i) - T_i)^{\alpha_i}(T_i - c_i)^{1-\alpha_i}$$

$$-\alpha_i(p_iD_i(p_i) - T_i)^{\alpha_i - 1}(T_i - c_i)^{1-\alpha_i} + (1-\alpha_i)(T_i - c_i)^{-\alpha_i}(p_iD_i(p_i) - T_i)^{\alpha_i} = 0$$

$$\alpha_i(p_iD_i(p_i) - T_i)^{\alpha_i - 1}(T_i - c_i)^{1-\alpha_i} = (1-\alpha_i)(T_i - c_i)^{-\alpha_i}(p_iD_i(p_i) - T_i)^{\alpha_i}$$

$$\tfrac{T_i - c_i}{p_iD_i(p_i) - T_i} = \tfrac{1-\alpha_i}{\alpha_i}$$

$$T_i - c_i = \tfrac{1-\alpha_i}{\alpha_i}(p_iD_i(p_i) - T_i)$$

$$T_i(1 + \tfrac{1-\alpha_i}{\alpha_i}) = \tfrac{1-\alpha_i}{\alpha_i}p_iD_i(p_i) + c_i$$

$$T_i^s = (1-\alpha_i)p_iD_i(p_i) + \alpha_i c_i$$

$$T_i^s = (1-\alpha_i)(p_iD_i(p_i) - c_i) + c_i \qquad for\ i = A, B$$

Marginal profits for selling channel i are:

-If it includes channel i, the cable operator gains: $(\Pi - d)^s = p_iD_i(p_i) - T_i^s = p_iD_i(p_i) - (1-\alpha_i)(p_iD_i(p_i) - c_i) - c_i = \alpha_i(p_iD_i(p_i) - c_i)$

-If it includes channel i, the supplier of i gains: $(\Pi_i - d_i)^s = T_i^s - c_i = (1-\alpha_i)(p_iD_i(p_i) - c_i)$

So long as total producer's marginal surplus is positive, $p_iD(p_i) \geqslant c_i$, channel i will be sold (notice that this is independent of α_i).

Taking this transfer payments into account, the cable operator chooses at what price to sell each channel by maximizing its profit:

$$\max_{p_B, p_A}\ (1-p_A)p_A + (1-p_B)p_B - T_A^s - T_B^s$$
$$\max_{p_B, p_A}\ (1-p_A)p_A + (1-p_B)p_B - [(1-\alpha_A)(p_A(1-p_A) - c_A) + c_A] - [(1-\alpha_B)(p_B(1-p_B) - c_B) + c_B]$$

$$\max_{p_B, p_A} \alpha_A(1-p_A)p_A + \alpha_B(1-p_B)p_B - \alpha_A c_A - \alpha_B c_B$$
$$\max_{p_B, p_A} \alpha_A\left[(1-p_A)p_A - c_A\right] + \alpha_B\left[(1-p_B)p_B - c_B\right]$$

$$(p_i) \quad \alpha_i\left[1-2p_i\right] = 0 \quad \text{for } i = A, B$$

$$p_A^* = p_B^* = \frac{1}{2}$$

Second order conditions are met to guarantee this is a maximum. We also need to check that $p_i D(p_i) \geqslant c_i$ for all channels to be included in the bundle. This is the case if: $\frac{1}{2}(1-\frac{1}{2}) \geqslant c_i \Rightarrow \frac{1}{4} \geqslant c_i$

With this prices, transfer payments are:

$T_i^{s*} = (1-\alpha_i)(p_i^*(1-p_i^*) - c_i) + c_i = (1-\alpha_i)(\frac{1}{2}\frac{1}{2} - c_i) + c_i$

$$T_i^{s*} = (1-\alpha_i)\frac{1}{4} + \alpha_i \ c_i \quad for \ i = A, B$$

The cable operator's profits are:

$\Pi_o^s = (1-p_A^*)p_A^* + (1-p_B^*)p_B^* - T_A^{s*} - T_B^{s*} = \frac{1}{4} + \frac{1}{4} - (1-\alpha_A)\frac{1}{4} - \alpha_A c_A - (1-\alpha_B)\frac{1}{4} - \alpha_B c_B$

$\Pi_o^s = \alpha_A(\frac{1}{4} - c_A) + \alpha_B(\frac{1}{4} - c_B)$

where Π_o^s is the marginal profit that the cable operator obtains when it sells channel A plus the marginal profit for selling channel B. The marginal profit of including channel i equals the total producer surplus if only that channel were sold.

The profits for each program provider are:

$\Pi_i^s = (1-\alpha_i)(\frac{1}{4} - c_i)$

where Π_i^s is the marginal profit that channel i gains when negotiating with the cable operator.

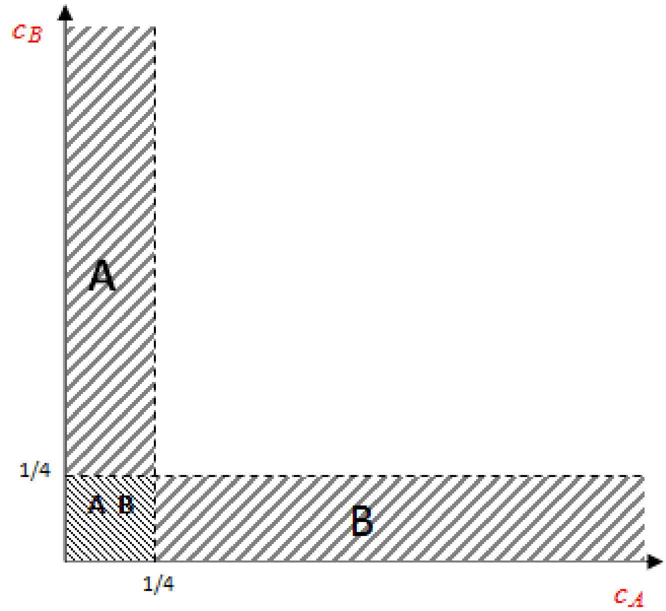
Total surplus is:

$\Pi_A^s + \Pi_B^s + \Pi_o^s = \ \frac{1}{2} - c_A - c_B$

Notice that prices and total producer surplus equal those obtained when the chain is integrated (under separate selling).

If $\frac{1}{4} \geq c_A$ and $\frac{1}{4} \geq c_B$, both channels will be included in the bundle. This case is represented in Figure 7 by the area AB.

Figure 7



If $\frac{1}{4} \geq c_i$ and $\frac{1}{4} < c_j$, there won't be a negotiation between the cable operator and channel j since the marginal profits are negative. Then, if $\frac{1}{4} < c_B \wedge \frac{1}{4} \leq c_A$, the cable operator only sells channel A (area A); if $\frac{1}{4} \geq c_B \wedge \frac{1}{4} < c_A$, it only sells channel B (area B).

Under bundling:

Assuming that j is being sold (we then need to autoconfirm this):

$$\max_{T_i}(p_{AB}D_{AB}(p_{AB}) - T_i - Tj - p_jD_j(p_j) + T_j)^{\alpha_i}(T_i - c_i)^{1-\alpha_i} \qquad \text{with } i \neq j$$

$$\max_{T_i}(p_{AB}D_{AB}(p_{AB}) - T_i - p_jD_j(p_j))^{\alpha_i}(T_i - c_i)^{1-\alpha_i}$$

$\alpha_i(p_{AB}D_{AB}(p_{AB}) - T_i - p_jD_j(p_j))^{\alpha_i-1}(-1)(T_i - c_i)^{1-\alpha_i} + (p_{AB}D_{AB}(p_{ij}) - T_i - p_jD_j(p_j))^{\alpha_i}(1-\alpha_i)(T_i - c_i)^{-\alpha_i} = 0$

$\alpha_i(p_{AB}D_{AB}(p_{AB}) - T_i - p_jD_j(p_j))^{\alpha_i-1}(T_i - c_i)^{1-\alpha_i} = (p_{AB}D_{AB}(p_{AB}) - T_i - p_jD_j(p_j))^{\alpha_i}(1-\alpha_i)(T_i - c_i)^{-\alpha_i}$

$$T_i - c_i = \frac{1-\alpha_i}{\alpha_i}(p_{AB}D_{AB}(p_{AB}) - T_i - p_jD_j(p_j))$$

$$T_i(1+\tfrac{1-\alpha_i}{\alpha_i}) = \tfrac{1-\alpha_i}{\alpha_i}(p_{AB}D_{AB}(p_{AB}) - p_jD_j(p_j)) + c_i$$

$$T_i^b = (1-\alpha_i)(p_{AB}D_{AB}(p_{AB}) - p_jD_j(p_j) - c_i) + c_i \quad \textbf{for } i = A, B$$

Marginal profits for including channel i in the bundle are:

- If it includes channel i, the cable operator gains: $(\Pi - d)^b = p_{AB}D_{AB}(p_{AB}) - T_i^b - p_jD_j(p_j) = p_{AB}D_{AB}(p_{AB}) - (1-\alpha_i)(p_{AB}D_{AB}(p_{AB}) - p_jD_j(p_j) - c_i) - c_i - p_jD_j(p_j) =$
$\alpha_i(p_{AB}D_{AB}(p_{AB}) - p_jD_j(p_j) - c_i)$

- If it includes channel i, the supplier of i gains: $(\Pi_i - d_i)^b = T_i^b - c_i = (1-\alpha_i)(p_{AB}D_{AB}(p_{AB}) - p_jD_j(p_j) - c_i) + c_i - c_i = (1-\alpha_i)(p_{AB}D_{AB}(p_{AB}) - p_jD_j(p_j) - c_i)$

So long as total producer's marginal surplus is positive, $p_{AB}D_{AB}(p_{AB}) - p_jD_j(p_j) - c_i \geqslant 0 \quad \forall i \neq j$, all channels will be included in the bundle (again, this is independent of α_i).

Taking this transfer payments into account, the cable operator chooses at what price to sell the bundle of channels by maximizing its profit:

$$\max_{p_{AB}} (1-\tfrac{1}{2}(p_{AB})^2)p_{AB} - T_A^b - T_B^b$$

$$\max_{p_{AB}} (1-\tfrac{1}{2}(p_{AB})^2)p_{AB} - [(1-\alpha_A)(p_{AB}(1-\tfrac{1}{2}(p_{AB})^2) - p_B(1-p_B) - c_A) + c_A] - [(1-\alpha_B)(p_{AB}(1-\tfrac{1}{2}(p_{AB})^2) - p_A(1-p_A) - c_B) + c_B]$$

$$\max_{p_{AB}} (1-\tfrac{1}{2}(p_{AB})^2)p_{AB}(\alpha_A+\alpha_B-1) + (1-\alpha_A)p_B(1-p_B) + (1-\alpha_B)p_A(1-p_A) - \alpha_Ac_A - \alpha_Bc_B$$

$$(p_{AB}) \quad 1-\tfrac{3}{2}p_{AB}^2 - (1-\alpha_A)(1-\tfrac{3}{2}p_{AB}^2) - (1-\alpha_B)(1-\tfrac{3}{2}p_{AB}^2) = 0$$

$$\left(1-\tfrac{3}{2}p_{AB}^2\right)(1-1+\alpha_A-1+\alpha_B) = 0$$

$$p_{AB}^2 = \tfrac{2}{3}$$

$$p_{AB}^* = \sqrt{\frac{2}{3}}$$

We need to check for second order conditions for this to be a maximum: $SCO : (\alpha_A + \alpha_B - 1)(-3)p_{AB}$

Since price is positive, $p_{AB} \geqslant 0$, depending on the $\alpha's$ we have two possible cases:

1 Case: $\alpha_A + \alpha_B < 1 : SOC > 0$

In this case, the solution found was not a maximum since the objective function is convex. The optimal choice for the cable operator is a price either equal to zero, or equal to or larger than $\sqrt{2}$ and its profit is: $\mathbf{0} + (1-\alpha_A)\frac{1}{2}(1-\frac{1}{2}) + (1-\alpha_B)\frac{1}{2}(1-\frac{1}{2}) - \alpha_A c_A - \alpha_B c_B = (1-\alpha_A)\frac{1}{4} + (1-\alpha_B)\frac{1}{4} - \alpha_A c_A - \alpha_B c_B = \frac{1}{2} - (\frac{1}{4}+c_A)\alpha_A - (\frac{1}{4}+c_B)\alpha_B$.

But none of these possible prices satisfy the condition $p_{AB}D_{AB}(p_{AB}) - p_j D_j(p_j) - c_i \geqslant 0$ (condition necessary to guarantee that all channels are included in the bundle):

$p(1-\frac{1}{2}p^2) - \frac{1}{2}(1-\frac{1}{2}) - c_i \geqslant 0$

$p - \frac{1}{2}p^3 - \frac{1}{4} - c_i \geqslant 0$

$p - \frac{1}{2}p^3 \geqslant \frac{1}{4} + c_i > 0$

This implies:

$(p - \frac{1}{2}p^3) > 0$, therefore price must be $0 < p < \sqrt{2}$.

But the cable operator would never choose these prices.

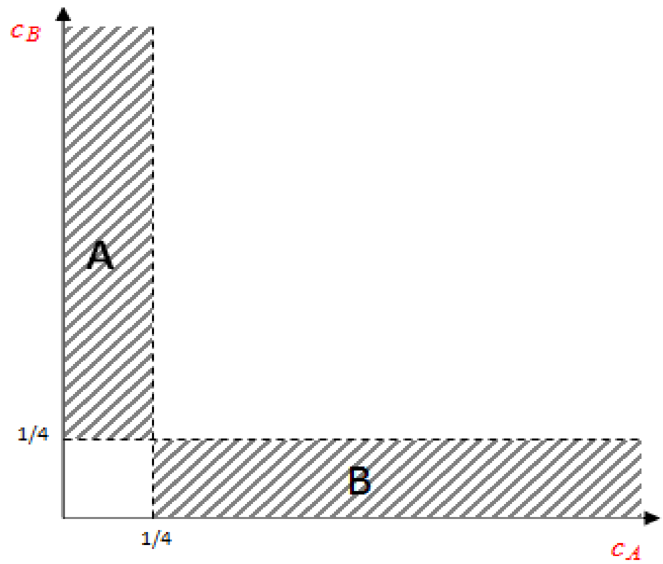
Since we can't find any price that satisfies this condition, the original problem $\max_{p_{AB}} (1-\frac{1}{2}(p_{AB})^2)p_{AB} - T_A^b - T_B^b$ can't be solved assuming the T_A^b and T_B^b are derived from the Nash bargaining problem that assumes that all channels will be included. Then, we need to consider the Nash bargainning problem when only one channel is included:

$\max_{Ti} (p_i D_i(p_i) - T_i)^{\alpha_i}(T_i - c_i)^{1-\alpha_i}$

Conditions that need to be met are: for one channel, $p_i D(p_i) \geqslant c_i$ and for the other $p_j D(p_j) < c_j$.Therefore, if $\frac{1}{4} \geq c_i$ and $\frac{1}{4} < c_j$, channel i will be offered. The areas A and B in Figure 8 refer to those cases.

But when $\frac{1}{4} \geq c_i$ and $\frac{1}{4} \geq c_j$ there is no Nash equilibrium to the bargaining problem given bundling.

Figure 8



2 Case: $\alpha_A + \alpha_B \geqslant 1 : SOC < 0$

In this case, the price obtained is indeed the one that maximizes the cable operator's profit. Moreover, it is the price that maximizes total producer surplus under bundling since the cable operator's problem results in maximizing $p_{AB}D_{AB}(p_{AB})$ multiplied by a positive constant that doesn't alter the solution from the case of total integration.

We need to check $p_{AB}D_{AB}(p_{AB}) - p_j D_j(p_j) - c_i \geqslant 0$ under this optimal price for all the channels to be included. (Remember p_j is the optimal price under separate selling). Indeed: $\sqrt{\frac{2}{3}}(1 - \frac{1}{2}\frac{2}{3}) - \frac{1}{2}(1 - \frac{1}{2}) - c_i \geqslant 0 \Rightarrow 0.29 \approx \frac{-9+8\sqrt{6}}{36} \geqslant c_i$

Transfer payments are $T_i^{b*} = (1-\alpha_i)(p_{AB}^*(1 - \frac{1}{2}(p_{AB}^*)^2) - p_j D_j(p_j) - c_i) + c_i = (1-\alpha_i)(\sqrt{\frac{2}{3}}(1 - \frac{1}{2}\frac{2}{3}) - \frac{1}{2}\frac{1}{2} - c_i) + c_i$ for $i = A, B$

$$T_i^{b*} = (1-\alpha_i)(\sqrt{\frac{2}{3}}\frac{2}{3} - \frac{1}{4} - c_i) + c_i$$

The cable operator's profits are $\Pi_o^b = (1 - \frac{1}{2}(p_{AB}^*)^2)p_{AB}^* - T_A^{b*} - T_B^{b*} = \alpha_A \left(\sqrt{\frac{2}{3}}\frac{2}{3} - \frac{1}{4} - c_A\right) + \alpha_B \left(\sqrt{\frac{2}{3}}\frac{2}{3} - \frac{1}{4} - c_B\right) - \sqrt{\frac{2}{3}}\frac{2}{3} + \frac{1}{2}$

where Π_o^b is the difference between total producer surplus under bundling and the share of marginal surplus created from selling each channel that the program suppliers keep for themselves.

The profits for each program provider are: $\Pi_i^b = (1-\alpha_i)(\sqrt{\frac{2}{3}\frac{2}{3}} - \frac{1}{4} - c_i)$

Total surplus is: $\Pi_A^b + \Pi_B^b + \Pi_o^b = \frac{2}{3}\sqrt{\frac{2}{3}} - c_A - c_B$. Again, we see that price and total producer surplus equal those obtained when the chain is integrated (under bundling).

If the total marginal surplus of including each channel in the bundle is positive, i.e. $c_i \leq \sqrt{\frac{2}{3}\frac{2}{3}} - \frac{1}{4} \simeq 0,29$ $\forall i = A, B$, both channels will be included.

But also for the cable operator to choose bundling, its benefits must be nonnegative. So, the following condition must be satisfied:

$$\Pi_o^b = \alpha_A\left(\sqrt{\tfrac{2}{3}\tfrac{2}{3}} - \tfrac{1}{4} - c_A\right) + \alpha_B\left(\sqrt{\tfrac{2}{3}\tfrac{2}{3}} - \tfrac{1}{4} - c_B\right) - \sqrt{\tfrac{2}{3}\tfrac{2}{3}} - \tfrac{1}{2} \geq 0$$
$$\alpha_B\left(\sqrt{\tfrac{2}{3}\tfrac{2}{3}} - \tfrac{1}{4} - c_B\right) \geq \sqrt{\tfrac{2}{3}\tfrac{2}{3}} - \tfrac{1}{2} - \alpha_A\left(\sqrt{\tfrac{2}{3}\tfrac{2}{3}} - \tfrac{1}{4} - c_A\right)$$
$$\sqrt{\tfrac{2}{3}\tfrac{2}{3}} - \tfrac{1}{4} - c_B \geq \tfrac{1}{\alpha_B}\left[\sqrt{\tfrac{2}{3}\tfrac{2}{3}} - \tfrac{1}{2} - \alpha_A\left(\sqrt{\tfrac{2}{3}\tfrac{2}{3}} - \tfrac{1}{4} - c_A\right)\right]$$
$$c_B \leq \sqrt{\tfrac{2}{3}\tfrac{2}{3}} - \tfrac{1}{4} - \tfrac{1}{\alpha_B}\left[\sqrt{\tfrac{2}{3}\tfrac{2}{3}} - \tfrac{1}{2} - \alpha_A\left(\sqrt{\tfrac{2}{3}\tfrac{2}{3}} - \tfrac{1}{4} - c_A\right)\right]$$
$$c_B \leq \sqrt{\tfrac{2}{3}\tfrac{2}{3}} - \tfrac{1}{4} - \tfrac{1}{\alpha_B}\left(\sqrt{\tfrac{2}{3}\tfrac{2}{3}} - \tfrac{1}{2}\right) + \tfrac{\alpha_A}{\alpha_B}\left(\sqrt{\tfrac{2}{3}\tfrac{2}{3}} - \tfrac{1}{4}\right) - \tfrac{\alpha_A}{\alpha_B}c_A$$
$$c_B \leq \left(\sqrt{\tfrac{2}{3}\tfrac{2}{3}} - \tfrac{1}{4}\right)\left(\tfrac{\alpha_A}{\alpha_B} + 1\right) - \tfrac{1}{\alpha_B}\left(\sqrt{\tfrac{2}{3}\tfrac{2}{3}} - \tfrac{1}{2}\right) - \tfrac{\alpha_A}{\alpha_B}c_A$$
$$c_B \leq \tfrac{0.29(\alpha_A+\alpha_B)-0.044-\alpha_A c_A}{\alpha_B}$$

Or, solving for c_A :

$$c_A \leq \tfrac{0.29(\alpha_A+\alpha_B)-0.044-\alpha_B c_B}{\alpha_A}$$

Since this condition depends on the value of the bargaining powers of the channels and their costs, we study extreme cases in order to gain some intuition:

A)

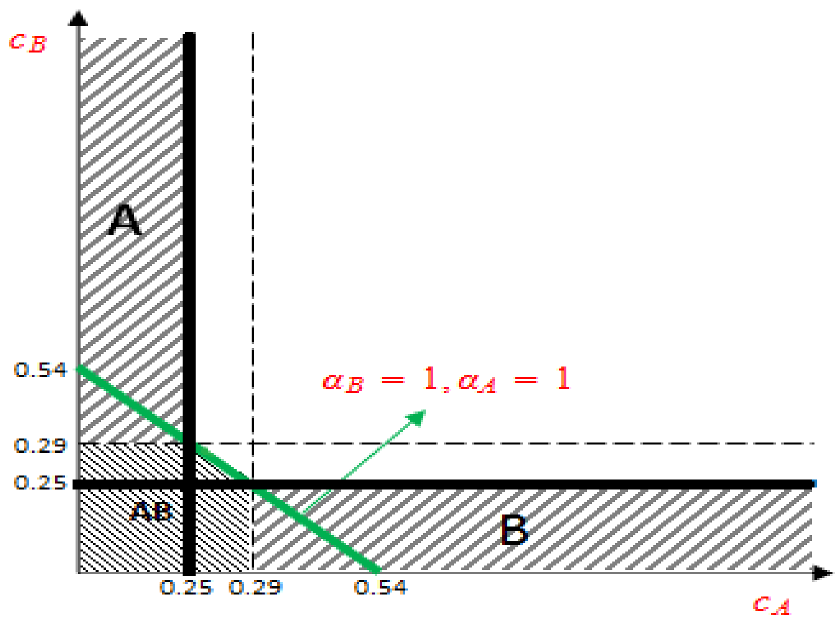
$$\alpha_A = \alpha_B = 1$$
$$c_B \leq \left(\sqrt{\tfrac{2}{3}\tfrac{2}{3}} - \tfrac{1}{4}\right)2 - \left(\sqrt{\tfrac{2}{3}\tfrac{2}{3}} - \tfrac{1}{2}\right) - c_A$$

$$c_B \leq 0.5443 - c_A$$

In Figure 9, $c_B = 0.5443 - c_A$ is represented by the green line. Then, both channels will be included in the bundle if their costs belong to the area AB.

If $\sqrt{\frac{2}{3}\frac{2}{3}} - \frac{1}{4} < c_j$, there will be no negotiation between the cable operator and channel j. But as long as $\frac{1}{4} \geqslant c_i$, the program provider i will sell its channel. Areas A and B refer to those cases.

Figure 9



B)

$\alpha_A \to 0 \wedge \alpha_B = 1$

$c_B \leq \left(\sqrt{\frac{2}{3}\frac{2}{3}} - \frac{1}{4}\right)\left(\frac{0}{1} + 1\right) - \frac{1}{1}\left(\sqrt{\frac{2}{3}\frac{2}{3}} - \frac{1}{2}\right) - \frac{0}{1}c_A$

$c_B \leq \left(\sqrt{\frac{2}{3}\frac{2}{3}} - \frac{1}{4}\right) - \left(\sqrt{\frac{2}{3}\frac{2}{3}} - \frac{1}{2}\right)$

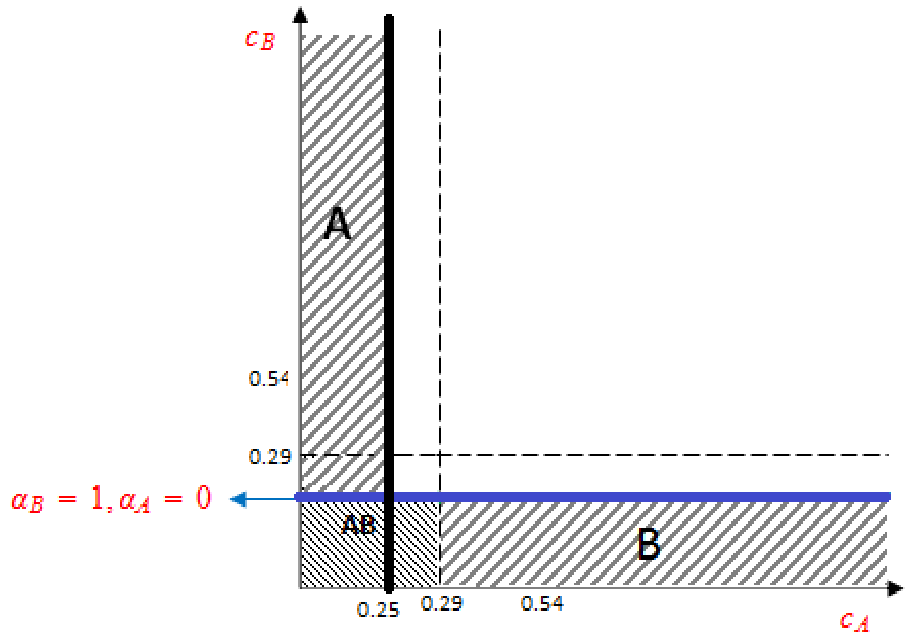
$c_B \leq 0.25$

In Figure 10, $c_B = 0.25$ is represented by the blue line. Then, both channels will be included in the bundle if their costs belong to the area AB.

If $\sqrt{\frac{2}{3}\frac{2}{3}} - \frac{1}{4} < c_A$ and $\frac{1}{4} \geq c_B$, only channel B will be offered (area B).

If $\frac{1}{4} \geq c_A$ and $\frac{1}{4} < c_B$, only channel A will be offered (area A).

Figure 10



C)

$\alpha_A = 1 \wedge \alpha_B \rightarrow 0$

$c_A \leq -\left(\sqrt{\frac{2}{3}\frac{2}{3}} - \frac{1}{2}\right) + \left(\sqrt{\frac{2}{3}\frac{2}{3}} - \frac{1}{4}\right)$

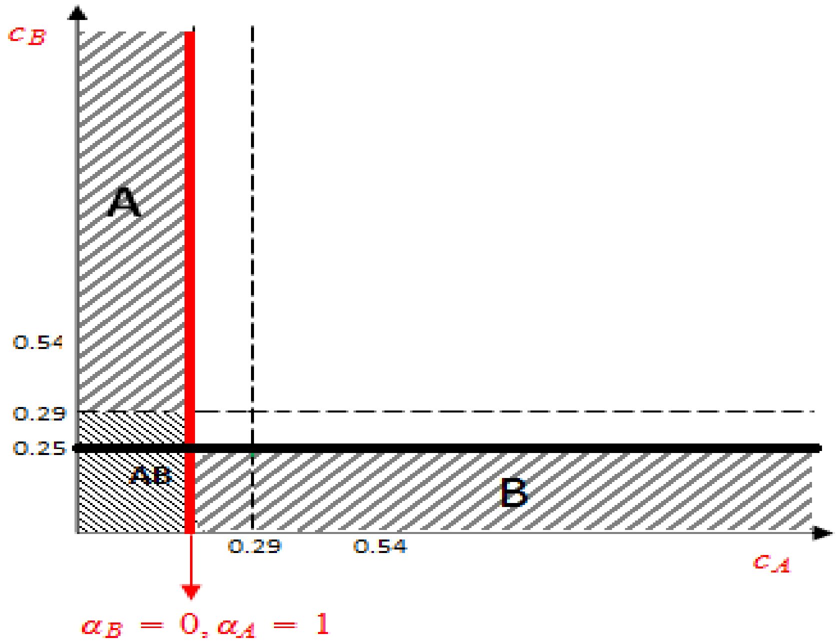
$c_A \leq 0.25$

In Figure 11, $c_A = 0.25$ is represented by the red line. Then, both channels will be included in the bundle if their costs belong to the area AB.

If $\sqrt{\frac{2}{3}\frac{2}{3}} - \frac{1}{4} < c_B$ and $\frac{1}{4} \geq c_A$, only channel A will be offered (area A).

If $\frac{1}{4} \geq c_A$ and $\frac{1}{4} < c_B$, only channel B will be offered (area B).

Figure 11



To sum up, regardless of the bargaining powers, when $\frac{1}{4} \geq c_i \ \ \forall i = A, B$, both channels will be included in the bundle; and when $\sqrt{\frac{2}{3}\frac{2}{3}} - \frac{1}{4} < c_j$ and $\frac{1}{4} \geq c_i$, channel i will be offered.

Comparing bundling vs. separate selling:

It is worth summarizing our two results related to the existence of an equilibrium of the bargaining problem given bundling:

- When $\alpha_A + \alpha_B < 1$ there is no Nash bargaining equilibrium in which the bundle includes both channels. The only possibility, under certain conditions of costs, is selling one channel.
- The only possible equilibrium in which all channels are included requires $\alpha_A + \alpha_B \geq 1$.

The cable operator chooses to bundle or sell channels separately by comparing its benefits under each scenario.

Case: $\alpha_A + \alpha_B \geq 1$

If $\frac{1}{4} \geq c_i \;\; \forall i = A, B$, both bundling and separate selling with both channels being offered are possible.The cable operator prefers bundling rather than separate selling:

$$\Pi_o^b \geq \Pi_o^s$$

$$\alpha_A\left(\sqrt{\frac{2}{3}\frac{2}{3}} - \frac{1}{4} - c_A\right) + \alpha_B\left(\sqrt{\frac{2}{3}\frac{2}{3}} - \frac{1}{4} - c_B\right) - \sqrt{\frac{2}{3}\frac{2}{3}} + \frac{1}{2} \geq \alpha_A(\frac{1}{4} - c_A) + \alpha_B(\frac{1}{4} - c_B)$$

$$\alpha_A\left(\sqrt{\frac{2}{3}\frac{2}{3}} - \frac{1}{4}\right) + \alpha_B\left(\sqrt{\frac{2}{3}\frac{2}{3}} - \frac{1}{4}\right) - \sqrt{\frac{2}{3}\frac{2}{3}} + \frac{1}{2} - \frac{1}{4}\alpha_A - \frac{1}{4}\alpha_B > 0$$

$$\sqrt{\frac{2}{3}\frac{2}{3}}(\alpha_A + \alpha_B - 1) - \frac{1}{2}(\alpha_A + \alpha_B) + \frac{1}{2} \geq 0$$

$$(\alpha_A + \alpha_B)(\sqrt{\frac{2}{3}\frac{2}{3}} - \frac{1}{2}) \geq \sqrt{\frac{2}{3}\frac{2}{3}} - \frac{1}{2}$$

$$\alpha_A + \alpha_B \geq 1$$

If $\sqrt{\frac{2}{3}\frac{2}{3}} - \frac{1}{4} < c_j$ and $\frac{1}{4} \geq c_i$, only channel i will be offered.

The remaining comparisons depend on the value of the bargaining powers. We will analyze the extreme cases that we presented above.

i)

$\alpha_A = \alpha_B = 1$

If $\frac{1}{4} < c_B \leq \sqrt{\frac{2}{3}\frac{2}{3}} - \frac{1}{4}$ and $c_A \leq \frac{1}{4}$, the cable operator chooses to sell both channels as a bundle over selling channel A:

$$\alpha_A\left(\sqrt{\frac{2}{3}\frac{2}{3}} - \frac{1}{4} - c_A\right) + \alpha_B\left(\sqrt{\frac{2}{3}\frac{2}{3}} - \frac{1}{4} - c_B\right) - \sqrt{\frac{2}{3}\frac{2}{3}} + \frac{1}{2} \geq \alpha_A\left(\frac{1}{4} - c_A\right)$$
$$\alpha_A\sqrt{\frac{2}{3}\frac{2}{3}} - \alpha_A\frac{1}{4} + \alpha_B\sqrt{\frac{2}{3}\frac{2}{3}} - \alpha_B\frac{1}{4} - \alpha_B c_B - \sqrt{\frac{2}{3}\frac{2}{3}} + \frac{1}{2} \geq \alpha_A\frac{1}{4}$$
$$(\alpha_A + \alpha_B)\left(\sqrt{\frac{2}{3}\frac{2}{3}} - \frac{1}{4}\right) - \sqrt{\frac{2}{3}\frac{2}{3}} + \frac{1}{2} - \frac{1}{4}\alpha_A \geq \alpha_B c_B$$
$$c_B \leq \frac{(\alpha_A+\alpha_B)}{\alpha_B}\left(\sqrt{\frac{2}{3}\frac{2}{3}} - \frac{1}{4}\right) - \frac{1}{\alpha_B}\left(\sqrt{\frac{2}{3}\frac{2}{3}} - \frac{1}{2}\right) - \frac{1}{4}\frac{\alpha_A}{\alpha_B}$$

$c_B \leq 2\left(\sqrt{\frac{2}{3}\frac{2}{3}} - \frac{1}{4}\right) - \sqrt{\frac{2}{3}\frac{2}{3}} + \frac{1}{2} - \frac{1}{4}$
$c_B \leq \sqrt{\frac{2}{3}\frac{2}{3}} - \frac{1}{4}$

In Figure 12, $c_B = \sqrt{\frac{2}{3}\frac{2}{3}} - \frac{1}{4}$ is represented by the blue line.

If $\frac{1}{4} < c_A \leq \sqrt{\frac{2}{3}\frac{2}{3}} - \frac{1}{4}$ and $c_B \leq \frac{1}{4}$, the cable operator chooses to sell both channels as a bundle over selling channel B:

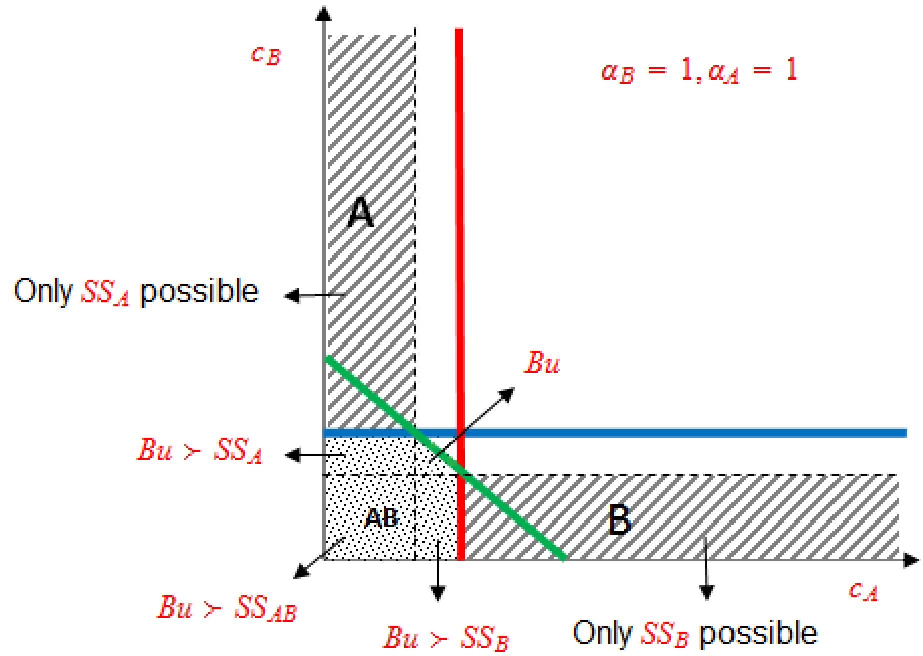
$\alpha_A\left(\sqrt{\frac{2}{3}\frac{2}{3}} - \frac{1}{4} - c_A\right) + \alpha_B\left(\sqrt{\frac{2}{3}\frac{2}{3}} - \frac{1}{4} - c_B\right) - \sqrt{\frac{2}{3}\frac{2}{3}} + \frac{1}{2} \geq \alpha_B\left(\frac{1}{4} - c_B\right)$
$\alpha_A\left(\sqrt{\frac{2}{3}\frac{2}{3}} - \frac{1}{4}\right) - \alpha_A c_A + \alpha_B\left(\sqrt{\frac{2}{3}\frac{2}{3}} - \frac{1}{4}\right) - \sqrt{\frac{2}{3}\frac{2}{3}} + \frac{1}{2} \geq \alpha_B \frac{1}{4}$
$(\alpha_A + \alpha_B)\left(\sqrt{\frac{2}{3}\frac{2}{3}} - \frac{1}{4}\right) - \sqrt{\frac{2}{3}\frac{2}{3}} + \frac{1}{2} - \frac{1}{4}\alpha_B \geq \alpha_A c_A$
$c_A \leq \frac{(\alpha_A+\alpha_B)}{\alpha_A}\left(\sqrt{\frac{2}{3}\frac{2}{3}} - \frac{1}{4}\right) - \frac{1}{\alpha_A}\left(\sqrt{\frac{2}{3}\frac{2}{3}} - \frac{1}{2}\right) - \frac{1}{4}\frac{\alpha_B}{\alpha_A}$
$c_A \leq \frac{(\alpha_A+\alpha_B)}{\alpha_A}\left(\sqrt{\frac{2}{3}\frac{2}{3}} - \frac{1}{4}\right) - \frac{1}{\alpha_A}\left(\sqrt{\frac{2}{3}\frac{2}{3}} - \frac{1}{2}\right) - \frac{1}{4}\frac{\alpha_B}{\alpha_A}$

$c_A \leq 2\left(\sqrt{\frac{2}{3}\frac{2}{3}} - \frac{1}{4}\right) - \left(\sqrt{\frac{2}{3}\frac{2}{3}} - \frac{1}{2}\right) - \frac{1}{4}$
$c_A \leq \sqrt{\frac{2}{3}\frac{2}{3}} - \frac{1}{4}$

In Figure 12, $c_A = \sqrt{\frac{2}{3}\frac{2}{3}} - \frac{1}{4}$ is represented by the red line.

Then, the area AB in the figure sums up different scenarios under which the cable operator prefers selling the channels as a bundle rather than selling them separately. When the channels' costs belong to the square delimited by $c_A = \frac{1}{4}$ and $c_B = \frac{1}{4}$, the cable operator chooses bundling over selling both channels a la carte. The rectangel delimited by $c_B \in [1/4,\ 0.29]$ and $c_A = 1/4$ refers to the case when the firm prefers bundling rather than selling only channel A. For channel B, the case is analogous. Finally, if the channels' costs belong to the triangle included in the area AB, the cable operator chooses bundling since it is the only alternative that brings nonnegative profits. Areas A and B represent those cases when only one channel is sold.

Figure 12



ii)

$\alpha_A \to 0 \wedge \alpha_B = 1$

If $\frac{1}{4} < c_B \le \sqrt{\frac{2}{3}\frac{2}{3}} - \frac{1}{4}$ and $c_A \le \frac{1}{4}$, only channel A will be offered since bundling would bring negative profits for the cable operator (as we analysed before in case B).

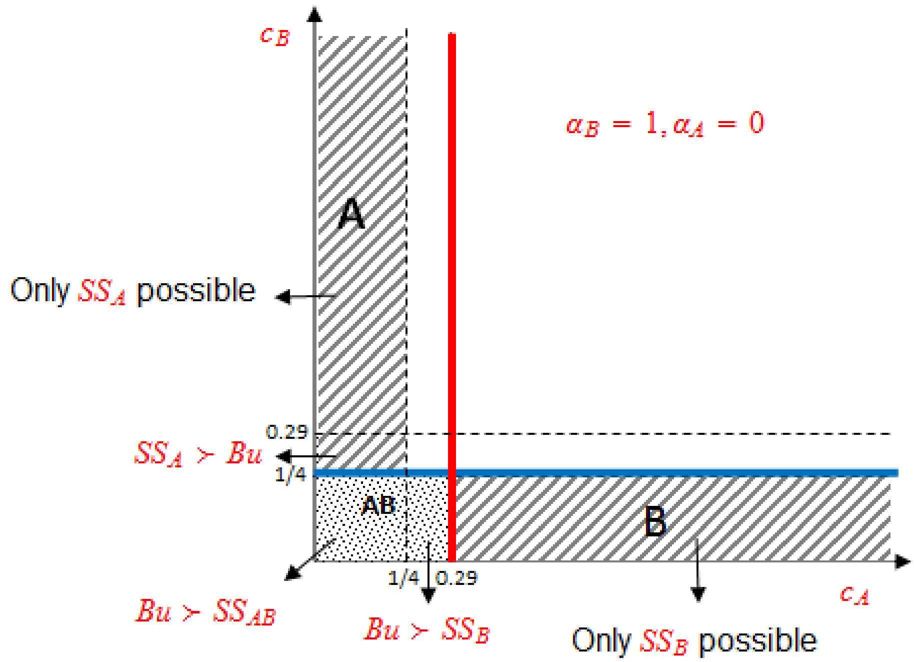
If $\frac{1}{4} < c_A \le \sqrt{\frac{2}{3}\frac{2}{3}} - \frac{1}{4}$ and $c_B \le \frac{1}{4}$, the cable prefers selling both channels as a bundle rather than selling only channel B.

$$c_A \le \lim_{\alpha_A \to 0} \frac{(\alpha_A+\alpha_B)}{\alpha_A}\left(\sqrt{\frac{2}{3}\frac{2}{3}} - \frac{1}{4}\right) - \frac{1}{\alpha_A}\left(\sqrt{\frac{2}{3}\frac{2}{3}} - \frac{1}{2}\right) - \frac{1}{4}\frac{\alpha_B}{\alpha_A} = \lim_{\alpha_A \to 0} \frac{(\alpha_A+\alpha_B)\left(\sqrt{\frac{2}{3}\frac{2}{3}} - \frac{1}{4}\right) - \left(\sqrt{\frac{2}{3}\frac{2}{3}} - \frac{1}{2}\right) - \frac{1}{4}\alpha_B}{\alpha_A} \underset{L'H}{=} \lim_{\alpha_A \to 0} \sqrt{\frac{2}{3}\frac{2}{3}} - \frac{1}{4} = \sqrt{\frac{2}{3}\frac{2}{3}} - \frac{1}{4}$$

In Figure 13, $c_A = \sqrt{\frac{2}{3}\frac{2}{3}} - \frac{1}{4}$ is represented by the red line.

Then, the area AB in the figure sums up different scenarios under which the cable operator prefers selling the channels as a bundle rather than selling them separately. When the channels' costs belong to the square delimited by $c_A = \frac{1}{4}$ and $c_B = \frac{1}{4}$, the cable operator chooses bundling over selling both channels a la carte as usual. The rectangel delimited by $c_A \in [1/4, 0.29]$ and $c_B = 1/4$ refers to the case when the firm prefers bundling rather than selling only channel B. Areas A and B represent those cases when only one channel is sold.

Figure 13



iii)

$\alpha_A = 1 \wedge \alpha_B \to 0$

If $\frac{1}{4} < c_A \leq \sqrt{\frac{2}{3}\frac{2}{3}} - \frac{1}{4}$ and $c_B \leq \frac{1}{4}$, only channel B will be offered since bundling would bring negative profits for the cable operator.

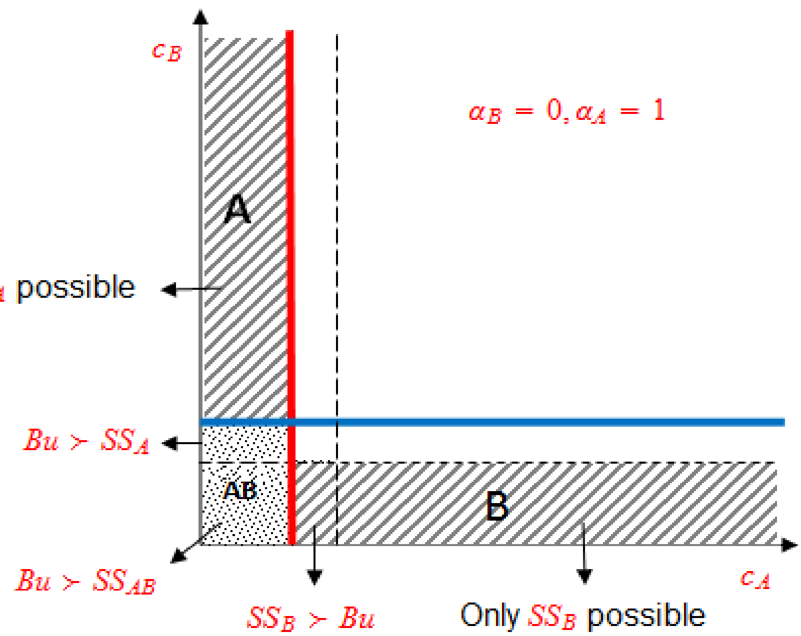
If $\frac{1}{4} < c_B \leq \sqrt{\frac{2}{3}\frac{2}{3}} - \frac{1}{4}$ and $c_A \leq \frac{1}{4}$, the cable prefers selling both channels as a bundle rather than selling only channel A.

$$c_B \leq \lim_{\alpha_A \to 0} \frac{(\alpha_A+\alpha_B)}{\alpha_A}\left(\sqrt{\frac{2}{3}\frac{2}{3}} - \frac{1}{4}\right) - \frac{1}{\alpha_A}\left(\sqrt{\frac{2}{3}\frac{2}{3}} - \frac{1}{2}\right) - \frac{1}{4}\frac{\alpha_B}{\alpha_A} = \sqrt{\frac{2}{3}\frac{2}{3}} - \frac{1}{4}$$

In Figure 14, $c_B = \sqrt{\frac{2}{3}\frac{2}{3}} - \frac{1}{4}$ is represented by the blue line.

The analysis of Figure 14 is analogous from the one done before for Figure 13.

Figure 14



Case: $\alpha_A + \alpha_B < 1$

Only separate selling is possible.

Therefore the cable operator chooses to bundle when its negotiation power is high enough ($\alpha_A + \alpha_B \geq 1$). We saw that under integration of the chain, the integrated firm prefers bundling over separate selling because profits are higher. But in this case, total producer surplus (which equals the profit of the integrated firm of case 1) is divided among the program providers and the cable operator. It is the cable operator who has the power to decide whether bundling or separate selling will take place. So it will compare its own benefits in each scenario, and it is possible that, although bundling maximizes total surplus, negotiation power of the cable operator is not high enough for bundling to be an equilibrium (the cable operator's share of the surplus is not high enough), and so separate selling takes place. Intuitively, as argued in Adilov et al. (2012), greater barganing power implies a greater share of surplus, and hence a greater incentive to maximize the surplus. However, a cable operator in a weak bargaining position has an incentive to cut down the amount of transfer payments paid to the channels by reducing program suppliers' marginal contributions to total surplus, eventhough this decision causes a decrease in total surplus.

4) Semi-bargaining

In stage one, the cable operator decides whether to practice bundling or provide channels à la carte (separate selling).

In stage two, the program provider that does not negotiate with the cable operator (B, without loss of generality) sets the price for its channel.

In stage three, the cable operator sets a price for each channel, if it has decided to sell separately, or for the bundle of channels.

In stage four, the cable operator enters into negotiations with only one program supplier (A, without loss of generality). The result of this negotiation is a transfer payment, T_A, that the cable operator pays to the program provider. The cable operator decides which channels to include in the package keeping its decision of whether to bundle or not.

In stage five, consumers decide whether to subscribe or not to the cable operator. If provision of channels is à la carte, it decides which channels to buy.

Under separate selling:

The assymmetric Nash bargaining problem between the cable operator and program provider A can be stated as follows:

$$\max_{T_A} \, (p_A D_A(p_A) - T_A^s + (p_B - w_B)D_B(p_B) - (p_B - w_B)D_B(p_B))^{\alpha_A}(T_A^s - c_A)^{1-\alpha_A}$$

$$\max_{T_A} \, (p_A D_A(p_A) - T_A^s)^{\alpha_A}(T_A^s - c_A)^{1-\alpha_A}$$

From were we get: $T_A^s = (1-\alpha_A)(p_A D_A(p_A) - c_A) + c_A$

The cable operator takes this transfer payment into account when setting the final price of the channels.

$$\max_{p_A, p_B} \, (1-p_A)p_A + (1-p_B)(p_B - w_B) - T_A^s$$

$$\max_{p_A, p_B} \, (1-p_A)p_A + (1-p_B)(p_B - w_B) - (1-\alpha_A)(p_A(1-p_A) - c_A) - c_A$$

$$\max_{p_A, p_B} (1-p_B)(p_B - w_B) + \alpha_A(p_A(1-p_A) - c_A)$$

$(p_A):\ \alpha_A(1-2p_A) = 0 \rightarrow\ p_A^* = \frac{1}{2}$

$(p_B):\ (1 - 2p_B + w_B) = 0 \rightarrow\ p_B = \frac{1+w_B}{2}$

Given this price for channel B and considering the amount of subscribers, program provider B solves:

$\max_{w_B} w_B \frac{1-w_B}{2} - c_B$
$(w_B): \ \frac{1}{2} - w_B = 0 \rightarrow \ w_B^* = \frac{1}{2} \quad ; \quad p_B^* = \frac{3}{4}$

We need to check that $p_A D(p_A) \geqslant c_A$ for all the channels to be sold. So, $\frac{1}{2}\frac{1}{2} \geqslant c_A \Longrightarrow \frac{1}{4} \geqslant c_A$

Profits are:

$\Pi_o^s = \frac{1}{2}\frac{1}{2} + \frac{1}{4}(\frac{3}{4} - \frac{1}{2}) - (1-\alpha_A)(\frac{1}{2}\frac{1}{2} - c_A) - c_A = \frac{1}{4} + \frac{1}{16} - (1-\alpha_A)(\frac{1}{4} - c_A) - c_A = \frac{5}{16} - (1-\alpha_A)(\frac{1}{4} - c_A) - c_A$
$\Pi_A^s = (1-\alpha_A)(\frac{1}{2}\frac{1}{2} - c_A) + c_A - c_A = (1-\alpha_A)(\frac{1}{4} - c_A)$
$\Pi_B^s = (\frac{1}{2}\frac{1}{4} - c_B) = \frac{1}{8} - c_B$

Program provider B will sell the channel so long as $\frac{1}{8} \geqslant c_B$.

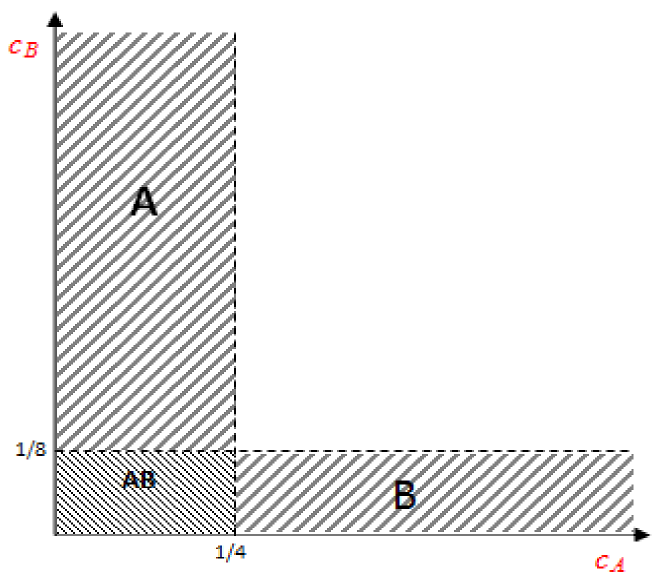
If we sum up the benefits of the cable operator and the two channels, we obtain total producer surplus:
$\Pi_o^s + \Pi_A^s + \Pi_B^s = \frac{5}{16} - c_A + \frac{1}{8} - c_B = \frac{7}{16} - c_A - c_B$

If $\frac{1}{4} \geq c_A$ and $\frac{1}{8} \geq c_B$, both channels will be offered. In Figure 15, this case is represented by the area AB.

If $\frac{1}{4} \geq c_A$ and $\frac{1}{8} < c_B$, the cable operator will negotiate with channel A (area A). The program provider B won't sell its channel.

If $\frac{1}{4} < c_A$ and $\frac{1}{8} \geq c_B$, only channel B will be offered (area B).

Figure 15



Under bundling:

The assymmetric Nash bargaining problem between the cable operator and program provider A can be stated as follows:

$$\max_{T_A} ((p_{AB} - w_B)D(p_{AB}) - T_A^b - (p_B - w_B^s)D_B(p_B))^{\alpha_A}(T_A^b - c_A)^{1-\alpha_A}$$

We distinguish between w_B, which is the price that program provider B sets in this setting, and w_B^s, the price that it sets under separate selling.

$$\max_{T_A} \left((p_{AB} - w_B)D(p_{AB}) - T_A^b - (\tfrac{3}{4} - \tfrac{1}{2})(1 - \tfrac{3}{4})\right)^{\alpha_A}(T_A^b - c_A)^{1-\alpha_A}$$

$$\max_{T_A} \left((p_{AB} - w_B)D(p_{AB}) - T_A^b - (\tfrac{3}{4} - \tfrac{1}{2})(1 - \tfrac{3}{4})\right)^{\alpha_A}(T_A^b - c_A)^{1-\alpha_A}$$

$$\max_{T_A} \left((p_{AB} - w_B)D(p_{AB}) - T_A^b - \tfrac{1}{16}\right)^{\alpha_A}(T_A^b - c_A)^{1-\alpha_A}$$

Solving for T_A :

$$-\alpha_A((p_{AB}-w_B)D(p_{AB})-T_A^b-\tfrac{1}{16})^{\alpha_A-1}(T_A^b-c_A)^{1-\alpha_A}+((p_{AB}-w_B)D(p_{AB})-T_A^b-\tfrac{1}{16})^{\alpha_A}(1-\alpha_A)(T_A^b-c_A)^{-\alpha_A}=0$$

$$\alpha_A((p_{AB} - w_B)D(p_{AB}) - T_A^b - \tfrac{1}{16})^{-1}(T_A^b - c_A) = (1 - \alpha_A)$$

$$\alpha_A T_A^b - \alpha_A c_A = (1 - \alpha_A)((p_{AB} - w_B)D(p_{AB}) - T_A^b - \tfrac{1}{16})$$

$$\alpha_A T_A^b - \alpha_A c_A = (1 - \alpha_A)((p_{AB} - w_B)D(p_{AB}) - \tfrac{1}{16}) - T_A^b + T_A^b\alpha_A$$

$$T_A^b = (1 - \alpha_A)((p_{AB} - w_B)D(p_{AB}) - \tfrac{1}{16}) + \alpha_A c_A$$

$$T_A^b = (1 - \alpha_A)((p_{AB} - w_B)D(p_{AB}) - \tfrac{1}{16} - c_A) + c_A$$

With this in mind, the cable operator solves:

$$\max_{p_{AB}} D(p_{AB})(p_{AB} - w_B) - T_A^b$$

$$\max_{p_{AB}} (1 - \tfrac{1}{2}p_{AB}^2)(p_{AB} - w_B) - (1 - \alpha_A)((p_{AB} - w_B)(1 - \tfrac{1}{2}p_{AB}^2) - \tfrac{1}{16} - c_A) - c_A$$

$$\max_{p_{AB}} \alpha_A(p_{AB}-w_B)(1-\tfrac{1}{2}p_{AB}^2)+(1-\alpha_A)(\tfrac{1}{16}+c_A)-c_A$$

Solving for p_{AB} :

$$-\alpha_A p_{AB}(p_{AB}-w_B)+\alpha_A(1-\tfrac{1}{2}p_{AB}^2)=0$$

$$-p_{AB}^2+p_{AB}w_B+1-\tfrac{1}{2}p_{AB}^2=0$$

$$-\tfrac{3}{2}p_{AB}^2+p_{AB}w_B+1=0$$

$$p_{AB}=\tfrac{1}{3}w_B+\tfrac{1}{3}\sqrt{w_B^2+6}$$
$$p_{AB}=\tfrac{1}{3}w_B-\tfrac{1}{3}\sqrt{w_B^2+6}$$

We need to check for second order conditions for this to be a maximum: $-3\alpha_A p_{AB}+\alpha_A w_B<0 \Longrightarrow p_{AB}>\frac{1}{3}w_B$, which is satisfied when $p_{AB}^*=\frac{1}{3}w_B+\frac{1}{3}\sqrt{w_B^2+6}$.

Number of subscribers will be: $q=1-\frac{1}{2}\left(\frac{1}{3}w_B+\frac{1}{3}\sqrt{w_B^2+6}\right)^2$

Finally, channel B solves:

$$\max_{w_B}\left(1-\tfrac{1}{2}\left(\tfrac{1}{3}w_B+\tfrac{1}{3}\sqrt{w_B^2+6}\right)^2\right)w_B-c_B$$

Solving for w_B :

$$\left(1-\tfrac{1}{2}\left(\tfrac{1}{3}w_B+\tfrac{1}{3}\sqrt{w_B^2+6}\right)^2\right)+\left(-\left(\tfrac{1}{3}w_B+\tfrac{1}{3}\sqrt{w_B^2+6}\right)\left(\tfrac{1}{3}+\tfrac{1}{3}\tfrac{1}{2}\tfrac{1}{\sqrt{w_B^2+6}}2w_B\right)w_B\right)=0$$
$$-\frac{1}{18\sqrt{w_B^2+6}}\left(24w_B-18\sqrt{w_B^2+6}+\left(w_B^2+6\right)^{\frac{3}{2}}+6w_B^3+5w_B^2\sqrt{w_B^2+6}\right)=0$$

Numeric solution is: $w_B^*=0.778\,17$

Therefore: $p_{AB}^*=\frac{1}{3}0.778\,17+\frac{1}{3}\sqrt{0.778\,17^2+6}=1.\,116\,1$

$T_A^{b*}=(1-\alpha_A)\left((p_{AB}-w_B)D(p_{AB})-\frac{1}{16}-c_A\right)+c_A$

$T_A^{b*}=(1-\alpha_A)(0.127\,45-\frac{1}{16}-c_A)+c_A=\alpha_A c_A-0.064\,95\alpha_A+0.064\,95$

We need to check that $(p_{AB}-w_B)D(p_{AB})-\frac{1}{16}-c_A\geq 0$ for channel A to be included in the bundle. So, $(1.\,116\,1-0.778\,17)0.37716-\frac{1}{16}-c_A\geq 0\Longrightarrow 0.064\,95\geq c_A$.

Profits are:

$\Pi_o^b = 0.064\,95\alpha_A - \alpha_A c_A + 0.062\,5$

$\Pi_A^b = \alpha_A c_A - 0.064\,95\alpha_A + 0.064\,95 - c_A$

$\Pi_B^b = 0.293\,50 - c_B$

Program provider B will sell the channel so long as $0.293\,50 \geqslant c_B$.

If we sum up the benefits of the cable operator and the two channels, we obtain total producer surplus:

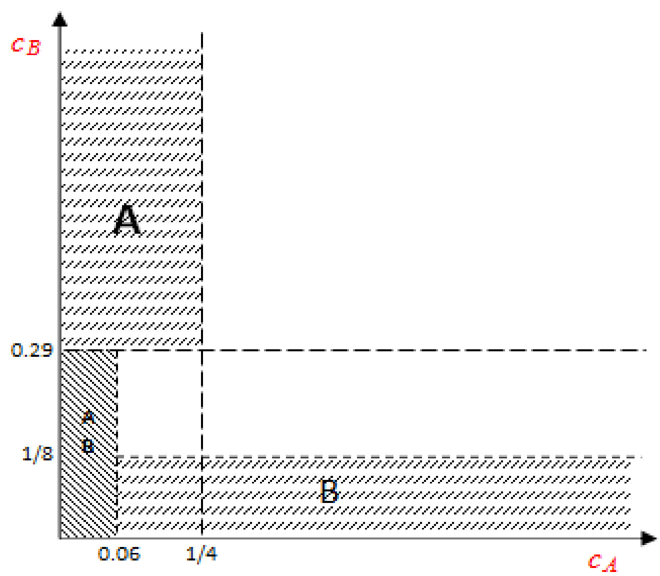
$\Pi_o^b + \Pi_A^b + \Pi_B^b = 0.420\,95 - c_A - c_B$

If $0.064\,95 \geq c_A$.and $0.293\,50 \geqslant c_B$, both channels will be included in the bundle. In Figure 16, this case is represented by the area AB.

If $0.064\,95 < c_A$, there will be no negotiation between the cable operator and channel A. But if $\frac{1}{8} \geqslant c_B$ (area B), the program provider B will sell its channel since $\Pi_B = \frac{1}{8} - c_B$.

If $0.293\,50 < c_B$, the program provider B won't sell its channel. But as long as $\frac{1}{4} \geq c_A$ (area A), channel A will be sold since the profits are: $\Pi_A = (1 - \alpha_A)(\frac{1}{4} - c_A)$ and $\Pi_o = \alpha_A(\frac{1}{4} - c_A)$.

Figure 16



Comparing bundling vs. separate selling:

If $0.064\,95 \geq c_A$ and $\frac{1}{8} \geqslant c_B$, the cable operator chooses to sell channels separately over selling them as a bundle:

$$\Pi_o^s > \Pi_o^b$$

$$\tfrac{5}{16} - (1-\alpha_A)(\tfrac{1}{4} - c_A) - c_A > 0.064\,95\alpha_A - \alpha_A c_A + 0.062\,5$$

$$\tfrac{5}{16} - \tfrac{1}{4} + c_A + \tfrac{1}{4}\alpha_A - \alpha_A c_A - c_A > 0.064\,95\alpha_A - \alpha_A c_A + 0.062\,5$$

$$\tfrac{1}{16} + \tfrac{1}{4}\alpha_A > 0.064\,95\alpha_A + \tfrac{1}{16}$$

$$\tfrac{1}{4} > 0.064\,95$$

If $0.064\,95 < c_A \leq \frac{1}{4}$ and $\frac{1}{8} \geqslant c_B$, the cable operator prefers selling both channels a la carte than selling channel B:

$$\Pi_o^s \geq \Pi_o^{s.B}$$

$$\tfrac{1}{16} + \alpha_A(\tfrac{1}{4} - c_A) \geq \tfrac{1}{16}$$

$$\tfrac{1}{4} \geq c_A$$

In Figure 17, the rectangle delimited by $c_A = 1/4$ and $c_B = 1/8$ sums up these two scenarios under which both channels are sold separately.

If $0.064\,95 \geq c_A$ and $\sqrt{\frac{2}{3}\frac{2}{3}} - \frac{1}{4} \geqslant c_B > \frac{1}{8}$, the cable operator decides between selling both channels as a bundle and selling channel A. Bundling will be practiced if:

$$\Pi_o^b \geq \Pi_o^{s.A}$$

$$\tfrac{1}{16} + \alpha_A(0.06495 - c_A) \geq \alpha_A(\tfrac{1}{4} - c_A)$$

$$\tfrac{1}{16} + \alpha_A(0.06495 - \tfrac{1}{4}) \geq 0$$

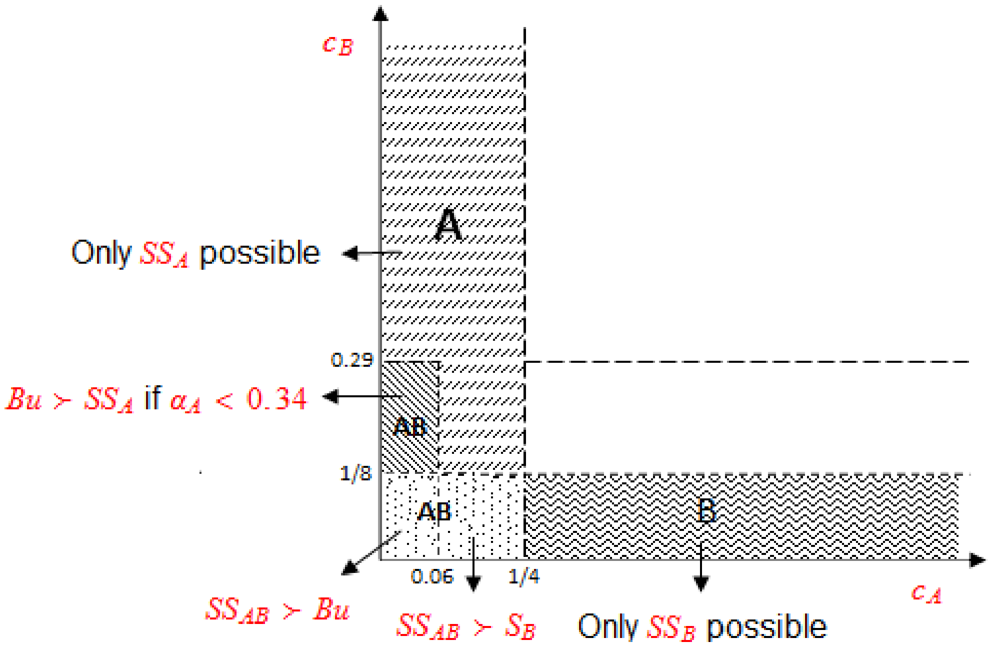
$$\alpha_A \leq 0.3377$$

When the cable operator doesn't have enough bargaining power with channel A, it decides to include channel B in the bundle even though there exists a double marginalization due to the fact that the cable operator doesn't negotiate with the program provider B. Then, if costs belong to the rectangle in Figure 17 delimited by $c_B \in [1/8,\, 0.29]$ and $c_A = 0.06$, both channels will be offered but as a bundle.

If $\frac{1}{4} < c_A$ and $\frac{1}{8} \geqslant c_B$, channel B will be offered. This case is represented by the area B in Figure 17.

If $\sqrt{\frac{2}{3}\frac{2}{3}} - \frac{1}{4} < c_B$ and $\frac{1}{4} \geq c_A$; if $\sqrt{\frac{2}{3}\frac{2}{3}} - \frac{1}{4} > c_B \geq \frac{1}{8}$ and $\frac{1}{4} > c_A \geq 0.06495$, channel A will be offered. These cases are represented by the area A in Figure 17.

Figure 17


c_B
A
Only SS_A possible
0.29
$Bu \succ SS_A$ if $\alpha_A < 0.34$
AB
1/8
AB
B
0.06
1/4
c_A
$SS_{AB} \succ Bu$
$SS_{AB} \succ S_B$
Only SS_B possible

Part III

Conclusions

Our first result is the difference in price and total producer surplus when the chain is desintegrated compared to when it is integrated. Given the decision to bundle or sell channels à la carte, the final price is higher and total producer surplus is smaller when the chain is not integrated. The reason behind this is the existence of a vertical externality referred to by the literature as 'double marginalization'. Simply put, when channel and cable operator set the price of their products (input or final product), they consider their own marginal income and costs instead of the chain's marginal cost. The result is a higher price than that which would maximize total producer surplus.

Remarkably, though, prices set under bundling and separate selling are the same if firms are vertically integrated or take part in an assymmetric Nash bargaining game. Total producer surplus is not less when firms are not integrated but bargain, compared to when they are. This is because the price that is set through the bargaining game, given the decision to bundle or not, maximizes total producer surplus and then is shared among the cable operator and the channels according to their respective bargaining powers. In other words, the Nash bargaining game solves the problem of double marginalization: firms replicate the price and total producer surplus achieved under integration of the chain and then establish a way to share marginal profits (α for the downstream, $1-\alpha$ for the upstream). Therefore when firms are not integrated but negotiate in this bargaining game, there are no vertical externalities to consider.

Our second results refer to the choice of bundling or separate selling. When the chain is integrated, preference for bundling is justified by the traditional reasons that point at the price discrimination that this practice allows. On the contrary, when the chain is desintegrated, the cable operator will choose separate selling: the interaction that bundling creates betweeen the upstream firms is insufficient to guarantee a price of the channels low enough for the cable operator to gain higher profits by selling a cheaper package given that it pays the program provider for each subscriber it gets. Finally, when we allow for a special kind of negotiation to take place between firms (the asymmetric Nash bargaining problem), we see that the power of negotiation of the cable operator is crucial in determining whether bundling will take place.

Part IV

Further extension

Signaling game

Once the firms' pay-offs under different scenarios have been obtained, we are now able to analyse how certain regulations of the media might affect the interaction among the firms. As presented in Part I, one of the main goals of the current audiovisual laws is the desintegration of 'media conglomerates'; this will be our starting point. We build a signaling game (see *Figure 18* for a diagram of the game) as our framework to analyse how the government's actions may modify the status-quo of the economy. Firms have the possibility to negotiate an asymmetric Nash bargaining in order to avoid such regulation. We concentrate on finding the equilibria when channels are being sold under separate selling. A further extension of our work would be to repeat this procedure when channels are being sold in a bundle, and then compare both cases to decide, given the equilibria of each game, if the cable operator would prefer to sell programmes à la carte or in a bundle.

There are five players in the game: nature, a cable operator, two channels and a government. The government has preferences over the dynamics of the chain of production: it preferes desintegration over integration. All firms maximize benefits. Initially, the cable operator is vertically integrated with both channels.

First, nature decides if the government is one of type regulator or not: a regulator government will impose a regulation to force the desintegration of the chain, a non regulator government will not; with probability $\Pr(R)$ it is of type regulator, with probability $\Pr(NR)$ it is of type non regulator, where $\Pr(R)+\Pr(NR) = 1$. Only the government knows its own type. Regulation, if passed, will be implemented in the following period; but at present the government sends a message to the firms: it can choose either to threat or not. Having received this message, firms have to decide now what the dynamics of the chain will be tomorrow. They do this simultaneously, by deciding whether to negotiate or not. Whatever their decision is in this period, they cannot change it tomorrow (due to bureaucracy and legal costs):

-If they all choose to maintain the status quo (NN), two things can happen: either no regulation is passed and so the chain remains totally integrated, or regulation is passed and forces the desintegration of the chain.

-If they all choose to desintegrate the chain but agree over a negotiation for the following period, an asymmetric Nash bargaining game will take place either if regulation is passed or not.

-If the cable operator and only one channel agree over a negotiation for the following period, an asymmetric Nash bargaining game will take place between them.

-If the cable operator refuses to agree over a negotiation for the following period (no matter what the channels decide), things are kept as they are: if regulation is passed, the chain remains totally integrated, otherwise regulation will a force its desintegration. We are not allowing for horizontal integration between channels. O'brien and Shaffer (2003) analysed the output and profit effects of horizontal mergers between upstream firms in intermediate-goods market.

A strategy for the government specifies the message it will send if it is of type regulator (R) and non regulator (NR). The possible strategies of the government are: (T, T), (T, NT), (NT, T), (NT, NT), where T stands for threat and NT, not threat.

A strategy for each firm specifies its decision when it receives message T and message NT. The possible strategies of each firm are: (N, N), (NN, NN), (N, NN), (NN, N), where N stands for negotiate and NN, not negotiate.

Given the message they observe, firms have beliefs on which type of government could have sent it. Such beliefs are represented by the probability distribution $\mu(type\ of\ government/message\ received)$ where $\mu \geqslant 0$ for both types of government.

$p = \mu(R/NT)$

$1 - p = \mu(NR/NT)$

$q = \mu(R/T)$

$1 - q = \mu(NR/T)$

Pay-offs of the firms under each possible scenario were already found and are summarized below. Given separate selling, four situations can take place: the chain is integrated; all the members of the chain negotiate through an asymmetric Nash bargaining game; the cable operator negotiates through an asymmetric Nash bargaining game with only one channel; or the chain is desintegrated.

* Integration (I)

$\Pi^s = \frac{1}{2} - c_A - c_B$ where Π^s are the benefitis of the chain.

We suppose that each member of the chain gets a fraction β_i of Π^s, with $i = O, A, B$ and $\sum \beta_i = 1$.

$\Pi^s_o = \beta_O(\frac{1}{2} - c_A - c_B)$

$\Pi^s_A = \beta_A(\frac{1}{2} - c_A - c_B)$

$\Pi^s_B = \beta_B(\frac{1}{2} - c_A - c_B)$

* Bargaining (B)

$\Pi_o^s = \alpha_A(\frac{1}{4} - c_A) + \alpha_B(\frac{1}{4} - c_B)$

$\Pi_A^s = (1 - \alpha_A)(\frac{1}{4} - c_A)$

$\Pi_B^s = (1 - \alpha_B)(\frac{1}{4} - c_B)$

* Semi bargaining between channel A and the cable operator (SB with C_A)

$\Pi_o^s = \frac{5}{16} - (1 - \alpha_A)(\frac{1}{4} - c_A) - c_A = \frac{1}{16} + \alpha_A(\frac{1}{4} - c_A)$

$\Pi_A^s = (1 - \alpha_A)(\frac{1}{4} - c_A)$

$\Pi_B^s = \frac{1}{8} - c_B$

* Semi bargaining between channel B and the cable operator (SB with C_B)

$\Pi_o^s = \frac{5}{16} - (1 - \alpha_B)(\frac{1}{4} - c_B) - c_B = \frac{1}{16} + \alpha_B(\frac{1}{4} - c_B)$

$\Pi_A^s = \frac{1}{8} - c_A$

$\Pi_B^s = (1 - \alpha_B)(\frac{1}{4} - c_B)$

* Desintegration (D)

$\Pi_o^s = \frac{1}{8}$

$\Pi_A^s = \frac{1}{8} - c_A$

$\Pi_B^s = \frac{1}{8} - c_B$

Notice that our analysis will focus on cases where both channels are being sold. Thus, taking into account the conditions over the firms' costs that were derived in our previous analysis, they should be no larger than $\frac{1}{8}$.

As for the government, its preferences are: desintegration $\succ$ semi-negotiation $\succ$ negotiation $\succ$ integration. Pay-offs are, accordingly: 3, 2, 1, 0. The government can know which scenario has taken place either by observing prices or because of legislation regarding registration of property rights.

In a signaling game, equilibria can be of two types: in a pooling equilibrium, the government chooses to send the same message, whether it be type regulator or non regulator. In a separating equilibrium, it sends a different message in each case. We derive conditions under which each equilibrium found can be hold and retrieve some intuition behind these conditions.

We begin by analysing the firms' decisions of the simultaneous game. Having observed message NT (therefore the relevant beliefs are p and $(1-p)$, we define the following matrices of pay-offs in order to illustrate this three-player simultaneous game (for reasons of clarity in the exposition, instead of rewriting pay-offs we indicate which of the four possible scenarios will take place; pay-offs can be looked up in the summary above).

A) Given that the cable operator chooses N,

$\frac{C_B}{C_A}$	N	NN
N	B	$SB\ with\ C_A$
NN	$SB\ with\ C_B$	$pD+(1-p)I$

B) Given that the cable operator chooses NN,

$\frac{C_B}{C_A}$	N	NN
N	$pD+(1-p)I$	$pD+(1-p)I$
NN	$pD+(1-p)I$	$pD+(1-p)I$

C) Given that channel A chooses N,

$\frac{C_B}{C_o}$	N	NN
N	B	$SB\ with\ C_A$
NN	$pD+(1-p)I$	$pD+(1-p)I$

D) Given that channel A chooses NN,

$\frac{C_B}{C_o}$	N	NN
N	$SB\ with\ C_B$	$pD+(1-p)I$
NN	$pD+(1-p)I$	$pD+(1-p)I$

E) Given that channel B chooses N,

$\frac{C_A}{C_o}$	N	NN
N	B	$SB\ with\ C_B$
NN	$pD+(1-p)I$	$pD+(1-p)I$

F) Given that channel B chooses NN,

$\frac{C_A}{C_o}$	N	NN
N	$SB\ with\ C_A$	$pD+(1-p)I$
NN	$pD+(1-p)I$	$pD+(1-p)I$

We introduce two additional costs: decision to negotitate comes at a cost G that can be interpreted as the price the firm pays for consultancy plus transactional costs if negotiation takes places; that is: $G = \{\underline{g}, \overline{g}\}$, where $\underline{g}$ is a fixed cost if the firm has decided to negotiate but bargaining fails and $\overline{g}$ is the cost if negotiations succeed, with $\underline{g} < \overline{g}$. Also if the government chooses to threat, whenever all the firms decide to negotiate each of them faces a cost of $2h$; and when the negotiation is between the cable operator and only one channel, the cost is h. These costs arise due to the fact that since the government is more aware of the behaviour of the firms, their arrangements are costly as they have to be more careful about not being caught by the government. Therefore this cost is increasing in the quantity of channels, as it is easier to the cable operator to negotiate with one channel than with two.

$\Longrightarrow$ *Possible equilibria under NT are:*

In each case, we present the conditions that must be satisfied in order to hold these equilibriums. For more detail, *see Appendix, part C.*

- $(C_O, C_A, C_B) : (N, N, N)$

$$(1-\alpha_B)(\tfrac{1}{4}-c_B)-\overline{g} > \tfrac{1}{8}-c_B$$
$$(1-\alpha_A)(\tfrac{1}{4}-c_A)-\overline{g} > \tfrac{1}{8}-c_A$$
$$\alpha_A(\tfrac{1}{4}-c_A)+\alpha_B(\tfrac{1}{4}-c_B)-\overline{g} > p\tfrac{1}{8}+(1-p)\beta_o(\tfrac{1}{2}-c_A-c_B)$$

- $(C_o, C_A, C_B) : (N, NN, N)$

$(1-\alpha_B)(\frac{1}{4}-c_B)-\overline{g} > p(\frac{1}{8}-c_B)+(1-p)\beta_B(\frac{1}{2}-c_A-c_B)$

$(1-\alpha_A)(\frac{1}{4}-c_A)-\overline{g} < \frac{1}{8}-c_A.$

$\frac{5}{16}-(1-\alpha_B)(\frac{1}{4}-c_B)-c_B-\overline{g} > p\frac{1}{8}+(1-p)\beta_o(\frac{1}{2}-c_A-c_B)$

- $(C_O, C_A, C_B) : (N, N, NN)$

$(1-\alpha_B)(\frac{1}{4}-c_B)-\overline{g} < \frac{1}{8}-c_B$

$(1-\alpha_A)(\frac{1}{4}-c_A)-\overline{g} > p(\frac{1}{8}-c_A)+(1-p)\beta_A(\frac{1}{2}-c_A-c_B)$

$\frac{5}{16}-(1-\alpha_A)(\frac{1}{4}-c_A)-c_A-\overline{g} > p\frac{1}{8}+(1-p)\beta_o(\frac{1}{2}-c_A-c_B)$

- $(C_O, C_A, C_B) : (NN, NN, NN)$

No conditions are needed.

Therefore, the equilibria that can take place in the simultaneous game among the firms when they have observed NT depend upon conditions on their baragaining powers (α_A and α_B), on how profits are distributed under integration of the chain (β_o, β_A, β_B), on costs (c_A, c_B, $\underline{g}$, $\overline{g}$) and beliefs.

$\Longrightarrow$ *Under T possible equilibria are:*

- $(C_O, C_A, C_B) : (N, N, N)$

$(1-\alpha_B)(\frac{1}{4}-c_B)-\overline{g}-2h > \frac{1}{8}-c_B$

$(1-\alpha_A)(\frac{1}{4}-c_A)-\overline{g}-2h > \frac{1}{8}-c_A$

$\alpha_A(\frac{1}{4}-c_A)+\alpha_B(\frac{1}{4}-c_B)-\overline{g}-2h > q\frac{1}{8}+(1-q)\beta_o(\frac{1}{2}-c_A-c_B)$

- $(C_o, C_A, C_B) : (N, NN, N)$

$(1-\alpha_B)(\frac{1}{4}-c_B)-\overline{g}-h > q(\frac{1}{8}-c_B)+(1-q)\beta_B(\frac{1}{2}-c_A-c_B)$

$(1-\alpha_A)(\frac{1}{4}-c_A)-\overline{g}-2h < \frac{1}{8}-c_A$

$\frac{5}{16}-(1-\alpha_B)(\frac{1}{4}-c_B)-c_B-\overline{g}-h > q\frac{1}{8}+(1-q)\beta_o(\frac{1}{2}-c_A-c_B)$

- $(C_O, C_A, C_B) : (N, N, NN)$

$(1-\alpha_B)(\frac{1}{4}-c_B)-\overline{g}-2h<\frac{1}{8}-c_B$
$(1-\alpha_A)(\frac{1}{4}-c_A)-\overline{g}-h>q(\frac{1}{8}-c_A)+(1-q)\beta_A(\frac{1}{2}-c_A-c_B).$
$\frac{5}{16}-(1-\alpha_A)(\frac{1}{4}-c_A)-c_A-\overline{g}-h>q\frac{1}{8}+(1-q)\beta_o(\frac{1}{2}-c_A-c_B).$

- $(C_O, C_A, C_B) : (NN, NN, NN)$

No conditions are needed.

Once the possible equilibriums of this simultaneous game between firms have been obtained, we analyse the government's strategy and, according to it, we specify the signaling game's possible equilibria. For clarity of exposition, we study which of the possible types of equilibria of a signaling game (pooling equilibrium, separating equilibrium) can be sustained and under which conditions. In the following pages, separating equilibriums will be derived. For pooling equilibriums, see *Appendix, part D.*

Separating equilibrium (NT,T): "the deceiving government"

We want to see if there exists a Perfect Bayesian Equilibrium (PBE) in which the government chooses the strategy (NT, T), what we call a "deceiving" strategy because a government that will not regulate sends a message of threat whereas a government that will indeed regulate does not threat. The firms' beliefs after observing the message from the government are: $p = 1$, $q = 0$. Depending on conditions over the channels' costs (c_A, c_B), the firm's negotiating powers (α_A, α_B) and their share of profits under integration $(\beta_O, \beta_A, \beta_B)$, the optimal strategies of the firms that could hold this separating equilibrium are:

We see some of this cases in detail. Note that these represent equilibria of the simultaneous game that takes place among the firms. Furthermore, notice that *not all of these are potential equilibria at the same time*, this will depend on conditons over the above-mentioned parameters.

1. $(C_O, C_A, C_B) : \{(N, NN), (N, NN), (N, NN)\}$: *"the risk-averse firms"*

This refers to an equilibrium of the simultaneous game in which all firms decide to negotiate only when the government threats. The general conditions that sustain this equilibrium were previously stated; under the beliefs that correspond to this separating equilibrium, they result in:

Condition 1 $(1-\alpha_A)(\frac{1}{4}-c_A)-\overline{g}-2h>\frac{1}{8}-c_A$

Condition 2 $(1-\alpha_B)(\frac{1}{4}-c_B)-\overline{g}-2h>\frac{1}{8}-c_B$

Condition 3 $\alpha_A(\frac{1}{4}-c_A)+\alpha_B(\frac{1}{4}-c_B)-\overline{g}-2h>\beta_o(\frac{1}{2}-c_A-c_B)$

From condition (1), $\alpha_A<\frac{1-8\overline{g}-16h}{8(\frac{1}{4}-c_A)}$

Analogously, from condition (2), $\alpha_B<\frac{1-8\overline{g}-16h}{8(\frac{1}{4}-c_B)}$

Given that channel B (A) negotiates with the cable operator, for channel A (B) to prefer to negotiate, its benefits from the asymmetric Nash Barganing must be higher than its benefits from the semi-negotiation scenario with B (A). In order to satisfy those conditions, the bargaining powers of both channels must be high enough. We should bear in mind that if, for example, channel A didn't negotiate given that the other channel did negotiate with the cable operator, channel A would be a leader when choosing w_A.

From condition (1), instead of solving for α_A, we can solve for c_A : $c_A>\frac{-1}{8\alpha_A}+\frac{1}{4}+\frac{\overline{g}+2h}{\alpha_A}$.

If $\frac{-1}{8\alpha_A}+\frac{1}{4}+\frac{\overline{g}+2h}{\alpha_A}<0\Rightarrow\alpha_A<\frac{1-8\overline{g}-16h}{2}$, so the condition on c_A is always satisfied since $c_A\geq 0$.

If $\frac{-1}{8\alpha_A}+\frac{1}{4}+\frac{\overline{g}+2h}{\alpha_A}>0\Rightarrow 1>\alpha_A>\frac{1-8\overline{g}-16h}{2}$, so c_A must be sufficiently high.

Given that the government is of type R, we need to check that it chooses NT.

If the government chooses T, the asymmetric Nash Bargaining will take place in the future because all firms have chosen N. The government's pay-off is 1.

If it chooses NT, firms will end up desintegrated because they have chosen NN and the government regulates. The government's pay-off is 3.

The government prefers NT rather than T.

Given that the government is of type NR, we need to check that it chooses T.

If the government chooses T, result is the asymmetric Nash Bargaining and the government's pay-off, 1.

If it chooses NT, firms will end up just as they are now, integrated, because they have chosen not to negotiate and the government does not regulate. The government's pay-off is 0.

The government prefers T rather than NT.

$\Longrightarrow$ There is an equilibrium:

$$\{(NT,T),(N,NN),(N,NN),(N,NN),\ q=0,\ p=1\}$$

We can think of this equilibrium as "deceiving": since firms decide to negotiate when the government threats, a government that in reality will not regulate takes advantage of this and sends a message of threat; this way, it deceives the firms into desintegration without actually passing a regulation. On the contrary, a government that will regulate, decides not to send a message of threat that would alert the firms and encourage them to negotiate in order to avoid regulation.

2. $(C_O, C_A, C_B) : \{(N,N),(N,N),(N,N)\}$: *"the untrusted government"*

In this case, firms negotiate regardless of the government's message.

Condition 4 $(1-\alpha_A)(\frac{1}{4}-c_A)-\overline{g} > \frac{1}{8}-c_A$

Condition 5 $(1-\alpha_B)(\frac{1}{4}-c_B)-\overline{g} > \frac{1}{8}-c_B$

Condition 6 $\alpha_A(\frac{1}{4}-c_A)+\alpha_B(\frac{1}{4}-c_B)-\overline{g} > \frac{1}{8}$

Condition 7 $(1-\alpha_B)(\frac{1}{4}-c_B)-\overline{g}-2h > \frac{1}{8}-c_B$

Condition 8 $(1-\alpha_A)(\frac{1}{4}-c_A)-\overline{g}-2h > \frac{1}{8}-c_A$

Condition 9 $\alpha_A(\frac{1}{4}-c_A)+\alpha_B(\frac{1}{4}-c_B)-\overline{g}-2h > \beta_o(\frac{1}{2}-c_A-c_B)-h$

Note that if conditions (4) and (5) hold, then conditions (1) and (2) are satisfied.

Given that the government is of type R, we need to check that it chooses NT.

If the government chooses either T or NT, firms negotiate and result is the asymmetric Nash Bargaining. The government's pay-off is 1.

Therefore, the government is indifferent between T and NT. Choosing NT is possible.

Given that the government is of type NR, we need to check that it chooses T.

If the government chooses either T or NT, firms negotiate and result is the asymmetric Nash Bargaining. The government's pay-off is 1.

The government is indifferent between T and NT. Choosing T is possible.

$\Rightarrow$There is an equilibrium:

$$\{(NT,T),(N,N),(N,N),(N,N),\ q=0,\ p=1\}$$

3. $(C_O, C_A, C_B) : \{(NN, NN), (NN, NN), (NN, NN)\}$: *"the passive firms"*

In this case, all firms decide to stay as they are, regardless of the government's message, so no negotiation takes place.

Given that the government is of type R, we need to check that it chooses NT.

If the government chooses either T or NT, firms do not negotiate and result is desintegration. The government's pay-off is 3.

Therefore, the government is indifferent between T and NT. Choosing NT is possible.

Given that the government is of type NR, we need to check that it chooses T.

If the government chooses either T or NT, firms do not negotiate and result is total integration of the chain. The government's pay-off is 0.

The government is indifferent between T and NT. Choosing T is possible.

⇒There is an equilibrium:

$$\{(NT, T), (NN, NN), (NN, NN), (NN, NN),\ q = 0,\ p = 1\}$$

4. $(C_O, C_A, C_B) : \{(N, N), (N, N), (NN, NN)\}$, i.e., both the cable operator and channel A negotiate whichever message they see, whereas channel B refrains from negotiating.

Condition 10 $(1 - \alpha_A)(\frac{1}{4} - c_A) - \overline{g} - h > \beta_A(\frac{1}{2} - c_A - c_B)$

Condition 11 $(1 - \alpha_B)(\frac{1}{4} - c_B) - \overline{g} < \frac{1}{8} - c_B$

Condition 12 $(1 - \alpha_A)(\frac{1}{4} - c_A) - \overline{g} > \frac{1}{8} - c_A$

Condition 13 $\frac{5}{16} - (1 - \alpha_A)(\frac{1}{4} - c_A) - c_A - \overline{g} - h > \beta_o(\frac{1}{2} - c_A - c_B)$

Condition 14 $(1 - \alpha_B)(\frac{1}{4} - c_B) - \overline{g} - 2h < \frac{1}{8} - c_B$

Condition 15 $\frac{5}{16} - (1 - \alpha_A)(\frac{1}{4} - c_A) - c_A - \overline{g} > \frac{1}{8}$

Note that if condition (2) holds, then condition (5) is satisfied.

From condition (8), since $\frac{1}{2} - c_A - c_B > 0$, $\beta_A < \frac{(1-\alpha_A)(\frac{1}{4}-c_A)-\overline{g}}{\frac{1}{2}-c_A-c_B}$. Given that channel B chooses not to negotiate, if channel A refuses to negotiate with the cable operator the result will be total integration if there is no regulation. Channel A would rather negotiate only if its share of the profits under integration is small enough.

Given that the government is of type R, we need to check that it chooses NT.

If the government chooses either T or NT, the cable operator ends up negotiating only with A. The government's pay-off is 2.

The government is therefore indifferent between T and NT.Choosing NT is possible.

Given that the government is of type NR, we need to check that it chooses T.

If the government chooses either T or NT, the cable operator ends up negotiating only with A. The government's pay-off is 2.

The government is indifferent between T and NT.Choosing T is possible.

$\Longrightarrow$ There is an equilibrium:

$$\{(NT,T),(N,N),(N,N),(NN,NN),\ q=0,\ p=1\}$$

5. (C_O, C_A, C_B) : $\{(N,N),(NN,NN),(N,N)\}$, i.e., in this case it is channel A who refrains from negotiation.

Condition 16 $(1-\alpha_A)(\frac{1}{4}-c_A)-\overline{g}<\frac{1}{8}-c_A$

Condition 17 $(1-\alpha_A)(\frac{1}{4}-c_A)-\overline{g}-2h<\frac{1}{8}-c_A$

Condition 18 $(1-\alpha_B)(\frac{1}{4}-c_B)-\overline{g}-h>\beta_B(\frac{1}{2}-c_A-c_B)$

Condition 19 $\frac{5}{16}-(1-\alpha_B)(\frac{1}{4}-c_B)-\overline{g}-h>\beta_o(\frac{1}{2}-c_A-c_B)$

Condition 20 $(1-\alpha_B)(\frac{1}{4}-c_B)-\overline{g}>\frac{1}{8}-c_B$

Condition 21 $\frac{5}{16}-(1-\alpha_B)(\frac{1}{4}-c_B)-c_B-\overline{g}>\frac{1}{8}$

Note that if condition (1) holds, then condition (2) is satisfied.

Given that the government is of type R, we need to check that it chooses NT.

If the government chooses either T or NT, the cable operator ends up negotiating only with B. The government's pay-off is 2.

The government is indifferent between T and NT.Choosing NT is possible.

Given that the government is of type NR, we need to check that it chooses T.

If the government chooses either T or NT, the cable operator ends up negotiating only with B. The government's pay-off is 2.

The government is indifferent between T and NT.Choosing T is possible.

$\Longrightarrow$ There is an equilibrium:

$$\{(NT, T), (N, N), (NN, NN), (N, N),\ q = 0,\ p = 1\}$$

6. $(C_O, C_A, C_B) : \{(N, NN), (N, NN), (NN, NN)\}$: *"the passive channel"*

In this equilibrium of the simultaneous game, the cable operator and channel A decide to negotiate if they observe a threat (channel B chooses not to negotiate) and all firms choose not to negotiate if they observe no threat.

Condition 22 $(1-\alpha_B)(\frac{1}{4}-c_B)-\overline{g}-2h<\frac{1}{8}-c_B$

Condition 23 $(1-\alpha_A)(\frac{1}{4}-c_A)-\overline{g}-h>\beta_A(\frac{1}{2}-c_A-c_B)$.

Condition 24 $\frac{5}{16}-(1-\alpha_A)(\frac{1}{4}-c_A)-c_A-\overline{g}-h>\beta_o(\frac{1}{2}-c_A-c_B)$.

Given that the government is of type R, we need to check that it chooses NT.

If the government chooses T, there will be an Nash negotiation between the cable operator and channel A only.The government's pay-off is 2.

If it chooses NT, since firms do not negotiate and regulation takes place, result is the desintegration of the chain. The government's pay-off is 3.

The government prefers NT rather than T.

Given that the government is of type NR, we need to check that it chooses T.

If the government chooses T, there will be an Nash negotiation between the cable operator and channel A only.The government's pay-off is 2.

If it chooses NT, since firms do not negotiate but regulation does not take place, the result is an integrated chain. The government's pay-off is 0

The government prefers T rather than NT.

$\Longrightarrow$ There is an equilibrium:

$$\{(NT,T),(N,NN),(N,NN),(NN,NN),\ q=0,\ p=1\}$$

7. $(C_O,C_A,C_B):\{(N,NN),(NN,NN),(N,NN)\}$: *"the passive channel"*

The cable operator and channel B decide to negotiate if they observe a threat (channel A chooses not to negotiate) and all firms choose not to negotiate if they observe no threat.

Condition 25 $(1-\alpha_B)(\frac{1}{4}-c_B)-\overline{g}-h>\beta_B(\frac{1}{2}-c_A-c_B)$

Condition 26 $(1-\alpha_A)(\frac{1}{4}-c_A)-\overline{g}-2h<\frac{1}{8}-c_A$

Condition 27 $\frac{5}{16}-(1-\alpha_B)(\frac{1}{4}-c_B)-c_B-\overline{g}-h>\beta_o(\frac{1}{2}-c_A-c_B)$

Since we have a symetric game, the choices of the government are identical from the ones stated above except except that in the semi-negotiation scenario, negotation takes place between the cable operator and channel B.

$\Longrightarrow$ There is an equilibrium:

$$\{(NT,T),(N,NN),(NN,NN),(N,NN),p=1,q=0\}$$

Separating equilibria $_{(T,NT):}$ "the explicit government"

We want to see if there exists a Perfect Bayesian Equilibrium (PBE) in which the government chooses the strategy (T,NT).The firms' beliefs after observing the message from the government are: $q=1$, $p=0$. The optimal strategies of the firms that could hold this separating equilibrium are:

1. $(C_O, C_A, C_B) : \{(NN, N), (NN, N), (NN, N)\}$

Condition 28 $(1-\alpha_A)(\frac{1}{4}-c_A)-\overline{g} > \frac{1}{8}-c_A$

Condition 29 $(1-\alpha_B)(\frac{1}{4}-c_B)-\overline{g} > \frac{1}{8}-c_B$

Condition 30 $\alpha_A(\frac{1}{4}-c_A)+\alpha_B(\frac{1}{4}-c_B)-\overline{g} > \beta_o(\frac{1}{2}-c_A-c_B)$

Given that the government is of type R, we need to check that it chooses T.

If the government chooses T, firms will end up desintegrated because they have chosen NN and the government regulates. The government's pay-off is 3.

If it chooses NT, result is the asymmetric Nash Bargaining and the government's pay-off, 1.

The government prefers T rather than NT.

Given that the government is of type NR, we need to check that it chooses NT.

If the government chooses T, firms will end up just as they are now, integrated, because they have chosen not to negotiate and the government does not regulate.

If it chooses NT, the asymmetric Nash Bargaining will take place in the future because all firms have chosen N. The government's pay-off is 1.

The government prefers NT rather than T.

$\Longrightarrow$ There is an equilibrium:

$$\{(T, NT), (NN, N), (NN, N), (NN, N),\ q=1,\ p=0\}$$

2. $(C_O, C_A, C_B) : \{(N, N), (N, N), (N, N)\}$: *"the untrusted government"*

Condition 31 $(1-\alpha_A)(\frac{1}{4}-c_A)-\overline{g} > \frac{1}{8}-c_A$

Condition 32 $(1-\alpha_B)(\frac{1}{4}-c_B)-\overline{g} > \frac{1}{8}-c_B$

Condition 33 $\alpha_A(\frac{1}{4}-c_A)+\alpha_B(\frac{1}{4}-c_B)-\overline{g} > \frac{1}{8}$

Condition 34 $(1-\alpha_A)(\frac{1}{4}-c_A)-\overline{g}-2h>\frac{1}{8}-c_A$

Condition 35 $(1-\alpha_B)(\frac{1}{4}-c_B)-\overline{g}-2h>\frac{1}{8}-c_B$

Condition 36 $\alpha_A(\frac{1}{4}-c_A)+\alpha_B(\frac{1}{4}-c_B)-\overline{g}-2h>\beta_o(\frac{1}{2}-c_A-c_B)$

Note that if conditions (4) and (5) hold, then conditions (1) and (2) are satisfied.

We need to check that the government chooses T when its type is R and that it chooses NT when its type is NR. Since firms negotiate no matter what message they receive from the government, result is the asymmetric Nash Bargaining. Therefore, the government is indifferent between T and NT.

$\Longrightarrow$There is an equilibrium:

$$\{(T,NT),(N,N),(N,N),(N,N),\ q=1,\ p=0\}$$

3. $(C_O,C_A,C_B):\{(NN,NN),(NN,NN),(NN,NN)\}$: *"the passive firms"*

In this case, all firms decide to stay as they are, regardless of the government's message, so no negotiation takes place.

Given that the government is of type R, we need to check that it chooses T.

If the government chooses either T or NT, firms do not negotiate and result is desintegration. The government's pay-off is 3.

Therefore, the government is indifferent between T and NT. Choosing T is possible.

Given that the government is of type NR, we need to check that it chooses NT.

If the government chooses either T or NT, firms do not negotiate and result is total integration of the chain. The government's pay-off is 0.

The government is indifferent between T and NT. Choosing NT is possible.

$\Rightarrow$There is an equilibrium:

$$\{(T,NT),(NN,NN),(NN,NN),(NN,NN),\ q=1,\ p=0\}$$

4. $(C_O, C_A, C_B) : \{(N, N), (N, N), (NN, NN)\}$: *"the left-out channel"*

The cable operator and channel A decide to negotiate either if they observe a threat or not. Channel B chooses not to negotiate.

Condition 37 $(1-\alpha_B)(\frac{1}{4}-c_B)-\overline{g}<\frac{1}{8}-c_B$

Condition 38 $(1-\alpha_A)(\frac{1}{4}-c_A)-\overline{g}-h>\frac{1}{8}-c_A$

Condition 39 $(1-\alpha_B)(\frac{1}{4}-c_B)-\overline{g}-2h<\frac{1}{8}-c_B$

Condition 40 $\frac{5}{16}-(1-\alpha_A)(\frac{1}{4}-c_A)-c_A-\overline{g}-h>\frac{1}{8}$

Condition 41 $(1-\alpha_A)(\frac{1}{4}-c_A)-\overline{g}>\beta_A(\frac{1}{2}-c_A-c_B)$

Condition 42 $\frac{5}{16}-(1-\alpha_A)(\frac{1}{4}-c_A)-c_A-\overline{g}>\beta_o(\frac{1}{2}-c_A-c_B)$

Note that if condition (1) holds, then condition (3) is satisfied.

Given that the government is of type R, we need to check that it chooses T.

If the government chooses either T or NT, the cable operator ends up negotiating only with A. The government's pay-off is 2. The government is therefore indifferent between T and NT.Choosing T is possible.

Given that the government is of type NR, we need to check that it chooses NT.

If the government chooses either T or NT, the cable operator ends up negotiating only with A. The government's pay-off is 2. The government is indifferent between T and NT.Choosing NT is possible.

⇒There is an equilibrium:

$$\{(T, NT), (N, N), (N, N), (NN, NN),\ q = 1,\ p = 0\}$$

5. $(C_O, C_A, C_B) : \{(N, N), (NN, NN), (N, N)\}$, i.e., in this case it is channel A who refrains from negotiation. The following conditions must be satisfied:

Condition 43 $(1-\alpha_B)(\frac{1}{4}-c_B)-\overline{g}-h>\frac{1}{8}-c_B$

Condition 44 $(1-\alpha_A)(\frac{1}{4}-c_A)-\overline{g}<\frac{1}{8}-c_A$

Condition 45 $(1-\alpha_A)(\frac{1}{4}-c_A)-\overline{g}-2h<\frac{1}{8}-c_A$

Condition 46 $\frac{5}{16} - (1-\alpha_B)(\frac{1}{4} - c_B) - c_B - \overline{g} - h > \frac{1}{8}$

Condition 47 $(1-\alpha_B)(\frac{1}{4} - c_B) - \overline{g} > \beta_B(\frac{1}{2} - c_A - c_B)$

Condition 48 $\frac{5}{16} - (1-\alpha_B)(\frac{1}{4} - c_B) - c_B - \overline{g} > \beta_o(\frac{1}{2} - c_A - c_B)$

Note that if condition (2) holds, then condition (3) is satisfied.

Given that the government is of type R, we need to check that it chooses T.

If the government chooses either T or NT, the cable operator ends up negotiating only with B. The government's pay-off is 2. The government is indifferent between T and NT.Choosing T is possible.

Given that the government is of type NR, we need to check that it chooses NT. If the government chooses either T or NT, the cable operator ends up negotiating only with B. The government's pay-off is 2. The government is indifferent between T and NT. Choosing NT is possible.

⇒There is an equilibrium:

$$\{(T, NT), (N, N), (NN, NN), (N, N),\ q = 1,\ p = 0\}$$

6. $(C_O, C_A, C_B) : \{(NN, N), (NN, N), (NN, NN)\}$

This refers to an equilibrium of the simultaneous game where the cable operator and channel A decide to negotiate if they observe no threat (channel B chooses not to negotiate) and an equilibrium where all the firms choose not to negotiate if they observe threat.The following conditions must be satisfied:

Condition 49 $(1-\alpha_A)(\frac{1}{4} - c_A) - \overline{g} > \beta_A(\frac{1}{2} - c_A - c_B)$

Condition 50 $(1-\alpha_B)(\frac{1}{4} - c_B) - \overline{g} < \frac{1}{8} - c_B$

Condition 51 $\frac{5}{16} - (1-\alpha_A)(\frac{1}{4} - c_A) - c_A - \overline{g} > \beta_o(\frac{1}{2} - c_A - c_B)$

Given that the government is of type R, we need to check that it chooses T.

If the government chooses T, since firms do not negotiate and regulation takes place, result is the desintegration of the chain. The government's pay-off is 3.

If it chooses NT, there will be an Nash negotiation between the cable operator and channel A only.The government's pay-off is 2.

The government prefers T rather than NT.

Given that the government is of type NR, we need to check that it chooses NT.

If the government chooses T, since firms do not negotiate but regulation does not take place, the result is an integrated chain. The government's pay-off is 0.

If it chooses NT, there will be an Nash negotiation between the cable operator and channel A.The government's pay-off is 2.

The government prefers NT rather than T.

$\Longrightarrow$ There is an equilibrium:

$$\{(T, NT), (NN, N), (NN, N), (NN, NN),\ q = 1,\ p = 0\}$$

7. $(C_O, C_A, C_B) : \{(NN, N), (NN, NN), (NN, N)\}$

The cable operator and channel B decide to negotiate if they observe no threat (channel A chooses not to negotiate) and all firms choose not to negotiate if they observe threat.

Condition 52 $(1 - \alpha_A)(\frac{1}{4} - c_A) - \overline{g} < \frac{1}{8} - c_A$

Condition 53 $(1 - \alpha_B)(\frac{1}{4} - c_B) - \overline{g} > \beta_B(\frac{1}{2} - c_A - c_B)$

Condition 54 $\frac{5}{16} - (1 - \alpha_B)(\frac{1}{4} - c_B) - c_B - \overline{g} > \beta_o(\frac{1}{2} - c_A - c_B)$

The choices of the government are identical from the ones stated above except that in the semi-negotiation scenario, negotation takes place between the cable operator and channel B.

$\Longrightarrow$ There is an equilibrium:

$$\{(T, NT), (NN, N), (NN, NN), (NN, N),\ q = 1,\ p = 0\}$$

In order to draw valid conclusions, a thorough analysis of the conditions under which each equilbrium holds should be carried out. The conditions are derived from the simultaneous game where the cable operator and the channels decide whether to negotiate or not after receiving a message from the government. As we have already analysed, despite the regulation the original situation could be replicated through bargaining. Given that the cable operator offers channels à la carte, total producer surplus is maximized and prices do not differ from the scenario where the whole chain of production is integrated. However, firms might not choose barganing over the desintegration of the chain; under certain values of parameters, they decide not to avoid regulation. Then, the firms' decisions should be analysed carefully. The equilibriums of the simultaneous game are diverse and depend on the relation among the parameters. Note that since we concentrate on separating equilibria, expected benefits become certain and this analysis is possible. The work done so far is a starting point for a further development on the interaction between the government and the firms under the latter's attempt to disolute the chain of production.

Appendix

A. Cuasilinear utilities

Continuous case:

Definition: the preference relation $\succsim$ on $X = (-\infty, \infty) \times \mathbb{R}_+^{L-1}$ is quasilinear (or linear) with respect to commodity 1 (called, in this case, de numeraire commodity) if:

(i) All the indifference sets are parallel displacements of each other along the axis of commodity 1. That is, if $x \sim y$, then $(x + \alpha e_1) \sim (y + \alpha e_1)$ for $e_1 = (1, 0, ..., 0)$ and any $\alpha \geqslant 0$.

(ii) Good 1 is desirable; that is, $x + \alpha e_1 \succ x$ for all x and $\alpha > 0$.

For a two commodity case, the indiference curves adopt the form $x_1 = k - v(x_2)$. Utility function is $u(x_1, x_2) = x_1 + v(x_2)$. Note that it is linear with respect to commodity 1.

This kind of utility functions are not particularly realistic, but it is relatively easy to operate with them. In which cases should we expect to find such preferences? Suppose x_2 is a good that does not represent a great share of the individual's expenditure, and the linear commodity, x_1, is money. Initially all expenditure goes to this good x_2 (since there is a fixed quantity that we would like to consume), but once the desirable level of consumption is reached, all extra money goes to other goods. This is why it is not entirely correct to say that x_2 has no income effect: indeed, the IE is zero but only when it has reached a certain level that allows the consumer to satisfy a required level of commodity 2.

In our model, we can interpret x_2 as the channels whereas x_1 is money spent on all of the other commodities the individual consumes.

Discrete goods:

Now suppose that good 2 is discrete. If p_2 is very high, consumer would like to consume little of this good, in particular, given demand is discrete, it would like to consume zero. At price r_1 (reservation price) he is indifferent between consuming one unit or zero; at price r_2, between consuming two units or one, etc.

$u(x_1, x_2) = x_1 + v(x_2)$

With $v(0) = 0$.

Taking good two as discrete, and good one as money, we get the following relations:

$u(m - r_1, 1) = u(m, 0)$

$m - r_1 + v(1) = m + v(0)$

$-r_1 + v(1) = 0$

$v(1) = r_1$

$u(m - 2.r_2, 2) = u(m - r_2, 1)$

$m - 2.r_2 + v(2) = m - r_2 + v(1)$

$v(2) = r_2 + v(1)$

$v(3) = r_3 + v(2)$, and so on.

So the reservation price measures the increase in utility due to consumption of good 2 that induces consumer to choose another unit of the good. In other words, it is the marginal utility corresponding to different levels of consumption of good 2.

Summing up:

In our case, good 2 is extremely discrete: one can consume up to 1 unit. Utility function is: $u(m, x_2) = m + v(x_2)$

$$\max_{m,x_2} u(m, x_2) = m + v(x_2)$$
$$s.a\ m + p_2 x_2 = \overline{m}$$
$$x_2 = \{0, 1\}$$

Notice the difference between: m (money spent at other goods appart from 2) and $\overline{m}$ (total income)

Since good 2 is discrete, the problem reduces to deciding whether to buy the good or not.

If consumer buys the channel: $u(\overline{m} - p_2, 1) = \overline{m} - p_2 + v(1) = \overline{m} - p_2 + \theta$

If consumer doesn't buy the channel: $u(\overline{m}, 0) = \overline{m} + v(0) = \overline{m}$

Consumer prefers to buy the channel whenever: $\overline{m} - p_2 + \theta > \overline{m} \Rightarrow \theta - p_2 > 0$. Remember we can interpret $v(1) = \theta$ as the reservation price of buying one unit.

This is the condition that we usually take into account to see whether he will purchase the good or not.

Notice that indirect utility is:

$$v^*(p_2,\overline{m}) = \begin{cases} \overline{m}-p_2+\theta \text{ if } \theta-p_2>0 \\ \overline{m} \text{ if } \theta-p_2<0 \end{cases}$$

B. Desintegration: implicit function theorem

$$\frac{\partial w_A}{\partial w_B} = -\frac{\frac{\partial g}{\partial w_B}}{\frac{\partial g}{\partial w_A}}$$

$$= -\frac{\frac{-\sqrt{w_B^2+2w_Aw_B+w_A^2+6}}{9}-\frac{(w_B+w_A)(w_B^3+5w_B^2w_A+w_B(7w_A^2+6)+3w_A(w_A^2+8))}{9(w_A^2+2w_Aw_B+w_B^2+6)^{\frac{3}{2}}}-\frac{2(2w_A+w_B)}{9}}{\frac{-2\sqrt{w_A^2+2w_Aw_B+w_B^2+6}}{9}-\frac{2(w_A+w_B)(2w_A^3+5w_A^2w_B+w_A(4w_B^2+15)+w_B(w_B^2+6))}{9(w_A^2+2w_Aw_B+w_B^2+6)^{\frac{3}{2}}}-\frac{2(3w_A+2w_B)}{9}}$$

$$= -\frac{5w_Aw_B^3+7w_A^3w_B+18w_A^2+9w_B^2+2w_A^4+w_B^4+9w_A^2w_B^2+27w_Aw_B+2w_A\left((w_A+w_B)^2+6\right)^{\frac{3}{2}}+w_B\left((w_A+w_B)^2+6\right)^{\frac{3}{2}}+18}{9w_Aw_B^3+11w_A^3w_B+27w_A^2+18w_B^2+3w_A^4+2w_B^4+15w_A^2w_B^2+45w_Aw_B+3w_A\left((w_A+w_B)^2+6\right)^{\frac{3}{2}}+2w_B\left((w_A+w_B)^2+6\right)^{\frac{3}{2}}+36} < 0$$

$$\frac{\partial w_B}{\partial w_A} = -\frac{\frac{\partial h}{\partial w_A}}{\frac{\partial h}{\partial w_B}}$$

$$= -\frac{\frac{-\sqrt{w_B^2+2w_Aw_B+w_A^2+6}}{9}-\frac{(w_B+w_A)(w_B^3+5w_B^2w_A+w_B(7w_A^2+6)+3w_A(w_A^2+8))}{9(w_A^2+2w_Aw_B+w_B^2+6)^{\frac{3}{2}}}-\frac{2(w_A+2w_B)}{9}}{\frac{-2\sqrt{w_A^2+2w_Aw_B+w_B^2+6}}{9}-\frac{2(w_A+w_B)(2w_A^3+5w_Aw_B^2+w_B(4w_A^2+15)+w_A(w_A^2+6))}{9(w_A^2+2w_Aw_B+w_B^2+6)^{\frac{3}{2}}}-\frac{2(2w_A+3w_B)}{9}}$$

$$= -\frac{5w_Aw_B^3+7w_A^3w_B+18w_A^2+9w_B^2+2w_A^4+w_B^4+9w_A^2w_B^2+27w_Aw_B+w_A\left((w_A+w_B)^2+6\right)^{\frac{3}{2}}+2w_B\left((w_A+w_B)^2+6\right)^{\frac{3}{2}}+18}{9w_Aw_B^3+11w_A^3w_B+18w_A^2+27w_B^2+4w_A^4+w_B^4+15w_A^2w_B^2+45w_Aw_B+2w_A((w_A+w_B)^2+6)^{\frac{3}{2}}+3w_B((w_A+w_B)^2+6)^{\frac{3}{2}}+36} < 0$$

The code we used to graph in Python an aproximation of both reaction functions is the following:

```
# w_a vs. w_b
import pylab
import sympy
from sympy.abc import x

w_a_list1 = list(pylab.arange(0, 1, 0.001))
w_b_list1 = []
"Función de reacción wb(wa)"
for w_a in w_a_list1:
    w_b = sympy.nsolve(2.0/3-1.0/9*(w_a**2+x**2)-2.0/9*w_a*x-1.0/9*(w_a+x)*
          ((w_a+x)**2+6)**0.5+x*(-2.0/9*(w_a+x)-1.0/9*(((w_a+x)**2+6)**0.5+
           1.0/((w_a+x)**2+6)**0.5*(w_a+x)**2)), 0.5)
    w_b_list1.append(w_b)

w_b_list2 = list(pylab.arange(0, 1, 0.001))
w_a_list2 = []
"Función de reacción wa(wb)"
for w_b in w_b_list2:
    w_a = sympy.nsolve(2.0/3-1.0/9*(w_b**2+x**2)-2.0/9*w_b*x-1.0/9*(w_b+x)*
          ((w_b+x)**2+6)**0.5+x*(-2.0/9*(w_b+x)-1.0/9*(((w_b+x)**2+6)**0.5+
           1.0/((w_b+x)**2+6)**0.5*(w_b+x)**2)), 0.5)
    w_a_list2.append(w_a)

pylab.figure(0)
pylab.plot(w_a_list1, w_b_list1, 'r-')
pylab.plot(w_a_list2, w_b_list2, 'b-')
pylab.axis([0,1,0,1])
pylab.title("Funciones de reaccion")
pylab.xlabel("Wa")
pylab.ylabel("Wb")
pylab.show()
```

C. Equilibria of the three-player simultaneous game

$\Longrightarrow$ *When the firms observe NT, possible equilibria are:*

- $(C_O, C_A, C_B) : (N, N, N)$

From A) we see that, given that the cable operator chooses N and channel A chooses N, channel B will choose N if: $(1-\alpha_B)(\frac{1}{4}-c_B)-\overline{g} > \frac{1}{8}-c_B$. And given that channel B chooses N, channel A will choose N if: $(1-\alpha_A)(\frac{1}{4}-c_A)-\overline{g} > \frac{1}{8}-c_A$.

From C) given that channel A chooses N and that the cable operator chooses N, channel B will choose N if: $(1-\alpha_B)(\frac{1}{4}-c_B)-\overline{g} > \frac{1}{8}-c_B$. And given that channel B chooses N, the cable operator will choose N if: $\alpha_A(\frac{1}{4}-c_A)+\alpha_B(\frac{1}{4}-c_B)-\overline{g} > p\frac{1}{8}+(1-p)\beta_o(\frac{1}{2}-c_A-c_B)$.

From E) given that channel B chooses N and that the cable operator chooses N, channel A will choose N if: $(1-\alpha_A)(\frac{1}{4}-c_A)-\overline{g} > \frac{1}{8}-c_A$. And given that channel A chooses N, the cable operator will choose N if: $\alpha_A(\frac{1}{4}-c_A)+\alpha_B(\frac{1}{4}-c_B)-\overline{g} > p\frac{1}{8}+(1-p)\beta_o(\frac{1}{2}-c_A-c_B)$.

- $(C_o, C_A, C_B) : (N, NN, N)$

From A), given that the cable operator chooses N and that channel A chooses NN, channel B will choose N if: $(1-\alpha_B)(\frac{1}{4}-c_B)-\overline{g} > p(\frac{1}{8}-c_B)+(1-p)\beta_B(\frac{1}{2}-c_A-c_B)$. And given that channel B chooses N, channel A will choose NN if: $(1-\alpha_A)(\frac{1}{4}-c_A)-\overline{g} < \frac{1}{8}-c_A$.

From D), given that channel A chooses NN and that the cable operator chooses N, channel B will choose N if: $(1-\alpha_B)(\frac{1}{4}-c_B)-\overline{g} > p(\frac{1}{8}-c_B)+(1-p)\beta_B(\frac{1}{2}-c_A-c_B)$. And given that channel B chooses N, the cable operator will choose N if: $\frac{5}{16}-(1-\alpha_B)(\frac{1}{4}-c_B)-c_B-\overline{g} > p\frac{1}{8}+(1-p)\beta_o(\frac{1}{2}-c_A-c_B)$.

From E), given that channel B chooses N and that the cable operator chooses N, channel A will choose NN if: $(1-\alpha_A)(\frac{1}{4}-c_A)-\overline{g} < \frac{1}{8}-c_A$. And given that channel A chooses NN, the cable operator will choose N if: $\frac{5}{16}-(1-\alpha_B)(\frac{1}{4}-c_B)-c_B-\overline{g} > p\frac{1}{8}+(1-p)\beta_o(\frac{1}{2}-c_A-c_B)$.

- $(C_O, C_A, C_B) : (N, N, NN)$

From A), given that the cable operator chooses N and that channel A chooses N, channel B will choose NN if: $(1-\alpha_B)(\frac{1}{4}-c_B)-\overline{g} < \frac{1}{8}-c_B$. And given that channel B chooses NN, channel A will choose N if: $(1-\alpha_A)(\frac{1}{4}-c_A)-\overline{g} > p(\frac{1}{8}-c_A)+(1-p)\beta_A(\frac{1}{2}-c_A-c_B)$.

From C), given that channel A chooses N and the cable operator chooses N, channel B will choose NN if: $(1-\alpha_B)(\frac{1}{4}-c_B)-\overline{g} < \frac{1}{8}-c_B$. And given that channel B chooses NN, the cable operator will choose N if: $\frac{5}{16}-(1-\alpha_A)(\frac{1}{4}-c_A)-c_A-\overline{g} > p\frac{1}{8}+(1-p)\beta_o(\frac{1}{2}-c_A-c_B)$.

From F), given that channel B chooses NN and that the cable operator chooses N, channel A will choose N if: $(1-\alpha_A)(\frac{1}{4}-c_A)-\overline{g} > p(\frac{1}{8}-c_A)+(1-p)\beta_A(\frac{1}{2}-c_A-c_B)$. And given that channel A chooses N, the cable operator will choose N if: $\frac{5}{16}-(1-\alpha_A)(\frac{1}{4}-c_A)-c_A-\overline{g} > p\frac{1}{8}+(1-p)\beta_o(\frac{1}{2}-c_A-c_B)$.

- $(C_O, C_A, C_B) : (NN, NN, NN)$

From B), given that the cable operator chooses NN and channel A chooses NN, channel B chooses NN because $p(\frac{1}{8} - c_B - \underline{g}) + (1-p)(\beta_B(\frac{1}{2} - c_A - c_B) - \underline{g}) < p(\frac{1}{8} - c_B) + (1-p)(\beta_B(\frac{1}{2} - c_A - c_B))$. And given that channel B chooses NN, channel A chooses NN because $p(\frac{1}{8} - c_A - \underline{g}) + (1-p)(\beta_A(\frac{1}{2} - c_A - c_B) - \underline{g}) < p(\frac{1}{8} - c_A) + (1-p)(\beta_A(\frac{1}{2} - c_A - c_B))$.

From D), given that channel A chooses NN and the cable operator chooses NN, channel B chooses NN because $p(\frac{1}{8} - c_B - \underline{g}) + (1-p)(\beta_B(\frac{1}{2} - c_A - c_B) - \underline{g}) < p(\frac{1}{8} - c_B) + (1-p)(\beta_B(\frac{1}{2} - c_A - c_B))$. And given that channel B chooses NN, the cable operator chooses NN because $p(\frac{1}{8} - \underline{g}) + (1-p)(\beta_O(\frac{1}{2} - c_A - c_B) - \underline{g}) < p\frac{1}{8} + (1-p)\beta_O(\frac{1}{2} - c_A - c_B)$.

From F), given that channel B chooses NN and channel A chooses NN, the cable operator chooses NN because $p(\frac{1}{8} - \underline{g}) + (1-p)(\beta_O(\frac{1}{2} - c_A - c_B) - \underline{g}) < p\frac{1}{8} + (1-p)\beta_O(\frac{1}{2} - c_A - c_B)$. And given that the cable operator chooses NN, channel A chooses NN because $p(\frac{1}{8} - c_A - \underline{g}) + (1-p)(\beta_A(\frac{1}{2} - c_A - c_B) - \underline{g}) < p(\frac{1}{8} - c_A) + (1-p)(\beta_A(\frac{1}{2} - c_A - c_B))$.

$\Longrightarrow$ *The following cases are not equlibria under NT:*

- $(C_O, C_A, C_B) : (NN, N, N)$

From B), given that the cable operator chooses NN and that channel A chooses N, channel B chooses NN because $p(\frac{1}{8} - c_B) + (1-p)\beta_B(\frac{1}{2} - c_A - c_B) > p(\frac{1}{8} - c_B - \underline{g}) + (1-p)(\beta_B(\frac{1}{2} - c_A - c_B) - \underline{g})$. And given that channel B chooses N, channel A chooses NN because: $p(\frac{1}{8} - c_A) + (1-p)\beta_A(\frac{1}{2} - c_A - c_B) > p(\frac{1}{8} - c_A - \underline{g}) + (1-p)(\beta_A(\frac{1}{2} - c_A - c_B) - \underline{g})$. So there is no autoconfrimation.

From C), given that channel A chooses N and that the cable operator chooses NN, channel B chooses NN because $p(\frac{1}{8} - c_B) + (1-p)\beta_B(\frac{1}{2} - c_A - c_B) > p(\frac{1}{8} - c_B - \underline{g}) + (1-p)(\beta_B(\frac{1}{2} - c_A - c_B) - \underline{g})$. And given that channel A chooses N and that channel B chooses N, the cable operator will choose NN if: $p\frac{1}{8} + (1-p)\beta_o(\frac{1}{2} - c_A - c_B) > \alpha_A(\frac{1}{4} - c_A) + \alpha_B(\frac{1}{4} - c_B) - \underline{g}$.

From E), given that channel B chooses N and that channel A chooses N, the cable operator will choose NN if the same condition stated in C) is satisfied. And given the cable operator chooses NN, channel A chooses NN because $p(\frac{1}{8} - c_A) + (1-p)\beta_A(\frac{1}{2} - c_A - c_B) > p(\frac{1}{8} - c_A - \underline{g}) + (1-p)(\beta_A(\frac{1}{2} - c_A - c_B) - \underline{g})$. So there is no autoconfirmation.

- $(C_o, C_A, C_B) : (N, NN, NN)$

From A) given that the cable operator chooses N and that channel A chooses NN, channel B will choose NN if: $(1-\alpha_B)\left(\frac{1}{4}-c_B\right)-\overline{g} < p(\frac{1}{8}-c_B)+(1-p)\beta_B(\frac{1}{2}-c_A-c_B)$. And given that channel B chooses NN, channel A will choose NN if: $(1-\alpha_A)(\frac{1}{4}-c_A)-\overline{g} < p(\frac{1}{8}-c_A)+(1-p)\beta_A(\frac{1}{2}-c_A-c_B)$.

From D) given that channel A chooses NN and that the cable operator chooses N, channel B will choose NN if: $(1-\alpha_B)(\frac{1}{4}-c_B)-\overline{g} < p(\frac{1}{8}-c_B)+(1-p)\beta_B(\frac{1}{2}-c_A-c_B)$. And given that channel B chooses NN, the cable operator chooses NN because $p(\frac{1}{8}-\underline{g})+(1-p)(\beta_O(\frac{1}{2}-c_A-c_B)-\underline{g}) < p\frac{1}{8}+(1-p)\beta_O(\frac{1}{2}-c_A-c_B)$, so there is no autoconfirmation.

From F), given that channel B chooses NN and that the cable operator chooses N, channel A will choose NN if: $(1-\alpha_A)(\frac{1}{4}-c_A)-\overline{g} < p(\frac{1}{8}-c_A)+(1-p)\beta_A(\frac{1}{2}-c_A-c_B)$. And given that channel A chooses NN, the cable operator chooses NN because $p(\frac{1}{8}-\underline{g})+(1-p)(\beta_O(\frac{1}{2}-c_A-c_B)-\underline{g}) < p\frac{1}{8}+(1-p)\beta_O(\frac{1}{2}-c_A-c_B)$, so there is no autoconfirmation.

- $(C_O, C_A, C_B) : (NN, N, NN)$

From B), given that the cable operator chooses NN, if channel A chooses N, channel B chooses NN because $p(\frac{1}{8}-c_B)+(1-p)\beta_B(\frac{1}{2}-c_A-c_B) > p(\frac{1}{8}-c_B-\underline{g})+(1-p)(\beta_B(\frac{1}{2}-c_A-c_B)-\underline{g})$. And given that channel B chooses NN, channel A chooses NN because $p(\frac{1}{8}-c_A-\underline{g})+(1-p)(\beta_A(\frac{1}{2}-c_A-c_B)-\underline{g}) < p(\frac{1}{8}-c_A)+(1-p)(\beta_A(\frac{1}{2}-c_A-c_B))$, so there is no autoconfirmation.

From C), given that channel A chooses N and that channel B chooses NN, the cable operator will choose NN if: $\frac{5}{16}-(1-\alpha_A)(\frac{1}{4}-c_A)-c_A-\overline{g} < p\frac{1}{8}+(1-p)\beta_o(\frac{1}{2}-c_A-c_B)$. And given that the cable operator chooses NN, channel B chooses NN.

From F), given that channel B chooses NN and that channel A chooses N, the cable operator will choose NN if the same condition stated above is satisfied. And given that the cable operator chooses NN, channel A chooses NN because $p(\frac{1}{8}-c_A-\underline{g})+(1-p)(\beta_A(\frac{1}{2}-c_A-c_B)-\underline{g}) < p(\frac{1}{8}-c_A)+(1-p)(\beta_A(\frac{1}{2}-c_A-c_B))$ so there is no autoconfirmation.

- $(C_O, C_A, C_B) : (NN, NN, N)$

From B), given that the cable operator chooses NN, if channel A chooses NN, channel B prefers NN because $p(\frac{1}{8}-c_B-\underline{g})+(1-p)(\beta_B(\frac{1}{2}-c_A-c_B)-\underline{g}) < p(\frac{1}{8}-c_B)+(1-p)(\beta_B(\frac{1}{2}-c_A-c_B))$, so there is no autoconfirmation. And if channel B chooses N, channel A chooses NN because $p(\frac{1}{8}-c_A-\underline{g})+(1-p)(\beta_A(\frac{1}{2}-c_A-c_B)-\underline{g})<p(\frac{1}{8}-c_A)+(1-p)(\beta_A(\frac{1}{2}-c_A-c_B))$,

From D), given that channel A chooses NN and that channel B chooses N, the cable operator will choose NN if: $\frac{5}{16}-(1-\alpha_B)(\frac{1}{4}-c_B)-c_B-\overline{g}<p\frac{1}{8}+(1-p)\beta_o(\frac{1}{2}-c_A-c_B)$. And given that the cable operator chooses NN, channel B chooses NN becuase $p(\frac{1}{8}-c_B-\underline{g})+(1-p)(\beta_B(\frac{1}{2}-c_A-c_B)-\underline{g})<p(\frac{1}{8}-c_B)+(1-p)(\beta_B(\frac{1}{2}-c_A-c_B))$, so there is no autoconfirmation.

From E), given that channel B chooses N and that channel A chooses NN, the cable operator will choose NN if the same condition stated above is satisfied. And given that the cable operator chooses NN, channel A chooses NN.

$\Longrightarrow$ *When the firms observe T, possible equilibria are:*

- $(C_O, C_A, C_B):(N,N,N)$

From A) we see that, given that the cable operator chooses N and channel A chooses N, channel B will choose N if: $(1-\alpha_B)(\frac{1}{4}-c_B)-\overline{g}-3h>\frac{1}{8}-c_B$. And given that channel B chooses N, channel A will choose N if: $(1-\alpha_A)(\frac{1}{4}-c_A)-\overline{g}-3h>\frac{1}{8}-c_A$.

From C) given that channel A chooses N and that the cable operator chooses N, channel B will choose N if: $(1-\alpha_B)(\frac{1}{4}-c_B)-\overline{g}-3h>\frac{1}{8}-c_B$. And given that channel B chooses N, the cable operator will choose N if: $\alpha_A(\frac{1}{4}-c_A)+\alpha_B(\frac{1}{4}-c_B)-\overline{g}-3h>q\frac{1}{8}+(1-q)\beta_o(\frac{1}{2}-c_A-c_B)$.

From E) given that channel B chooses N and that the cable operator chooses N, channel A will choose N if: $(1-\alpha_A)(\frac{1}{4}-c_A)-\overline{g}-3h>\frac{1}{8}-c_A$. And given that channel A chooses N, the cable operator will choose N if: $\alpha_A(\frac{1}{4}-c_A)+\alpha_B(\frac{1}{4}-c_B)-\overline{g}-3h>q\frac{1}{8}+(1-q)\beta_o(\frac{1}{2}-c_A-c_B)$.

- $(C_o, C_A, C_B):(N,NN,N)$

From A), given that the cable operator chooses N and that channel A chooses NN, channel B will choose N if: $(1-\alpha_B)(\frac{1}{4}-c_B)-\overline{g}-2h>q(\frac{1}{8}-c_B)+(1-q)\beta_B(\frac{1}{2}-c_A-c_B)$. And given that channel B chooses N, channel A will choose NN if: $(1-\alpha_A)(\frac{1}{4}-c_A)-\overline{g}-3h<\frac{1}{8}-c_A$.

From D), given that channel A chooses NN and that the cable operator chooses N, channel B will choose N if: $(1-\alpha_B)(\frac{1}{4}-c_B)-\overline{g}-2h>q(\frac{1}{8}-c_B)+(1-q)\beta_B(\frac{1}{2}-c_A-c_B)$. And given that channel B chooses N, the cable operator will choose N if: $\frac{5}{16}-(1-\alpha_B)(\frac{1}{4}-c_B)-c_B-\overline{g}-2h>q\frac{1}{8}+(1-q)\beta_o(\frac{1}{2}-c_A-c_B)$.

From E), given that channel B chooses N and that the cable operator chooses N, channel A will choose NN if: $(1-\alpha_A)(\frac{1}{4}-c_A)-\overline{g}-3h<\frac{1}{8}-c_A$. And given that channel A chooses NN, the cable operator will choose N if: $\frac{5}{16}-(1-\alpha_B)(\frac{1}{4}-c_B)-c_B-\overline{g}-2h>q\frac{1}{8}+(1-q)\beta_o(\frac{1}{2}-c_A-c_B)$.

- $(C_O, C_A, C_B) : (N, N, NN)$

From A), given that the cable operator chooses N and that channel A chooses N, channel B will choose NN if: $(1-\alpha_B)(\frac{1}{4}-c_B)-\overline{g}-3h<\frac{1}{8}-c_B$. And given that channel B chooses NN, channel A will choose N if: $(1-\alpha_A)(\frac{1}{4}-c_A)-\overline{g}-2h>q(\frac{1}{8}-c_A)+(1-q)\beta_A(\frac{1}{2}-c_A-c_B)$.

From C), given that channel A chooses N and the cable operator chooses N, channel B will choose NN if:$(1-\alpha_B)(\frac{1}{4}-c_B)-\overline{g}-3h<\frac{1}{8}-c_B$. And given that channel B chooses NN, the cable operator will choose N if: $\frac{5}{16}-(1-\alpha_A)(\frac{1}{4}-c_A)-c_A-\overline{g}-2h>q\frac{1}{8}+(1-q)\beta_o(\frac{1}{2}-c_A-c_B)$.

From F), given that channel B chooses NN and that the cable operator chooses N, channel A will choose N if: $(1-\alpha_A)(\frac{1}{4}-c_A)-\overline{g}-2h>q(\frac{1}{8}-c_A)+(1-q)\beta_A(\frac{1}{2}-c_A-c_B)$. And given that channel A chooses N, the cable operator will choose N if: $\frac{5}{16}-(1-\alpha_A)(\frac{1}{4}-c_A)-c_A-\overline{g}-2h>q\frac{1}{8}+(1-q)\beta_o(\frac{1}{2}-c_A-c_B)$.

- $(C_O, C_A, C_B) : (NN, NN, NN)$

From B), given that the cable operator chooses NN and channel A chooses NN, channel B chooses NN because $q(\frac{1}{8}-c_B-\underline{g})+(1-q)(\beta_B(\frac{1}{2}-c_A-c_B)-\underline{g})<q(\frac{1}{8}-c_B)+(1-q)(\beta_B(\frac{1}{2}-c_A-c_B))$. And given that channel B chooses NN, channel A chooses NN because $q(\frac{1}{8}-c_A-\underline{g})+(1-q)(\beta_A(\frac{1}{2}-c_A-c_B)-\underline{g})<q(\frac{1}{8}-c_A)+(1-q)(\beta_A(\frac{1}{2}-c_A-c_B))$.

From D), given that channel A chooses NN and the cable operator chooses NN, channel B chooses NN because $q(\frac{1}{8}-c_B-\underline{g})+(1-q)(\beta_B(\frac{1}{2}-c_A-c_B)-\underline{g})<q(\frac{1}{8}-c_B)+(1-q)(\beta_B(\frac{1}{2}-c_A-c_B))$. And given that channel B chooses NN, the cable operator chooses NN because $q(\frac{1}{8}-\underline{g})+(1-q)(\beta_O(\frac{1}{2}-c_A-c_B)-\underline{g})<q\frac{1}{8}+(1-q)\beta_O(\frac{1}{2}-c_A-c_B)$.

From F), given that channel B chooses NN and channel A chooses NN, the cable operator chooses NN because $q(\frac{1}{8}-\underline{g})+(1-q)(\beta_O(\frac{1}{2}-c_A-c_B)-\underline{g})<q\frac{1}{8}+(1-q)\beta_O(\frac{1}{2}-c_A-c_B)$. And given that the cable operator chooses NN, channel A chooses NN because $q(\frac{1}{8}-c_A-\underline{g})+(1-q)(\beta_A(\frac{1}{2}-c_A-c_B)-\underline{g})<q(\frac{1}{8}-c_A)+(1-q)(\beta_A(\frac{1}{2}-c_A-c_B))$.

$\Longrightarrow$*The following cases are not equlibria under T:*

- $(C_O, C_A, C_B) : (NN, N, N)$

From B), given that the cable operator chooses NN and that channel A chooses N, channel B chooses NN because $q(\frac{1}{8} - c_B) + (1-q)\beta_B(\frac{1}{2} - c_A - c_B) > q(\frac{1}{8} - c_B - \underline{g}) + (1-q)(\beta_B(\frac{1}{2} - c_A - c_B) - \underline{g})$. And given that channel B chooses N, channel A chooses NN because: $q(\frac{1}{8} - c_A) + (1-q)\beta_A(\frac{1}{2} - c_A - c_B) > q(\frac{1}{8} - c_A - \underline{g}) + (1-q)(\beta_A(\frac{1}{2} - c_A - c_B) - \underline{g})$. So there is no autoconfrimation.

From C), given that channel A chooses N and that the cable operator chooses NN, channel B chooses NN because $q(\frac{1}{8} - c_B) + (1-q)\beta_B(\frac{1}{2} - c_A - c_B) > q(\frac{1}{8} - c_B - \underline{g}) + (1-q)(\beta_B(\frac{1}{2} - c_A - c_B) - \underline{g})$. And given that channel A chooses N and that channel B chooses N, the cable operator will choose NN if: $q\frac{1}{8} + (1-q)\beta_o(\frac{1}{2} - c_A - c_B) > \alpha_A(\frac{1}{4} - c_A) + \alpha_B(\frac{1}{4} - c_B) - \underline{g} - 3h$.

From E), given that channel B chooses N and that channel A chooses N, the cable operator will choose NN if the same condition stated in C) is satisfied. And given the cable operator chooses NN, channel A chooses NN because $q(\frac{1}{8} - c_A) + (1-q)\beta_A(\frac{1}{2} - c_A - c_B) > q(\frac{1}{8} - c_A - \underline{g}) + (1-q)(\beta_A(\frac{1}{2} - c_A - c_B) - \underline{g})$. So there is no autoconfirmation.

- $(C_o, C_A, C_B) : (N, NN, NN)$

From A) given that the cable operator chooses N and that channel A chooses NN, channel B will choose NN if: $(1-\alpha_B)\ (\frac{1}{4} - c_B) - 2h - \overline{g} < q(\frac{1}{8} - c_B) + (1-q)\beta_B(\frac{1}{2} - c_A - c_B)$. And given that channel B chooses NN, channel A will choose NN if: $(1-\alpha_A)(\frac{1}{4} - c_A) - \overline{g} - 2h < q(\frac{1}{8} - c_A) + (1-q)\beta_A(\frac{1}{2} - c_A - c_B)$.

From D) given that channel A chooses NN and that the cable operator chooses N, channel B will choose NN if: $(1-\alpha_B)(\frac{1}{4} - c_B) - \overline{g} - 2h < q(\frac{1}{8} - c_B) + (1-q)\beta_B(\frac{1}{2} - c_A - c_B)$. And given that channel B chooses NN, the cable operator chooses NN because $q(\frac{1}{8} - \underline{g}) + (1-q)(\beta_O(\frac{1}{2} - c_A - c_B) - \underline{g}) < q\frac{1}{8} + (1-q)\beta_O(\frac{1}{2} - c_A - c_B)$, so there is no autoconfirmation.

From F), given that channel B chooses NN and that the cable operator chooses N, channel A will choose NN if: $(1-\alpha_A)(\frac{1}{4} - c_A) - \overline{g} - 2h < q(\frac{1}{8} - c_A) + (1-q)\beta_A(\frac{1}{2} - c_A - c_B)$. And given that channel A chooses NN, the cable operator chooses NN because $q(\frac{1}{8} - \underline{g}) + (1-q)(\beta_O(\frac{1}{2} - c_A - c_B) - \underline{g}) < q\frac{1}{8} + (1-q)\beta_O(\frac{1}{2} - c_A - c_B)$, so there is no autoconfirmation.

- $(C_O, C_A, C_B) : (NN, N, NN)$

From B), given that the cable operator chooses NN, if channel A chooses N, channel B chooses NN because $q(\frac{1}{8} - c_B) + (1-q)\beta_B(\frac{1}{2} - c_A - c_B) > q(\frac{1}{8} - c_B - \underline{g}) + (1-q)(\beta_B(\frac{1}{2} - c_A - c_B) - \underline{g})$. And given that channel B chooses NN, channel A chooses NN because $q(\frac{1}{8} - c_A - \underline{g}) + (1-q)(\beta_A(\frac{1}{2} - c_A - c_B) - \underline{g}) < q(\frac{1}{8} - c_A) + (1-q)(\beta_A(\frac{1}{2} - c_A - c_B))$, so there is no autoconfirmation.

From C), given that channel A chooses N and that channel B chooses NN, the cable operator will choose NN if: $\frac{5}{16} - (1-\alpha_A)(\frac{1}{4} - c_A) - c_A - \overline{g} - 2h < q\frac{1}{8} + (1-q)\beta_o(\frac{1}{2} - c_A - c_B)$. And given that the cable operator chooses NN, channel B chooses NN.

From F), given that channel B chooses NN and that channel A chooses N, the cable operator will choose NN if the same condition stated above is satisfied. And given that the cable operator chooses NN, channel A chooses NN because $q(\frac{1}{8} - c_A - \underline{g}) + (1-q)(\beta_A(\frac{1}{2} - c_A - c_B) - \underline{g}) < q(\frac{1}{8} - c_A) + (1-q)(\beta_A(\frac{1}{2} - c_A - c_B))$ so there is no autoconfirmation.

- $(C_O, C_A, C_B) : (NN, NN, N)$

From B), given that the cable operator chooses NN, if channel A chooses NN, channel B prefers NN because $q(\frac{1}{8} - c_B - \underline{g}) + (1-q)(\beta_B(\frac{1}{2} - c_A - c_B) - \underline{g}) < q(\frac{1}{8} - c_B) + (1-q)(\beta_B(\frac{1}{2} - c_A - c_B))$, so there is no autoconfirmation. And if channel B chooses N, channel A chooses NN because $q(\frac{1}{8} - c_A - \underline{g}) + (1-q)(\beta_A(\frac{1}{2} - c_A - c_B) - \underline{g}) < q(\frac{1}{8} - c_A) + (1-q)(\beta_A(\frac{1}{2} - c_A - c_B))$,

From D), given that channel A chooses NN and that channel B chooses N, the cable operator will choose NN if: $\frac{5}{16} - (1-\alpha_B)(\frac{1}{4} - c_B) - c_B - \overline{g} - 2h < q\frac{1}{8} + (1-q)\beta_o(\frac{1}{2} - c_A - c_B)$. And given that the cable operator chooses NN, channel B chooses NN becuase $q(\frac{1}{8} - c_B - \underline{g}) + (1-q)(\beta_B(\frac{1}{2} - c_A - c_B) - \underline{g}) < q(\frac{1}{8} - c_B) + (1-q)(\beta_B(\frac{1}{2} - c_A - c_B))$, so there is no autoconfirmation.

From E), given that channel B chooses N and that channel A chooses NN, the cable operator will choose NN if the same condition stated above is satisfied. And given that the cable operator chooses NN, channel A chooses NN.

D. Pooling equilibriums of the signaling game

Pooling equilibria 1

If the government chooses the strategy (T, T), the firms' beliefs after observing the message from the government are:

$q = \Pr(R)$

$1 - q = \Pr(NR)$

since the message does not provide any additional information about the type of government.

The information set corresponding to the government's decision not to threat is off the equilibrium path i.e. it is certain not to be reached if the game is played according to the equilibrium strategies. Since the firms don't conceive the idea of not being threat, the belief p is arbitrary.

1. $(C_O, C_A, C_B) : \{(N, N), (N, N), (N, N)\}$: *"the untrusted government"*

Condition 55 $(1 - \alpha_A)(\frac{1}{4} - c_A) - \overline{g} - 2h > \frac{1}{8} - c_A$

Condition 56 $(1 - \alpha_B)(\frac{1}{4} - c_B) - \overline{g} - 2h > \frac{1}{8} - c_B$

Condition 57 $(1 - \alpha_B)(\frac{1}{4} - c_B) - \overline{g} > \frac{1}{8} - c_B$

Condition 58 $(1 - \alpha_A)(\frac{1}{4} - c_A) - \overline{g} > \frac{1}{8} - c_A$

Condition 59 $\alpha_A(\frac{1}{4} - c_A) + \alpha_B(\frac{1}{4} - c_B) - \overline{g} > p\frac{1}{8} + (1 - p)\beta_o(\frac{1}{2} - c_A - c_B)$

Condition 60 $\alpha_A(\frac{1}{4} - c_A) + \alpha_B(\frac{1}{4} - c_B) - \overline{g} - 2h > \Pr(R)\frac{1}{8} + \Pr(NR)\beta_o(\frac{1}{2} - c_A - c_B)$

If conditions (1) and (2) hold, then conditions (4) y (3) are satisfied, respectively.

Given that the government is of type R, we need to check that it chooses T.

If the government chooses either T or NT, firms negotiate and result is the asymmetric Nash Bargaining. The government's pay-off is 1. Therefore, the government is indifferent between T and NT. Choosing T is possible.

Given that the government is of type NR, we need to check that it chooses T.

If the government chooses either T or NT, the result is the asymmetric Nash Bargaining. The government's pay-off is 1.

The government is indifferent between T and NT. Choosing T is possible.

$\Longrightarrow$ There is an equilibrium:

$$\{(T,T),(N,N),(N,N),(N,N),\ q=\Pr(R),\ p\in[0,1]:$$
$$\alpha_A(\tfrac{1}{4}-c_A)+\alpha_B(\tfrac{1}{4}-c_B)-\overline{g}>p\tfrac{1}{8}+(1-p)\beta_o(\tfrac{1}{2}-c_A-c_B)\}$$

2. $(C_O,C_A,C_B):\{(NN,NN),(NN,NN),(NN,NN)\}$: *"the passive firms"*

The firms decide not to negotiate either if they observe a threat or not. In terms of the simultaneous game, the firms are indifferent between N and NN so (NN,NN,NN) is a possible equilibrium of that static game.

Given that the government is of type R, we need to check that it chooses T. If the government chooses either T or NT, firms do not negotiate and result is desintegration. The government's pay-off is 3.

Therefore, the government is indifferent between T and NT. Choosing T is possible.

Given that the government is of type NR, we need to check that it also chooses T. If the government chooses either T or NT, firms do not negotiate and result is total integration of the chain. The government's pay-off is 0.

The government is indifferent between T and NT. Choosing T is possible.

$\Rightarrow$There is an equilibrium:

$$\{(T,T),(NN,NN),(NN,NN),(NN,NN),\ q=\Pr(R),\ p\in[0,1]\}$$

3. $(C_O,C_A,C_B):\{(N,N),(N,NN),(NN,N)\}$

Condition 61 $(1-\alpha_B)(\frac{1}{4}-c_B)-\overline{g}-2h<\frac{1}{8}-c_B.$

Condition 62 $(1-\alpha_A)(\frac{1}{4}-c_A)-\overline{g}-h>\Pr(R)(\frac{1}{8}-c_A)+(1-\Pr(R))\beta_A(\frac{1}{2}-c_A-c_B)$

Condition 63 $\frac{5}{16}-(1-\alpha_A)(\frac{1}{4}-c_A)-c_A-\overline{g}-h>\Pr(R)\frac{1}{8}+(1-\Pr(R))\beta_o(\frac{1}{2}-c_A-c_B)$

Condition 64 $(1-\alpha_B)(\frac{1}{4}-c_B)-\overline{g}>p(\frac{1}{8}-c_B)+(1-p)\beta_B(\frac{1}{2}-c_A-c_B)$

Condition 65 $(1-\alpha_A)(\frac{1}{4}-c_A)-\overline{g}<\frac{1}{8}-c_A.$

Condition 66 $\frac{5}{16}-(1-\alpha_B)(\frac{1}{4}-c_B)-c_B-\overline{g}>p\frac{1}{8}+(1-p)\beta_o(\frac{1}{2}-c_A-c_B).$

We need to check that the government chooses T whichever type it is.

If the government chooses T, the cable operator negotiates with channel A. The pay-off of the government is 2.

If it chooses NT, the cable operator doesn't negotiate with channel A but with channel B. The pay-off of the government is 2.

The government is indiferent between T and NT. Choosing T is possible.

$\Longrightarrow$ There is an equilibrium:

$$\{(T,T),(N,N),(N,NN),(NN,N),q=\Pr(R),p\in[0,1]:$$
$$(1-\alpha_B)(\tfrac{1}{4}-c_B)-\overline{g}>p(\tfrac{1}{8}-c_B)+(1-p)\beta_B(\tfrac{1}{2}-c_A-c_B)\ \wedge$$
$$\tfrac{5}{16}-(1-\alpha_B)(\tfrac{1}{4}-c_B)-c_B-\overline{g}>p\tfrac{1}{8}+(1-p)\beta_o(\tfrac{1}{2}-c_A-c_B)\}$$

4. $(C_O,C_A,C_B):\{(N,N),(NN,N),(N,NN)\}$

This refers to an equilibrium of the simultaneous game where channel A chooses not to negotiate but channel B and the cable operator do negotiate if they happen to observe a threat from the government, and an equilibrium where channel B does not enter in negotiations with the cable operator but channel A does negotiate if there is no threat. The following conditions must be satisfied:

Condition 67 $(1-\alpha_A)(\frac{1}{4}-c_A)-\overline{g}-2h<\frac{1}{8}-c_A$

Condition 68 $(1-\alpha_B)(\frac{1}{4}-c_B)-\overline{g}-h>\Pr(R)(\frac{1}{8}-c_B)+\Pr(NR)\beta_B(\frac{1}{2}-c_A-c_B)$

Condition 69 $\frac{5}{16}-(1-\alpha_B)(\frac{1}{4}-c_B)-c_B-\overline{g}-h>\Pr(R)\frac{1}{8}+\Pr(NR)\beta_o(\frac{1}{2}-c_A-c_B)$

Condition 70 $(1-\alpha_A)(\frac{1}{4}-c_A)-\overline{g}>p(\frac{1}{8}-c_A)+(1-p)\beta_A(\frac{1}{2}-c_A-c_B)$

Condition 71 $\frac{5}{16}-(1-\alpha_A)(\frac{1}{4}-c_A)-c_A-\overline{g}>p\frac{1}{8}+(1-p)\beta_o(\frac{1}{2}-c_A-c_B)$

Condition 72 $(1-\alpha_B)(\frac{1}{4}-c_B)-\overline{g}<\frac{1}{8}-c_B$

The equilibrium is symmetric with respect to the one just explained.

$\Longrightarrow$ There is an equilibrium:

$$\{(T,T),(N,N),(NN,N),(N,NN),q=\Pr(R),p\in[0,1]:$$
$$(1-\alpha_A)(\tfrac{1}{4}-c_A)-\overline{g}>p(\tfrac{1}{8}-c_A)+(1-p)\beta_A(\tfrac{1}{2}-c_A-c_B)\ \wedge$$
$$\tfrac{5}{16}-(1-\alpha_A)(\tfrac{1}{4}-c_A)-c_A-\overline{g}>p\tfrac{1}{8}+(1-p)\beta_o(\tfrac{1}{2}-c_A-c_B)\}$$

5. $(C_O, C_A, C_B) : \{(N,N),(N,N),(NN,NN)\}$: *"the left-out channel"*

This refers to an equilibrium of the simultaneous game where the cable operator and channel A decide to negotiate either if they observe a threat or not.

Condition 73 $(1-\alpha_A)(\frac{1}{4}-c_A)-\overline{g}-h > \Pr(R)(\frac{1}{8}-c_A)+\Pr(NR)\beta_A(\frac{1}{2}-c_A-c_B)$

Condition 74 $(1-\alpha_B)(\frac{1}{4}-c_B)-\overline{g}-2h < \frac{1}{8}-c_B$

Condition 75 $(1-\alpha_B)(\frac{1}{4}-c_B)-\overline{g} < \frac{1}{8}-c_B$

Condition 76 $\frac{5}{16}-(1-\alpha_A)(\frac{1}{4}-c_A)-c_A-\overline{g}-h > \Pr(R)\frac{1}{8}+\Pr(NR)\beta_o(\frac{1}{2}-c_A-c_B)$

Condition 77 $(1-\alpha_A)(\frac{1}{4}-c_A)-\overline{g} > p(\frac{1}{8}-c_A)+(1-p)\beta_A(\frac{1}{2}-c_A-c_B)$

Condition 78 $\frac{5}{16}-(1-\alpha_A)(\frac{1}{4}-c_A)-c_A-\overline{g} > p\frac{1}{8}+(1-p)\beta_o(\frac{1}{2}-c_A-c_B)$

Given that the government is of type R, we need to check that it chooses T.

No matter what the government chooses, the cable operator negotiates with channel A.The pay-off of the government is 2. The government is indifferent between T and NT.Choosing T is possible.

Given that the government is of type NR, we need to check that it chooses T.

The result is the same, so the government is indifferent between T and NT.Choosing T is possible.

$\Longrightarrow$ There is an equilibrium:

$$\{(T,T),(N,N),(N,N),(NN,NN),\ q=\Pr(R),\ p\in[0,1]:$$
$$(1-\alpha_A)(\tfrac{1}{4}-c_A)-\overline{g} > p(\tfrac{1}{8}-c_A)+(1-p)\beta_A(\tfrac{1}{2}-c_A-c_B)\ \wedge$$
$$\tfrac{5}{16}-(1-\alpha_A)(\tfrac{1}{4}-c_A)-c_A-\overline{g} > p\tfrac{1}{8}+(1-p)\beta_o(\tfrac{1}{2}-c_A-c_B)\}$$

6. $(C_O, C_A, C_B) : \{(N,N),(NN,NN),(N,N)\}$: *"the left-out channel"*

This refers to an equilibrium of the simultaneous game where the cable operator and channel B decide to negotiate either if they observe a threat or not. This case is analogous to the case just presented but with channel B negotiating with the cable operator instead of channel A.

Condition 79 $(1-\alpha_A)(\frac{1}{4}-c_A)-\overline{g}-2h < \frac{1}{8}-c_A$

Condition 80 $(1-\alpha_B)(\frac{1}{4}-c_B)-\overline{g}-h > \Pr(R)(\frac{1}{8}-c_B)+\Pr(NR)\beta_B(\frac{1}{2}-c_A-c_B)$

Condition 81 $\frac{5}{16} - (1-\alpha_B)(\frac{1}{4} - c_B) - c_B - \overline{g} - h > \Pr(R)\frac{1}{8} + \Pr(NR)\beta_o(\frac{1}{2} - c_A - c_B)$

Condition 82 $(1-\alpha_A)(\frac{1}{4} - c_A) - \overline{g} < \frac{1}{8} - c_A$

Condition 83 $(1-\alpha_B)(\frac{1}{4} - c_B) - \overline{g} > p(\frac{1}{8} - c_B) + (1-p)\beta_B(\frac{1}{2} - c_A - c_B)$

Condition 84 $\frac{5}{16} - (1-\alpha_B)(\frac{1}{4} - c_B) - c_B - \overline{g} > p\frac{1}{8} + (1-p)\beta_o(\frac{1}{2} - c_A - c_B)$

$\Longrightarrow$ There is an equilibrium:

$$\{(T,T),(N,N),(NN,NN),(N,N),q=\Pr(R),p\in[0,1]:$$
$$(1-\alpha_B)(\tfrac{1}{4} - c_B) - \overline{g} > p(\tfrac{1}{8} - c_B) + (1-p)\beta_B(\tfrac{1}{2} - c_A - c_B)\wedge$$
$$\tfrac{5}{16} - (1-\alpha_B)(\tfrac{1}{4} - c_B) - c_B - \overline{g} > p\tfrac{1}{8} + (1-p)\beta_o(\tfrac{1}{2} - c_A - c_B)\}$$

Pooling equilibria 2

If the government chooses the strategy (NT, NT),the firms' beliefs after observing the message from the government are:

$p = \Pr(R)$

$1 - p = \Pr(NR)$

Since the firms do not expect a threat from the government, the belief q is arbitrary.

1. $(C_O, C_A, C_B) : \{(N,N),(N,N),(N,N)\}$

Condition 85 $(1-\alpha_A)(\frac{1}{4} - c_A) - \overline{g} > \frac{1}{8} - c_A$

Condition 86 $(1-\alpha_B)(\frac{1}{4} - c_B) - \overline{g} > \frac{1}{8} - c_B$

Condition 87 $(1-\alpha_A)(\frac{1}{4} - c_A) - \overline{g} - 2h > \frac{1}{8} - c_A$

Condition 88 $(1-\alpha_B)(\frac{1}{4} - c_B) - \overline{g} - 2h > \frac{1}{8} - c_B$

Condition 89 $\alpha_A(\frac{1}{4} - c_A) + \alpha_B(\frac{1}{4} - c_B) - \overline{g} > \Pr(R)\frac{1}{8} + \Pr(NR)\beta_o(\frac{1}{2} - c_A - c_B)$

Condition 90 $\alpha_A(\frac{1}{4}-c_A)+\alpha_B(\frac{1}{4}-c_B)-\overline{g}-h>q\frac{1}{8}+(1-q)\beta_o(\frac{1}{2}-c_A-c_B)$

We need to check that the government chooses NT whichever type it is. Since firms negotiate no matter what message they receive from the government, result is the asymmetric Nash Bargaining. Therefore, the government is indifferent between T and NT. Choosing NT is possible.

$\Longrightarrow$ There is an equilibrium:

$$\{(NT,NT),(N,N),(N,N),(N,N),p=\Pr(R),q\in[0,1]:$$
$$\alpha_A(\tfrac{1}{4}-c_A)+\alpha_B(\tfrac{1}{4}-c_B)-\overline{g}-h>q\tfrac{1}{8}+(1-q)\beta_o(\tfrac{1}{2}-c_A-c_B)\}$$

2. $(C_O,C_A,C_B):\{(NN,NN),(NN,NN),(NN,NN)\}$

In this case, all firms decide to stay as they are, regardless of the government's message, so no negotiation takes place.

Given that the government is of type R, we need to check that it chooses NT. If the government chooses either T or NT, firms do not negotiate and result is desintegration. The government's pay-off is 3. Therefore, the government is indifferent between T and NT. Choosing NT is possible.

Given that the government is of type NR, we need to check that it chooses NT. If the government chooses either T or NT, firms do not negotiate and result is total integration of the chain. The government's pay-off is 0.

$\Rightarrow$There is an equilibrium:

$$\{(NT,NT),(NN,NN),(NN,NN),(NN,NN),\ p=\Pr(R),\ q\in[0,1]\}$$

3. $(C_O,C_A,C_B):\{(N,N),(N,NN),(NN,N)\}$

In this case, channel B chooses not to negotiate but channel A and the cable operator do negotiate if they happen to observe a threat from the government; and channel B doesn't enter in negotiations with the cable operator but channel A does negotiate if there is no threat.

Condition 91 $(1-\alpha_B)(\frac{1}{4}-c_B)-\overline{g}>\Pr(R)(\frac{1}{8}-c_B)+\Pr(NR)\beta_B(\frac{1}{2}-c_A-c_B)$

Condition 92 $\frac{5}{16}-(1-\alpha_B)(\frac{1}{4}-c_B)-c_B-\overline{g}>\Pr(R)\frac{1}{8}+\Pr(NR)\beta_o(\frac{1}{2}-c_A-c_B)$

Condition 93 $(1-\alpha_A)(\frac{1}{4}-c_A)-\overline{g}<\frac{1}{8}-c_A$

Condition 94 $(1-\alpha_B)(\frac{1}{4}-c_B)-\overline{g}-2h<\frac{1}{8}-c_B$

Condition 95 $\frac{5}{16}-(1-\alpha_A)(\frac{1}{4}-c_A)-c_A-\overline{g}-h>q\frac{1}{8}+(1-q)\beta_o(\frac{1}{2}-c_A-c_B)$

Condition 96 $(1-\alpha_A)(\frac{1}{4}-c_A)-\overline{g}-h>q(\frac{1}{8}-c_A)+(1-q)\beta_A(\frac{1}{2}-c_A-c_B)$

We need to check that the government chooses NT whichever type it is.

If the government chooses T, the cable operator ends up negotiating with channel A. The pay-off of the government is 2.

If it chooses NT, the negotiation is between the cable operator and channel B.The pay-off of the government is 2. The government is indiferent between T and NT. Choosing NT is possible.

$\Longrightarrow$ There is an equilibrium:

$$\{(NT,NT),(N,N),(N,NN),(NN,N),\ p=\Pr(R),\ q\in[0,1]:$$
$$\frac{5}{16}-(1-\alpha_A)(\frac{1}{4}-c_A)-c_A-\overline{g}-h>q\frac{1}{8}+(1-q)\beta_o(\frac{1}{2}-c_A-c_B)\ \wedge$$
$$(1-\alpha_A)(\frac{1}{4}-c_A)-\overline{g}-h>q(\frac{1}{8}-c_A)+(1-q)\beta_A(\frac{1}{2}-c_A-c_B)\}$$

4. $(C_O,C_A,C_B):\{(N,N),(NN,N),(N,NN)\}$

This case analogous to the one just explained but the roles of the channels are reversed.

Condition 97 $(1-\alpha_A)(\frac{1}{4}-c_A)-\overline{g}-2h<\frac{1}{8}-c_A$

Condition 98 $(1-\alpha_B)(\frac{1}{4}-c_B)-\overline{g}-h>q(\frac{1}{8}-c_B)+(1-q)\beta_B(\frac{1}{2}-c_A-c_B)$

Condition 99 $\frac{5}{16}-(1-\alpha_B)(\frac{1}{4}-c_B)-c_B-\overline{g}-h>q\frac{1}{8}+(1-q)\beta_o(\frac{1}{2}-c_A-c_B)$

Condition 100 $(1-\alpha_B)(\frac{1}{4}-c_B)-\overline{g}<\frac{1}{8}-c_B$.

Condition 101 $(1-\alpha_A)(\frac{1}{4}-c_A)-\overline{g}>\Pr(R)(\frac{1}{8}-c_A)+(1-\Pr(R))\beta_A(\frac{1}{2}-c_A-c_B)$.

Condition 102 $\frac{5}{16}-(1-\alpha_A)(\frac{1}{4}-c_A)-c_A-\overline{g}>\Pr(R)\frac{1}{8}+(1-\Pr(R))\beta_o(\frac{1}{2}-c_A-c_B)$.

$\Longrightarrow$ There is an equilibrium:

$$\{(NT,NT),(N,N),(NN,N),(N,NN),\ p=\Pr(R), q\in[0,1]:$$
$$(1-\alpha_B)(\tfrac{1}{4}-c_B)-\overline{g}-h>q(\tfrac{1}{8}-c_B)+(1-q)\beta_B(\tfrac{1}{2}-c_A-c_B)\ \wedge$$
$$\tfrac{5}{16}-(1-\alpha_B)(\tfrac{1}{4}-c_B)-c_B-\overline{g}-h>q\tfrac{1}{8}+(1-q)\beta_o(\tfrac{1}{2}-c_A-c_B)\}$$

5. $(C_O,C_A,C_B):\{(N,N),(N,N),(NN,NN)\}$

This refers to an equilibrium of the simultaneous game where the cable operator and channel A decide to negotiate either if they observe a threat or not. Channel B chooses not to negotiate.

Condition 103 $(1-\alpha_A)(\frac{1}{4}-c_A)-\overline{g}>\Pr(R)(\frac{1}{8}-c_A)+\Pr(NR)\beta_A(\frac{1}{2}-c_A-c_B)$

Condition 104 $(1-\alpha_B)(\frac{1}{4}-c_B)-\overline{g}-2h<\frac{1}{8}-c_B$

Condition 105 $(1-\alpha_B)(\frac{1}{4}-c_B)-\overline{g}<\frac{1}{8}-c_B$

Condition 106 $\frac{5}{16}-(1-\alpha_A)(\frac{1}{4}-c_A)-c_A-\overline{g}>\Pr(R)\frac{1}{8}+\Pr(NR)\beta_o(\frac{1}{2}-c_A-c_B)$

Condition 107 $(1-\alpha_A)(\frac{1}{4}-c_A)-\overline{g}-h>q(\frac{1}{8}-c_A)+(1-q)\beta_A(\frac{1}{2}-c_A-c_B)$

Condition 108 $\frac{5}{16}-(1-\alpha_A)(\frac{1}{4}-c_A)-c_A-\overline{g}-h>q\frac{1}{8}+(1-q)\beta_o(\frac{1}{2}-c_A-c_B)$

Note that if condition (2) holds, then condition 3 is satisfied.

Given that the government is of type R, we need to check that it chooses NT.

No matter what the government chooses, the cable operator negotiates with channel A.The pay-off of the government is 2.

The government is indifferent between T and NT. Choosing NT is possible.

Given that the government is of type NR, we need to check that it chooses NT.

No matter what the government chooses, the cable operator negotiates with channel A.

The government is indifferent between T and NT.Choosing NT is possible.

$\Longrightarrow$ There is an equilibrium:

$$\{(NT,NT),(N,N),(N,N),(NN,NN),p=\Pr(R), q\in[0,1]:$$
$$(1-\alpha_A)(\tfrac{1}{4}-c_A)-\overline{g}-h>q(\tfrac{1}{8}-c_A)+(1-q)\beta_A(\tfrac{1}{2}-c_A-c_B)\ \wedge$$
$$\tfrac{5}{16}-(1-\alpha_A)(\tfrac{1}{4}-c_A)-c_A-\overline{g}-h>q\tfrac{1}{8}+(1-q)\beta_o(\tfrac{1}{2}-c_A-c_B)\}$$

6. $(C_O, C_A, C_B) : \{(N, N), (NN, NN), (N, N)\}$

This refers to an equilibrium of the simultaneous game where the cable operator and channel B decide to negotiate either if they observe a threat or not. Channel A chooses not to negotiate. So, this equilibirum is analogous from the one just presented.

Condition 109 $(1 - \alpha_A)(\frac{1}{4} - c_A) - \overline{g} < \frac{1}{8} - c_A$

Condition 110 $(1 - \alpha_A)(\frac{1}{4} - c_A) - \overline{g} - 2h < \frac{1}{8} - c_A$

Condition 111 $(1 - \alpha_B)(\frac{1}{4} - c_B) - \overline{g} > \Pr(R)(\frac{1}{8} - c_B) + \Pr(NR)\beta_B(\frac{1}{2} - c_A - c_B)$

Condition 112 $\frac{5}{16} - (1 - \alpha_B)(\frac{1}{4} - c_B) - c_B - \overline{g} > \Pr(R)\frac{1}{8} + \Pr(NR)\beta_o(\frac{1}{2} - c_A - c_B)$

Condition 113 $(1 - \alpha_B)(\frac{1}{4} - c_B) - \overline{g} - h > q(\frac{1}{8} - c_B) + (1 - q)\beta_B(\frac{1}{2} - c_A - c_B)$

Condition 114 $\frac{5}{16} - (1 - \alpha_B)(\frac{1}{4} - c_B) - c_B - \overline{g} - h > q\frac{1}{8} + (1 - q)\beta_o(\frac{1}{2} - c_A - c_B)$

$\Longrightarrow$ There is an equilibrium:

$$\{(NT, NT), (N, N), (NN, NN), (N, N), p = \Pr(R), q \in [0, 1] :$$
$$(1 - \alpha_B)(\tfrac{1}{4} - c_B) - \overline{g} - h > q(\tfrac{1}{8} - c_B) + (1 - q)\beta_B(\tfrac{1}{2} - c_A - c_B) \wedge$$
$$\tfrac{5}{16} - (1 - \alpha_B)(\tfrac{1}{4} - c_B) - c_B - \overline{g} - h > q\tfrac{1}{8} + (1 - q)\beta_o(\tfrac{1}{2} - c_A - c_B)\}$$

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