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From Solow to Schumpeter: A Two-Stage Endogenous Model of Economic Growth

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Abstract

This paper studies the determinants which enable an economy to enter an innovation-driven growth stage. We present a model in which the final good is produced with labor and an intermediate good. This intermediate good is produced by default in a competitive market, but a firm can have the possibility to invest in research and development and, if successful, become the monopolist in the market for a period. Successful research generates improvements in productivity that make long term economic growth possible. We derive a condition to be met in order to initiate innovation, and additionally we analyze specific policies that may help commence innovation in an economy which originally does not meet the precedent condition.

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"*Innovation is the outstanding fact in the economic history of capitalist society or in what is purely economic in that history, and also it is largely responsible for most of what we would at first sight attribute to other factors.*"
J. A. Schumpeter

1 Introduction

In the *Global Innovation Index 2013*[1], innovation is recognized to play "a key role as a driver of economic growth and prosperity". *The Economist*[2], in its own innovation report, defines innovation to be "the application of knowledge in a novel way, primarily for economic benefit". Furthermore, reports studying innovation in companies, in the public sector and in academia appear on a regular basis. Innovation is the engine that enables differentiation from the competition, which is the main source of opportunities and profits. One common conclusion of these reports is that there exist a positive correlation between countries with high indexes of innovation and their per capita GDP.

This evidence motivated our interest in further investigating the role played by innovation in the process of economic growth. According to the economic literature, there are two basic ways of growing: the first one is by factor accumulation and the second one is by innovation. The neoclassical model, first presented by Solow (1956) and Swan (1956) and then modified by Cass (1965) and Koopmans (1965), emphasizes the role of capital accumulation in the growth process. Because of diminishing marginal productivity of capital, this form of growth cannot last indefinitely. For this reason, the model introduces an exogenous technological growth rate to avoid the stagnation. Further research focused on developing endogenous growth models, where the technological growth rate would be determined by forces internal to the economic system. In these endogenous growth models, innovation was placed at the center of research.

Technological progress – made possible due to innovation - is what makes economic growth sustainable in the long run. Factor accumulation – human or physical – is subject to the law of diminishing returns. Without technological improvements, an economy can perhaps grow for a while by accumulating capital, but will eventually stagnate. Seminal models of endogenous growth are, among others, Romer (1990), where innovation causes productivity growth by creating new varieties of products and Aghion and Howitt (1992), where quality-improving innovations render old products obsolete through a Schumpeterian creative destruction process.

Vast differences in per capita GDP exist across the world. The question we will try to answer in this paper is whether any economy can enter an innovation-driven growth stage, in which innovation – and not factor accumulation – is the key driver of growth. An economic model that would pretend to make the point on the importance of innovation in the long run should therefore (1) contain both forms of economic growth and (2) enable economic stagnation. In other words, this economic model should be a model of growth stages, with a first stage of factor accumulation-driven growth and a second stage of innovation-driven growth. Empirical evidence shows that reaching the second stage of growth is not self-evident.

The importance of innovation for an economy has been first pointed out by Schumpeter, who made a case against both the Marxist division of society into capitalists and proletariat and the static Walrasian equilibrium, claiming for the introduction of a third character – the entrepreneur – whose role was to disturb that equilibrium due to his innovations. Schumpeter's argument was that innovation and technological change come from entrepreneurs, usually large companies which have the resources and capital to invest in research and development as well as some sort of market power. He realized that monopolies were necessary to provide incentives to innovate, well before endogenous growth models with imperfect competition were invented.
Hansen and Prescott (2002) model a two-stage economy to explain the transition from a stagnated Malthusian economy to a growing modern Solow economy. Motivated by this idea, we present in this paper an endogenous growth model with two stages of development, which adds to the Solow model the

[1] *The Global Innovation Index 2013*, The Local Dynamics of Innovation, Cornell University, INSEAD, WIPO, 2013
[2] Economist Intelligence Unit, *A new ranking of the world's most innovative countries*, 2009

innovation effect emphasized by Schumpeter as the key factor explaining sustainable economic growth. The result is an economy in which no exogenous growth rate has to be assumed in order to generate economic growth in the long run. Also, in contrast with Hansen and Prescott, the economies we study are already capitalist economies.

Our objective is to create a model in which both ways of economic growth could coexist, and where the transition from one stage of another would not be inevitable, as it happens to be in Hansen and Prescott. In particular, the transition from one stage to the other occurs when the economy reaches a capital level that makes the benefits of a monopolist who produces intermediate goods big enough so that he has the incentives to invest in research. We identify four important parameters that influence this minimum capital level necessary to invest: the savings rate of the economy, the cost on investment, the probability of success of the research and the quality of the innovation. Differences in these parameters among countries can serve as an explanation to differences in per capita GDP.

The rest of the paper is organized as follows. In the next Section we describe the model economy and we define and characterize the equilibrium. In Section 3 we analyze empirical evidence and we calibrate the model. In Section 4 we discuss some policies a government could implement and their effect on long run growth. Some concluding comments are provided in Section 5.

2 The Model

2.1 Production of the Final Good

Each period, a competitive firm produces a final good Y_t, according to the following production function

$$Y_t = (A_t L)^{1-\alpha}(X_t)^{\alpha}$$

where A_t is the productivity of the economy, L are hours of labor, X_t is an intermediate good and Y_t is increasing, concave, differentiable and homogenous of degree one, and Inada conditions hold.

Taking prices as given, the final firm maximizes

$$(A_t L)^{1-\alpha}(X_t)^{\alpha} - w_t L - p_t X_t$$

First order conditions are

$$(X_t) \qquad (A_t L)^{1-\alpha}\alpha(X_t)^{\alpha-1} = p_t$$

$$(L) \qquad (1-\alpha)(A_t L)^{-\alpha}A_t(X_t)^{\alpha} = w_t$$

2.2 Production of the Intermediate Good

There is a continuum [0,1] of firms who can produce the intermediate good. Their production function is extremely capital-intensive and can be written as

$$X_t = K_t$$

Each period, one of these intermediate firms is randomly chosen and given the possibility to innovate. In order to do so, it has to invest R_t in research, knowing that, with probability μ_t, it will be successful, rising the productivity of the economy from A_{t-1} to γA_{t-1}, where $\gamma > 1$, and becoming a monopolist in providing the intermediate good to the final firm. The rise in productivity benefits him because of the rise in the demand for the intermediate good. If it fails, or if it decides not to invest in research, productivity remains the same as in the precedent period, there is free entry to the market, and therefore the intermediate good is produced competitively. If it takes place, the rise in productivity happens in the same period in which the research is done. For simplicity, we will assume that for every period $R_t = R$ and $\mu_t = \mu$. We also assume that the intermediate firms are risk neutral.

The reason to introduce a monopolist is that some kind of imperfect competition is needed in order to generate endogenous growth. For an incentive to invest in research to exist, resources have to be available and the benefits of the monopolist have to be positive. Within a competitive market structure, after paying the marginal productivity of the inputs, the economy has nothing more available to allocate to those who invest in innovation.

If the chosen firm decides to invest in research, it maximizes his expected benefits

$$E(\pi_t^*) = \mu\pi_t^m(\gamma A_{t-1}) + (1-\mu)\pi_t^c(A_{t-1}) - R \qquad (2.1)$$

where $\pi_t^m(\gamma A_{t-1}) = p_t X_t - r_t K_t$ are the benefits of the monopolist if it succeeds in innovating, with $p_t = \alpha(\gamma A_{t-1}L)^{1-\alpha}X_t^{\alpha-1}$ being the inverse demand for the intermediate good, and $\pi_t^c(A_{t-1}) = 0$ are the benefits under perfect competition. From (2.1) and using that $X_t = K_t$

$$E(\pi_t^*) = \mu(\alpha(\gamma A_{t-1}L)^{1-\alpha}K_t^\alpha - r_t K_t) - R$$

$$(K_t) \qquad r_t = \alpha^2(\gamma A_{t-1}L)^{1-\alpha}K_t^{\alpha-1} \qquad (2.2)$$

The intermediate firm produces (and supplies the final firm) the quantity

$$X_t^s = \gamma A_{t-1}L(\frac{\alpha^2}{r_t})^{\frac{1}{1-\alpha}}$$

which is also the quantity of capital K_t^d he demands. The supply of capital is determined by the motion law of capital.

Combining equations (2.1) and (2.2) yields

$$E(\pi_t^*) = \mu\alpha(1-\alpha)(\gamma A_{t-1}L)^{1-\alpha}K_t^\alpha - R$$

The profit of the monopolist is an increasing function of the capital stock.

<u>2.3 Description of the Household</u>

There is one representative household, who lives infinitely and is endowed each period with L hours of time which it supplies inelastically as labor and earns $w_t L$ in return. It can also save part of his income and earn $r_t K_t$. Moreover, it owns the benefits π_t^* of the firm that produces the intermediate goods. Each period, it saves a fraction s of his income, and consumes a fraction $(1-s)$. Capital evolves according to its classic motion law $K_{t+1} = sY_t + (1-\delta)K_t$.

2.4 The Innovation Decision Problem

If the randomly chosen firm decides not to invest in research, its benefits are those of perfect competition $\pi_t^c = 0$. Each period, this firm decides to invest in research if its expected profits of doing so are bigger than zero, such that

$$E(\pi_t^*) > 0$$

$$\mu\pi_t^m(\gamma A_{t-1}) + (1-\mu)\pi_t^c(A_{t-1}) - R > 0$$

From where we can isolate K_t, to obtain a level of capital we call K^{inn} (innovation)

$$K_t > \left(\frac{R}{\alpha(1-\alpha)\mu(\gamma A_0 L)^{1-\alpha}}\right)^{\frac{1}{\alpha}} = K^{inn}$$

As A_t is fixed in every period before innovation begins, we replace it by A_0, the initial level of productivity. Notice the interesting fact that K^{inn} does not depend on time. Investment in research will take place only if the capital stock of the economy is bigger than K^{inn}.

2.5 Growth

The expected productivity of the economy each period is

$$E(A_t) = \mu\gamma A_{t-1} + (1-\mu)A_{t-1}$$

Therefore, growth will be random. Let g_t be the growth rate of A_t. In each period, with probability μ g_t will equal $(\gamma - 1)$ and with probability $(1-\mu)$ g_t will equal 0. By the law of large numbers, the mean of this distribution - and therefore the expected productivity growth rate - is given by the frequency of innovations times the size of the innovation, such that

$$E(g_t) = \frac{A_{t+1} - A_t}{A_t} = (\gamma - 1)\mu$$

If there is no rise in productivity, the economy reaches a steady state where capital remains constant at the level

$$K^{ss} = A_0 L\left(\frac{s}{\delta}\right)^{\frac{1}{1-\alpha}}$$

In an economy where $K^{inn} < K^{ss}$, the monopolist has enough incentives to invest in research before the steady state of the economy is reached. This type of economy grows not only by accumulating capital, but also, when the conditions are met, by innovating. In an economy where the opposite is true, the steady state is reached before the economy begins to innovate. As a consequence, in this second case the economy grows only for some time before stagnating, and the only source of growth is factor accumulation, as in a standard Solow model.

The expected growth rate of the economy can be written as

$$E[1 + Z_t] = E\left[\frac{Y_{t+1}}{Y_t}\right] = E\left[\left(\frac{A_{t+1}}{A_t}\right)^{1-\alpha}\left(\frac{K_{t+1}}{K_t}\right)^{\alpha}\right]$$

The model yields two types of economies: an economy which innovates and an economy which stagnates. Let ξ_t be the growth rate of capital.

If $K^{inn} < K^{ss}$ (2.4) holds, the economy innovates and its expected growth rate is given by

$$E[1+Z_t] = \begin{cases} (\frac{K_{t+1}}{K_t})^{\alpha} = {\xi_t}^{\alpha} & if\ t < t^{inn} \\ (\frac{A_{t+1}}{A_t})^{1-\alpha}(\frac{K_{t+1}}{K_t})^{\alpha} = \{\mu(\gamma-1)+1\}^{1-\alpha}{\xi_t}^{\alpha} & if\ t^{inn} < t \end{cases}$$

Where t^{inn} holds that $K_t = K^{inn}$.

If $K^{inn} > K^{ss}$ (2.5) holds, the economy stagnates. Its growth rate is

$$1+Z_t = \begin{cases} (\frac{K_{t+1}}{K_t})^{\alpha} = {\xi_t}^{\alpha} & if\ t < t^{ss} \\ 0 & if\ t^{ss} < t \end{cases}$$

Where t^{ss} holds that $K_t = K^{ss}$.

Figure 1 shows the evolution of income of Economy 1 (in which condition (2.4) holds) and Economy 2 (in which condition (2.5) holds).

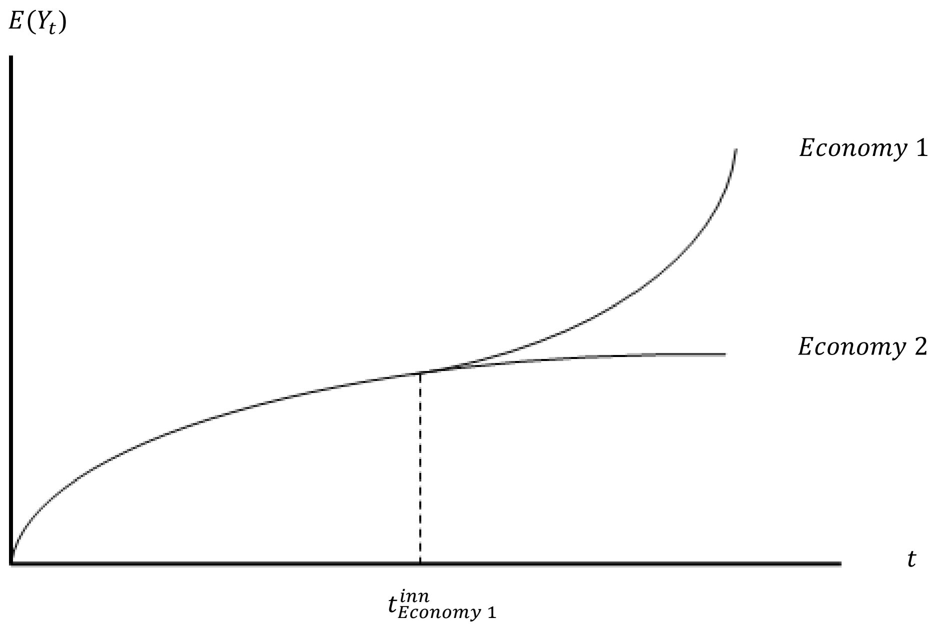

FIGURE 1

In the first periods, when the capital level of the economy is low, every economy grows due to capital accumulation. Then, after some periods and if condition (2.4) holds, the economy begins to innovate, its growth rate rises and growth is sustainable in the long run. In contrast, if it is condition (2.5) which holds, the economy reaches its steady state before the incentives to innovate are in place. As a consequence, the economy stagnates. The level of capital K^{inn} is what gives the monopolist the incentives to invest in research. In other words, every economy behaves in the beginning like in a standard Solow model. Only if certain conditions are met, the economy then accesses the Schumpeterian stage.

In a stagnated economy, intermediate goods are produced competitively, and no resources are left available to invest in research. As a result, this stage lacks a monopolist, which in our model plays the role of the entrepreneur. The division of society into workers and capitalists, and the consequent nonexistence of entrepreneurs, is the flaw Schumpeter found in the Marxist description of the economy. According to Schumpeter, entrepreneurs are the ones who foster innovation, which is the critical dimension of economic change. In our model, innovation occurs only when the monopolist-entrepreneur appears. Its existence depends on broad conditions of the economy, which can be reduced to a minimum level of capital necessary to provide the right incentives to innovate.

2.6 Equilibrium

An equilibrium of the economy is an allocation $\{K_t, X_t, Y_t\}_{t=0}^{\infty}$, a distribution of profits $\{\pi_t^*\}_{t=0}^{\infty}$ and a price path $\{r_t, w_t, p_t\}_{t=0}^{\infty}$, such that

(i) Given $\{r_t, w_t, p_t\}_{t=0}^{\infty}$ and$\{\pi_t^*\}_{t=0}^{\infty}$, the path $\{K_t, X_t, Y_t\}_{t=0}^{\infty}$ is consistent with the behavior of the individuals

(ii) $\{K_t, X_t\}_{t=0}^{\infty}$ maximize firms' profits

(iii) The capital market and the intermediate good market clear every period

2.7 Determinants of Growth

As we said before, an economy in which $K^{ss} < K^{inn}$ stagnates. The difference between an economy which innovates and one that does not is explained in this model by differences in the parameters, as both crucial levels of capital are determined by them. A high initial productivity level or big population lower K^{inn}. But four other parameters are more important to our analysis: the savings rate s, the probability of success μ, the fixed cost of research R and the factor γ by which productivity raises when innovation takes place.

A lower saving rate s implies higher consumption. In the steady state, ceteris paribus, a country with a lower s attains a capital level lower than that of a country with a higher s. The smaller the saving rate, the bigger the possibility of reaching K^{ss} before K^{inn}, which means that long run growth is eliminated. For this situation to be avoided, the saving rate should be higher than

$$s > \delta[(\frac{R}{\alpha(1-\alpha)\mu\gamma^{1-\alpha}})]^{\frac{1-\alpha}{\alpha}}$$

The probability of the investment in innovation to be successful μ is another key variable. For K^{inn} to be lower than K^{ss}, μ has to be bigger than

$$\mu > \left(\frac{\delta}{s}\right)^{\frac{\alpha}{1-\alpha}} \frac{R}{\alpha(1-\alpha)\gamma^{1-\alpha}}$$

A high R is also a risk: the benefits of the monopolist may not be big enough to encourage him to invest in innovation, thus postponing the period t^{inn} in the best scenario or eliminating all possibility for this period to occur in the worst scenario. Therefore, R should not be higher than

$$R < \left(\frac{s}{\delta}\right)^{\frac{\alpha}{1-\alpha}} \alpha(1-\alpha)\mu\gamma^{1-\alpha}$$

Following the same reasoning, the raise in productivity should be bigger than

$$\gamma > (\frac{\delta}{s})^{\frac{1}{\alpha}}(\frac{R}{\alpha(1-\alpha)\mu})^{\frac{1}{1-\alpha}}$$

2.8 Short Run Trade Off

In the long run, there is no doubt that the household is better off in an innovative economy than in a stagnated one. Nevertheless, in the first periods after innovation begins, the household is not necessarily better off. Household's income in the periods before innovation happens is given by $Y_t^s = w_t L + r_t K_t = (A_0 L)^{1-\alpha}(K_t)^{\alpha}$.

Once investment in research and development begins, household's income is given by $w_t L + r_t K_t + \pi_t^*$, which equals

$$\begin{cases} (\gamma A_t L)^{1-\alpha}(K_t)^{\alpha} - R & \textit{with probability } \mu \\ (A_t L)^{1-\alpha}(K_t)^{\alpha} - R & \textit{with probability } 1-\mu \end{cases}$$

In particular, expected income is given by

$$E(Y_t) = (A_t L)^{1-\alpha}(K_t)^{\alpha}\{1 - \mu + \mu\gamma^{1-\alpha}\} - R$$

With innovation, the household´s expected income increases because of the expected rise in productivity, but at the same time, the cost R hast to be paid. In order to analyze what happens with the household´s income when the economy innovates, both opposite effects have to be compared. The household will be better off if

$$E(Y_t) - R > Y_t^s$$

The precedent inequality is satisfied if:

$$K^{sr} > (\frac{R}{\mu(\gamma^{1-\alpha}-1)A_0 L})^{\frac{1}{\alpha}}$$

Assuming reasonable values for α, the following inequality should hold

$$\gamma < (\frac{1}{1-\alpha+\alpha^2})^{\frac{1}{\alpha}}$$

If the previous inequality is satisfied,

$$K^{sr} > K^{inn}$$

Thus until the capital reaches K^{sr}

$$E(Y_t) - R < Y_t^s$$

When innovation begins, the income of the household is reduced because the cost of research exceeds the rise in income originated by the rise in productivity. Therefore, our model predicts that the sooner the economy innovates, the larger its income in the long run, but also the higher the reduction of its income in the short term.

2.9 Summary

The model we presented to study the difficulties for an economy to reach an innovation-driven growth stage predicts that a minimum level of capital K^{inn} is necessary in order to generate the incentives to invest in research. The minimum level on capital depends on different parameters, consisting of the saving rate of the economy, the fixed cost of investment, the probability of success and the quality of the improvement in productivity. This suggests that the existence of the right incentives to innovate depends on broad conditions of the economy. Our model contains both requirements we identified in the introduction as indispensable: (i) it includes both forms of economic growth (factor accumulation and innovation) and (ii) it enables economic stagnation. The model succeeds therefore in emphasizing the importance of innovation in the long run and above all in accounting for the difficulties to enter the growth stage in which innovation is the primary driver of economic growth. Also, the model generates endogenous growth by the introduction of imperfect competition in the Solow model, which yields a Schumpeterian economy in which innovation, and not factor accumulation as in the Solow model, is the key driver of growth. In the following Section we contrast the model with empirical evidence and we calibrate it.

3 Empirical Evidence and Calibration

3.1 Empirical Evidence

Innovation is undoubtedly the buzzword of our time. In a competitive economy, those who innovate are rewarded with more opportunities and profits. Innovation creates progress and improves the well-being of the people. This is recognized by individuals, companies and governments all across the world, who are putting innovation at the center of their growth strategies, and is one of the reasons why innovation remains always a subject of intensive studies. In this Section we compare the conclusions of our model with the results of the Global Innovation Index (GII) 2013, co-published by Cornell University, INSEAD and the World Intellectual Property Organization.

The GII was launched motivated by three main objectives: the importance of innovation in driving economic progress and competitiveness - for both developed and developing economies -, the awareness that the definition of innovation has broadened to include not only expenditure in R&D but also social, political and economic conditions, and the inspiration that innovation creates in human beings. The GII ranks 142 countries according to their innovation capacity, and gives them a score from 0 (the worst) to 100 (the best), with the first country being Switzerland (with a score of 66.6) and the last country being Yemen (with a score of 19.3). As we said before, a strong correlation exists between innovative countries and their income levels, as shown in Figure 2.

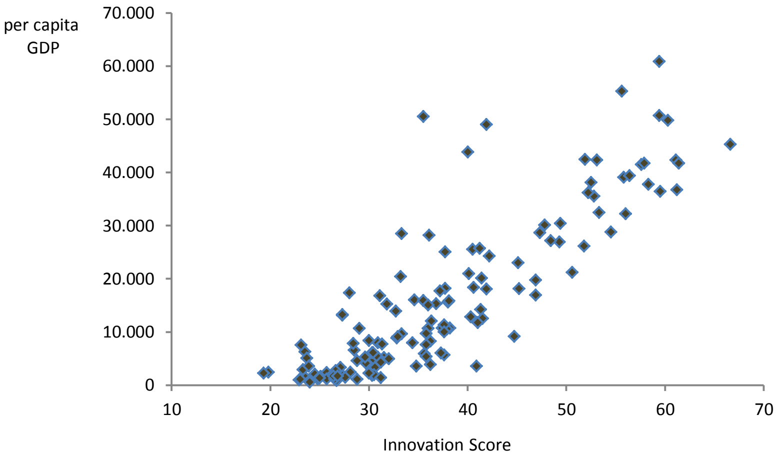

FIGURE 2

Source: The Global Innovation Index 2013 (Per capita GDP in PPP$)

As shown in the following table, we divide the 142 countries ranked from most to least innovative in four groups:

Quartile	Number of Countries	Innovation Index's Rank
1st quartile	35	44.7 – 66.6
2nd quartile	35	35.8 – 42.2
3rd quartile	35	29 – 35.8
4th quartile	37	19.3 – 28.8

If we take the average evolution of per capita GDP of these four groups of countries, we obtain the graph in Figure 3.

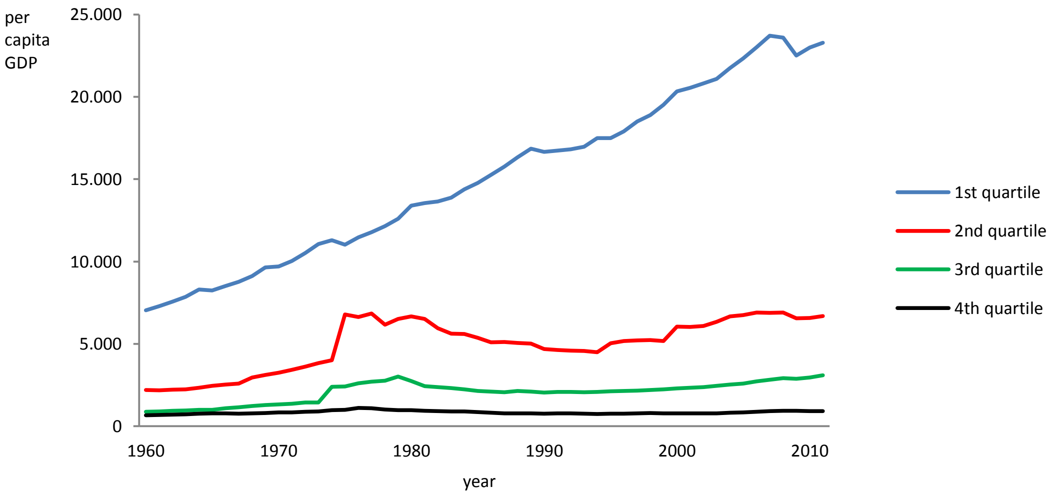

FIGURE 3

Source: World Bank (per capita GDP in constant 2000 US$) and Global Innovation Index 2013

As we can see, the countries of the 1st quartile, whose Innovation Score goes from 44.7 to 66.6, have grown steadily in the past 50 years. The rest of the quartiles, although their performance differs between them, look more to be stagnated around a specific level of income. For these three groups, growth has been modest at best, and non existent at worst during this time.

The Global Innovation Index is constructed with measures concerning the seven pillars we now describe:

- Institutions

The GII argues that "nurturing an institutional framework that attracts businesses and fosters growth by providing good governance and the correct levels of protection and incentives is essential to innovation". This pillar considers the political environment, the regulatory framework and the business environment of each country. The "institutional hypothesis" has been emphasized, among others, by Acemoglu, Johnson and Robinson (2002) to explain economic performance, arguing that societies that provide incentives and opportunities for investment will be richer than those that fail to do so. Institutions of private property, which secure property rights for a majority of the population, are essential for investment incentives and successful economic performance. In contrast, extractive institutions, which concentrate power in the hands of the elite and create a high risk of expropriation for the majority of the population, are likely to discourage investment and economic performance.

- Human Capital and Research

This pillar measures the human capital of the countries. Education and research activity are prime determinants of the innovation capacity and are crucial elements in the process of adding value in the economic system.

- Infrastructure

This pillar considers information and communication technologies, general infrastructure and ecological sustainability, which "facilitate the production and exchange of ideas, services and goods", increasing productivity and lowering transactions costs.

- Market sophistication

Availability of credit, investment funds, access to international markets and free trade are essential for business to prosper. This pillar takes into account the market conditions and the volume of financial transactions.

- Business sophistication

This pillar analyses how businesses perform when it comes to using available resources like human capital, technology and how they succeed in establishing strategic associations with research centers and universities, as well as in which measure do business involve themselves in research expenditure. Flows of foreign investment in also considered in this pillar.

- Knowledge and technology outputs

Patent applications, scientific and technical publications and all that has to do with knowledge creation is summarized in this pillar, as well as elements that foster knowledge diffusion and technological penetration.

- Creative Outputs

Trademark registration and creativity-driven production like culture, arts and services is measured in this pillar.

As we see, innovation depends on a mix of institutions, skills, infrastructure, integration with global markets and linkages to the business community. Innovation has to spread across all the dimensions of society and touch large segments of the population to foster growth and prosperity. Although our model is not so explicit about all the pillars which form the GII, the parameters we identified in 2.7 as important can be related in some way or another to these elements. The savings rate, the cost of investment, the probability of success of research and the quality of innovation are actually variables that directly depend on the performance in these different pillars.

3.2 Calibration

We now calibrate our model to obtain the income evolution of two theoretical economies: an innovative one and a non-innovative one. We assume that both economies have similar initial conditions in productivity, stock of capital and population. We calibrate μ and γ in a way for the model to yield an average long run growth of 2%, which is a reasonable long run growth rate for a developed economy. These two economies therefore differ only in their savings rate and in the cost of investment in research, as shown in the following table:

Innovative Economy			
α	1/3	s	0.20
δ	0.05	R	0.2
A_0	1	μ	0.4
K_0	1	γ	1.05
L	1		

Non-Innovative Economy			
α	1/3	s	0.25
δ	0.05	R	0.4
A_0	1	μ	0.4
K_0	1	γ	1.05
L	1		

Figure 4 shows a possible path of the evolution of income of both economies.

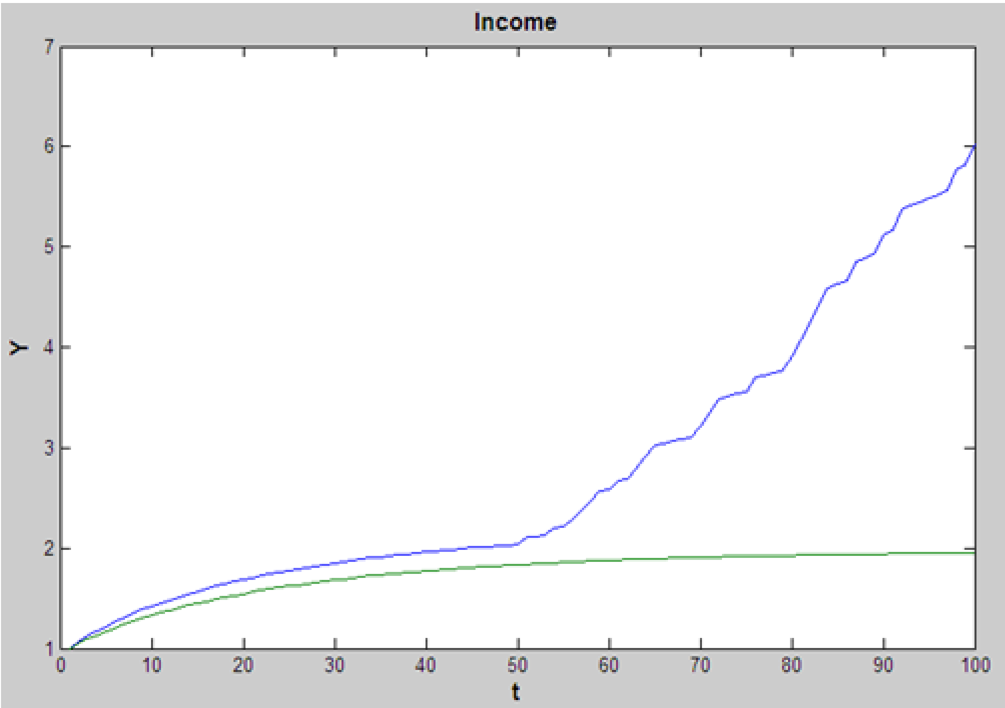

FIGURE 4

As can be seen in the graph, the evolution of income during the Solow stage has no randomness. This is not the case in the Schumpeter stage. It is worth noting that the graph is a realization of one possible path of the Schumpeter stage. This is consistent with the fact that innovation is a chaotic process, which occurs randomly and is not evenly distributed across different periods. Growth by factor accumulation is far more predictable and therefore doesn't show randomness. In addition to this, the innovative economy exhibits clearly higher growth rates once it starts to innovate and a great divergence between both economies can be observed.

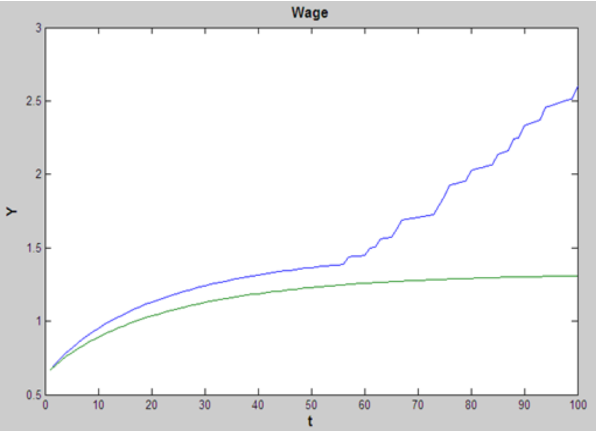

FIGURE 5

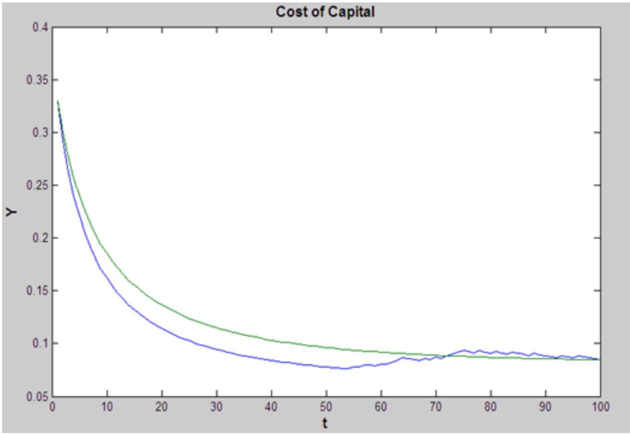

FIGURE 6

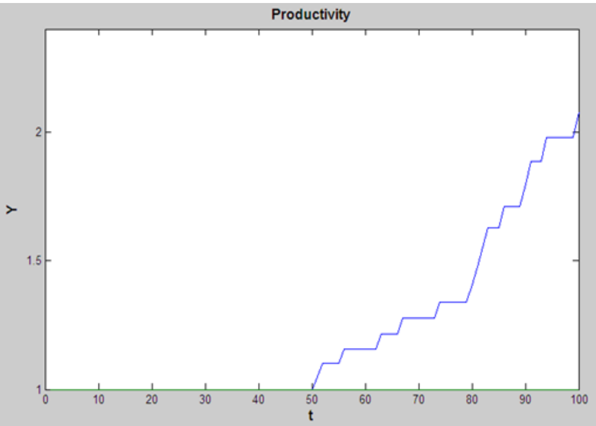

FIGURE 7

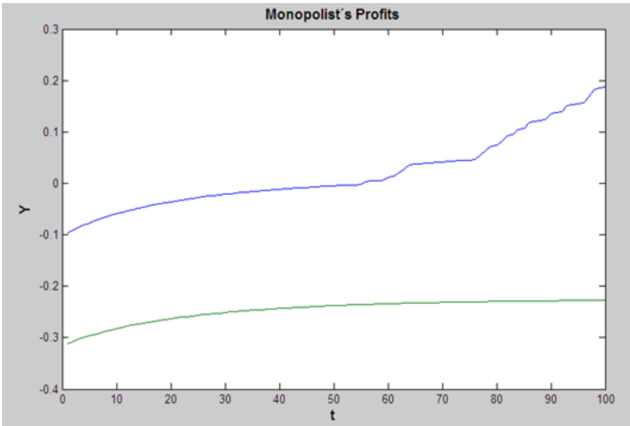

FIGURE 8

Figure 5 graphs the wage of both the innovative and the non-innovative economy. It can be noted that at the beginning both economies´ wages are increasing due to the accumulation of capital. This accumulation raises the marginal productivity of labor and as a result the wage. However, later on the wage in the non-innovative economy stagnates due to the stoppage of capital accumulation. In contrast, the wage in the innovative economy continues to grow, and at a higher pace, due to the productivity improvements generated by the research and development.

Figure 6 graphs the cost of capital. Originally, the cost of capital at both economies shows a declining trend due to capital accumulation and consequently smaller marginal productivity of capital. It is worth noting that the cost of capital at the innovative economy is consistently smaller than that of the non-innovative economy. This happens as a result of a higher savings rate which provokes higher capital accumulation and consequently a bigger stock of capital. Furthermore, it can be observed that once the innovative economy starts to invest in research and development its cost of capital starts to increase. When research is successful and productivity increases, the marginal product of capital grows and so does the cost of capital.

Figure 7 shows the productivity of the non-innovative economy, which remains constant at 1, and the productivity of the innovative economy, which grows randomly. Here it can be clearly observed that there are some moments in which research and development exhibits success, and therefore productivity increases, and moments where it is not successful and as a consequence the productivity remains the same as the previous period.

Finally, Figure 8 graphs the potential monopolist's profits. In the non-innovative economy, these profits are always negative, so the monopolist never decides to invest in research. In the innovative economy, profits are negative until t^{inn}. Once they become positive the monopolist decides to begin investing in research and development.

The model presented in Section 2 yields two different growth paths: if the economy reaches K^{inn} before its steady state, then growth is possible in the long run; otherwise, the model predicts stagnation. Empirical evidence shows that both paths exist. As we have seen in Figure 3, some economies have been growing constantly, and others have not. In the following Section, we study some policies and their effect on economic growth.

4 Political Economy

Transition to the innovation-driven growth stage can be fastened by policies that affect the important parameters of the model (s, R, μ and γ). The objective of a good government should be to implement policies that lower K^{inn}. If innovation depends on such diverse elements as those described in the precedent Section, then the government has an interesting room for action in promoting or hampering economic growth. The implementation of a given policy can therefore have long lasting consequences. In this section we analyze three policies and their effect on long term growth.

4.1 Pro-Innovation Tax Policy

Let assume an economy where originally $K^{inn} > K^{ss}$, that is, the context of a potential stagnated economy, without any kind of government intervention. The government could implement some kind of tax policy to modify this situation. Let τ be the tax rate. With an income tax on the household, the government would be able to subsidize the monopolist, rising his benefits and thereby lowering K^{inn}. Let K^{inn}_{gov} be the innovation capital level the government chooses, and K^{ss}_{gov} the steady state capital level that results from government intervention. Of course $K^{inn}_{gov} < K^{ss}_{gov} < K^{ss} < K^{inn}$. An income tax on the household and a subsidy to the monopolist lowers K^{inn} but has opposing effects on K^{ss}: on the one hand, it lowers K^{ss} because it reduces the accumulation of capital, and on the other hand, it rises it because of the increase in productivity that results from innovation. For this reason, were the right tax rate not adopted, the inequality the government wants to change would be maintained. Let K^{inn}_{new} and K^{ss}_{new} be the new levels of capital that result from government intervention. As productivity can potentially increase every period, this levels might change every period too. The tax rate for each period that would generate innovation earlier is

$$\tau_t = \begin{cases} 0 & if\ K_t < K^{inn}_{gov} \\ \tau_{t^{inn}_{gov}} & if\ K_t = K^{inn}_{gov} \\ \tau^{dec}_t & if\ K^{inn}_{gov} < K_t < K^{inn}_{new} \\ 0 & if\ K^{inn}_{new} < K_t \end{cases}$$

Let t_{gov}^{inn} be the period t where $K_t = K_{gov}^{inn}$. In this period the government would raise an income tax to the households, $\tau_{t_{gov}^{inn}}(A_0L)^{1-\alpha}(K_{gov}^{inn})^{\alpha}$. To have the desired effect, $\tau_{t_{gov}^{inn}}$ must meet the following two conditions:

1. $\{\mu\alpha(1-\alpha)+\tau_{t_{gov}^{inn}}\}(\gamma A_0L)^{1-\alpha}(K_{gov}^{inn})^{\alpha} - R > 0$

2. $s(\gamma A_0L)^{1-\alpha}(K_{gov}^{inn})^{\alpha-1} - \delta > 0$

In words, the tax rate should be high enough in order to make the monopolist's benefits big enough to invest in research, but low enough in order not to lower the steady state capital in a magnitude that the economy will reach in that same period its steady state. The following range can be obtained:

$$\tau_{t_{gov}^{inn}} \in [\frac{R}{(\gamma A_0L)^{1-\alpha}(K_t)^{\alpha}} - \mu\alpha(1-\alpha)\ ;\ 1 - \frac{\delta}{s(\gamma A_0L)^{1-\alpha}(K_t)^{\alpha-1}}]$$

When capital level is between K_{gov}^{inn} and K_{new}^{inn}, the tax rate τ_t^{dec} decreases, until the economy reaches K_{new}^{inn} and there is no more need of taxing income. The rate of decrease in the tax rate should accompany the growth of capital and technology, so that incentives to invest in research are maintained.

The model predicts thus that in early stages of growth, income should be taxed and monopolist rent should be subsidized. As capital level rises, tax rates would tend to decrease progressively, until they lastly disappear because they are not useful any more.

4.2 Conditional License

As we have seen before, the only way in which the randomly chosen firm becomes the sole provider of the intermediate good is if it is successful in its research. In this subsection we study the case in which a government is capable of ensuring the firm its monopoly power even if he is not successful in innovating, provided he takes the risk (which means paying the fixed cost R).

The benefits of the monopolist if it does not take the risk are the same as before, namely $\pi_t^c = 0$. However, now, if it decides to pay the fix cost R, his expected benefits are:

$$E(\pi_t^m) = \mu\pi_t^m(\gamma A_{t-1}) + (1-\mu)\pi_t^m(A_{t-1}) - R$$

$$E(\pi_t^m) = \alpha\,\mu(\gamma A_{t-1}L)^{1-\alpha}X_t^{\alpha} + \alpha(1-\mu)(A_{t-1}L)^{1-\alpha}X_t^{\alpha} - R$$

$$E(\pi_t^m) = \alpha(1-\alpha)[\,\mu\gamma^{1-\alpha} + (1-\mu)](A_{t-1}L)^{1-\alpha}X_t^{\alpha} - R$$

The entrepreneur innovates if:

$$E(\pi_t^m) = \alpha(1-\alpha)[\,\mu\gamma^{1-\alpha} + (1-\mu)](A_{t-1}L)^{1-\alpha}X_t^{\alpha} - R > \pi_t^c = 0$$

From where we isolate the minimum level of capital needed to innovate

$$K_t = (\frac{R}{\alpha(1-\alpha)[\,\mu\gamma^{1-\alpha} + (1-\mu)](A_{t-1}L)^{1-\alpha}})^{\frac{1}{\alpha}}$$

which is lower than the K^{inn} of Section 2. The reason for this is that this type of license increases the expected benefits of the monopolist. As a consequence, the level of capital needed in order to invest in research is lower.

4.3 Competition between Two Firms

We now analyze the case where the regulator permits the competition in research of two firms at each specific period. There is still a continuum [0,1] of firms that produce the intermediate good, with the difference that now at the beginning of each period two firms instead of one are randomly chosen to have the possibility to invest in research and development. Therefore, these firms face each others' competition and as a result have to decide whether to invest in research or not, taking into account what the other player plans to do. To study this scenario we use a game theory approach.

In order to understand what the possible outcomes of this game are, we have to analyze the consequences of each of the decisions that the firms may take. These possibilities are shown in the matrix below:

	I	**NI**
I	A,A	B,C
NI	C,B	D,D

If none of the firms invest in research, then none of them will become a monopolist and both firms will consequently compete with the rest of the firms which were not picked. This competition will lead to zero profits for all the firms in the market.

In case one of the firms decides to invest in research, but the other does not, two possible scenarios arise. Should the firm be successful in its research, then it will become the monopolist in the market. In contrast, if its research doesn´t generate any advancement, the firm, along with the other one picked, will compete with all the others and will not gain a positive profit.

In addition to this, we also have to study the case where the two firms randomly picked decide to invest in research. With the same rationale as the one explained above, if one of the firms is successful and the other is not, then the successful firm will become the market´s monopolist. Moreover, provided both firms fail in its research, none of them will become a monopolist and will compete with the remaining firms. Lastly, in case both firms are successful in its research, they will collude and equally share the monopolist´s profit.

We show the expected profits of each decision below:

A: $E_t(\pi_t^A) = \mu^2 \frac{1}{2}\alpha(1-\alpha)(\gamma A_{t-1}L)^{1-\alpha}K_t^\alpha + \mu(1-\mu)\alpha(1-\alpha)(\gamma A_{t-1}L)^{1-\alpha}K_t^\alpha + (1-\mu)\mu * 0 + (1-\mu)^2 * 0 - R$

This is the expected profit in case the firm decides to invest in research, taking into account that the competitor also decided to invest.

B: $E_t(\pi_t^B) = \mu\alpha(1-\alpha)(\gamma A_{t-1}L)^{1-\alpha}K_t^\alpha - R$

This is the expected profit in case the firm decides to invest in research, taking into account that the competitor decided not to invest.

C: $E_t(\pi_t^C) = 0$

This is the expected profit in case the firm decides not to invest in research, taking into account that the competitor decided to invest.

D: $E_t(\pi_t^D) = 0$

This is the expected profit in case the firm decides not to invest in research, taking into account that the competitor also decided not to invest.

Having this game setting, we proceed to solve for the Nash equilibrium. Since the expected profits vary as the level of capital changes, we analyze each case taking into account the different levels of capital that the economy may have.

1) Case 1: A>C & B>D
The Nash equilibrium of this game is (I,I): both firms decide to invest in research.

2) Case 2: A>C & B<D
The Nash equilibria of this game are (I,I) & (NI,NI): either both firms decide to invest in research or none of them invests.

3) Case 3: A<C & B>D
The Nash equilibria of this game are (I,NI) & (NI,I): one of the firms invests in research, and the other does not.

4) Case 4: A<C & B<D
The Nash equilibrium of this game is (NI,NI): none of the firms decides to invest in research.

Now, knowing the different equilibria we want to understand the equilibria´s path and determine under which level of capital each of them prevails.

Therefore, we look for which levels of capital A>C, and for which levels B>D.

- A>C if:

$$E_t(\pi_t^A) = \mu^2 \frac{1}{2}\alpha(1-\alpha)(\gamma A_t L)^{1-\alpha} K_t^\alpha + \mu(1-\mu)\alpha(1-\alpha)(\gamma A_t L)^{1-\alpha} K_t^\alpha + \mu(1-\mu) * 0 + (1-\mu)^2 * 0 - R > E_t(\pi_t^C) = 0$$

$$K_t > \left(\frac{R}{\alpha(1-\alpha)(\gamma A_t L)^{1-\alpha}[\mu\left(1-\frac{\mu}{2}\right)]}\right)^{\frac{1}{\alpha}}$$

When the economy has a capital level below this one, A<C, and in contrast, when the economy has a higher level of capital, A>C.

- B>D if:

$$E_t(\pi_t^B) = \mu\alpha(1-\alpha)(\gamma A_t L)^{1-\alpha} K_t^\alpha - R > E_t(\pi_t^D) > E_t(\pi_t^D) = 0$$

$$K_t > \left(\frac{R}{\alpha(1-\alpha)(\gamma A_t L)^{1-\alpha}\mu}\right)^{\frac{1}{\alpha}}$$

When the economy has a capital level below this one, B<D, and in contrast, when the economy has a higher level of capital, B>D.

Comparing these two threshold levels we find that the threshold for A>C is higher than for B>D. As a result, since C=D (both equal zero), it is impossible to simultaneously have A>C and B<D and consequently we discard Case 2.

Finally, we get the following equilibria:

$$\begin{cases} \text{NI, NI} & if\ 0 < K_t < \left(\frac{R}{\alpha(1-\alpha)(\gamma A_t L)^{1-\alpha}\mu}\right)^{\frac{1}{\alpha}} \\ \text{I,NI and NI, I} & if\ \left(\frac{R}{\alpha(1-\alpha)(\gamma A_t L)^{1-\alpha}\mu}\right)^{\frac{1}{\alpha}} < K_t < \left(\frac{R}{\alpha(1-\alpha)(\gamma A_t L)^{1-\alpha}(\mu(1-\frac{\mu}{2}))}\right)^{\frac{1}{\alpha}} \\ \text{I,I} & if\ \left(\frac{R}{\alpha(1-\alpha)(\gamma A_t L)^{1-\alpha}(\mu(1-\frac{\mu}{2}))}\right)^{\frac{1}{\alpha}} < K_t \end{cases}$$

These equilibria are associated with the following growth rates:

$$\begin{cases} \left(\frac{K_{t+1}}{K_t}\right)^{\alpha} & if\ 0 < K_t < \left(\frac{R}{\alpha(1-\alpha)(\gamma A_t L)^{1-\alpha}\mu}\right)^{\frac{1}{\alpha}} \\ \left(\frac{K_{t+1}}{K_t}\right)^{\alpha}(\mu(\gamma-1)+1)^{1-\alpha} & if\ \left(\frac{R}{\alpha(1-\alpha)(\gamma A_t L)^{1-\alpha}\mu}\right)^{\frac{1}{\alpha}} < K_t < \left(\frac{R}{\alpha(1-\alpha)(\gamma A_t L)^{1-\alpha}(\mu(1-\frac{\mu}{2}))}\right)^{\frac{1}{\alpha}} \\ \left(\frac{K_{t+1}}{K_t}\right)^{\alpha}(2\mu-\mu^2)\,(\gamma-1)+1)^{1-\alpha} & if\ \left(\frac{R}{\alpha(1-\alpha)(\gamma A_t L)^{1-\alpha}(\mu(1-\frac{\mu}{2}))}\right)^{\frac{1}{\alpha}} < K_t \end{cases}$$

It can be interpreted from the results shown above that if the steady state level of capital is higher thanK^{inn}, then the economy will undergo through three stages of growth. At first the economy will only grow due to capital accumulation. At this stage, the firms´ potential profits from investing in research and becoming monopolists are still small and do not compensate for the fixed cost R that they have to pay. As a result, only when a certain stock of capital is reached the firms decide to start investing in research. At this point, only one of the firms will be investing and as a consequence, the economy growth will be driven by two factors: capital accumulation and productivity improvements due to the research performed. Lastly, the third stage of growth begins when, due to the capital accumulation and the technology improvements, the profits of being a monopolist (or colluding if both researches are successful) are big enough to compensate for the fixed cost that has to be paid in order to perform the research. At this stage, growth is driven by the technology improvements (which are considerably more likely than in the previous stage) and the growth in the stock of capital.

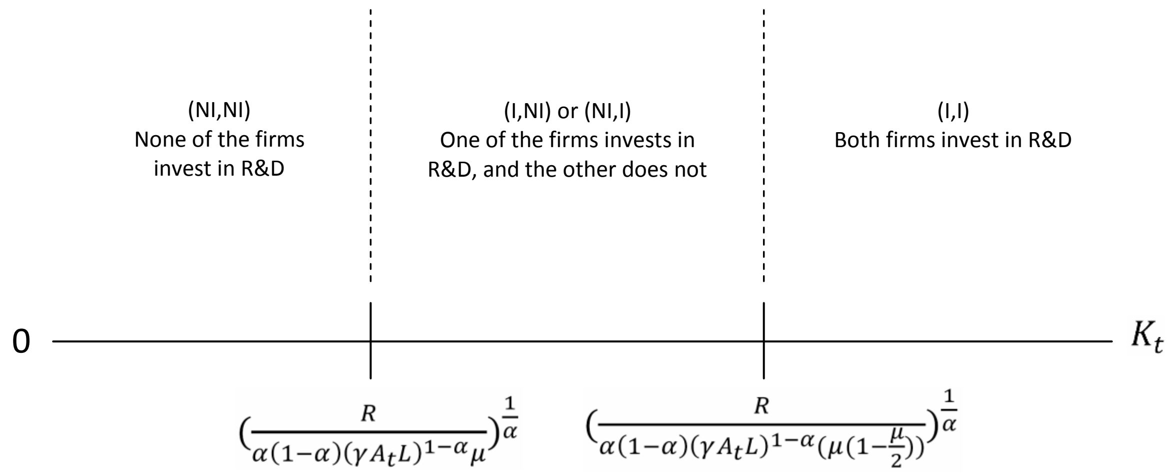

FIGURE 9

We can see that the threshold for innovation is the same as in the base case. This shows us that if a regulator decides to let another firm compete, this will not change the moment at which the economy starts to innovate. Consequently, if the steady state level of capital is lower than the capital required to kick-start innovation, this policy will not help solve the problem and throw the economy into an innovation-driven economic growth stage. However, in case the economy is able to start investing in research and therefore innovates, it will at some point reach a level of capital that will provoke that both firms begin to invest in research. Once this moment is reached, the growth rate of the economy will be higher than before since the probability that some research is successful is considerably higher than when only one firm was investing in research.

5 Concluding Remarks

Like Marx, Schumpeter believed that capitalism was a self-destroying system. The seeds of that destruction, nevertheless, didn't lie in the class warfare that would result from the appropriation of the surplus value by the owners of the means of production, but rather in the very success of the system: innovative entrepreneurs would tend to a form of corporatism with hostile values towards capitalism, which would in the end be replaced by socialism. Modern capitalism has certainly evolved into a corporate structure, dominated by multinational firms. But innovation is still the main force that drives these companies in particular and the world economy in general.

The model we presented in this paper introduces in the setting of the Solow model a monopolist who plays the role of the entrepreneur, and who, owing to his monopolist situation, has available resources to invest in research and development. This monopolist enables the model to escape the logic of stagnation, and its innovations allow, as economic literature and empirical evidence show, sustainable long run growth. Our model can thus account for the basic growth facts of two different stages of development, one driven by factor accumulation and the other by innovation, as well as for the transition between the two. The model concludes that economic growth by factor accumulation can only be temporary, and that innovation has to be present in an economy in order for it to avoid stagnation. The transition from one stage to the other is made possible by a minimum level of capital, which is in turn dependent on four important parameters: the savings rate, the probability of success of the research, the cost of investment and the quality of the innovation.

Although in reality these parameters are variables that change constantly over time, the calibration we performed in Section 3 shows that a model based on innovation can explain divergence in income across countries. Empirical evidence seems to support the idea that innovation is what makes countries grow. Of course both the report on innovation we worked with (the GII) and our model define innovation in a general way, and include many different components in it. But the fact that each of these components is measurable and directly comparable between countries constitutes a sufficient strong case in favor of the connections between innovation and growth. Also, in Section 4 we studied three policies and their effects on growth: the tax rate necessary to foster innovation, a case in which competition is reduced and a situation in which two firms instead of one try to innovate.

In conclusion, entering an innovation-driven growth stage depends on how an economy is positioned on the essential variables described in this paper. Despite this, stagnation should not be viewed as an unavoidable. Diverse policies can help an economy build the necessary incentives for innovation to take place. The main challenge of economic growth still is, now and ever, how to allocate factors in the most difficult way: the way that creates, that invents - products, services, models and processes - nobody has yet invented.

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Annex I

In this annex we develop the model in more detail.

I.1 Competitive Setting

Let the production function Y_t be

$$Y_t = (A_t L)^{1-\alpha}(X_t)^{\alpha}$$

When there is not investment in research, or when innovation is not successful, the model behaves as a competitive economy.

Equilibrium in the final good market: $Y_t = C_t + I_t$, where $C_t = (1-s)Y_t$ and $I_t = sY_t$

The final good firm maximizes

$$(A_t L)^{1-\alpha}(X_t)^{\alpha} - w_t L - p_t X_t$$

(K_t) $\qquad (A_t L)^{1-\alpha}\alpha(X_t)^{\alpha-1} = p_t$

(L) $\qquad (1-\alpha)(A_t L)^{-\alpha}A_t(X_t)^{\alpha} = w_t$

As there is perfect competition, $p_t = r_t$.

His demand for X_t is $X_t^d = A_t L(\frac{\alpha}{p_t})^{\frac{1}{1-\alpha}}$

As $X_t = K_t$, the previous equation is also the demand for capital. The supply of K_t is given by the motion law of capital.

In equilibrium, $K_t^d = A_t L(\frac{\alpha}{r_t})^{\frac{1}{1-\alpha}} = K_t^s$

Because of perfect competition, the firm takes the price r_t as given. r_t adjusts to clear the market of capital, and can be rewritten as

$$r_t = \alpha(\frac{A_t L}{K_t^s})^{1-\alpha}$$

I.2 Setting with research

When there is investment in research, the equilibrium of the economy is $Y_t = C_t + I_t + R$.

The competitive firm maximizes

$$(A_t L)^{1-\alpha}(X_t)^{\alpha} - w_t L - p_t X_t$$

(X_t) $\qquad (A_t L)^{1-\alpha}\alpha(X_t)^{\alpha-1} = p_t$

(L) $\qquad (1-\alpha)(A_t L)^{-\alpha}A_t(X_t)^{\alpha} = w_t$

His demand for X_t is $X_t^d = \begin{cases} \gamma A_{t-1} L (\frac{\alpha}{p_t})^{\frac{1}{1-\alpha}} & \textit{with probability } \mu \\ A_{t-1} L (\frac{\alpha}{p_t})^{\frac{1}{1-\alpha}} & \textit{with probability } 1-\mu \end{cases}$

The monopolist supplies X_t^s. Therefore he maximizes

$$E(\pi_t^*) = \mu \pi_t^m(\gamma A_{t-1}) + (1-\mu)\pi_t^c(A_{t-1}) - R \quad \text{(I.1)}$$

Where $\pi_t^m(\gamma A_{t-1}) = p_t X_t - r_t K_t$ and $\pi_t^c(A_{t-1}) = 0$, where $p_t = \alpha(\gamma A_{t-1} L)^{1-\alpha} X_t^{\alpha-1}$ is the inverse demand for the intermediate good.

He produces the quantity

$$X_t^s = \begin{cases} \gamma A_{t-1} L (\frac{\alpha^2}{r_t})^{\frac{1}{1-\alpha}} & \textit{with probability } \mu \\ 0 & \textit{with probability } 1-\mu \end{cases} \quad \text{(I.2)}$$

To produce this quantity, he demands

$$K_t^d = \begin{cases} \gamma A_{t-1} L \left(\frac{\alpha^2}{r_t}\right)^{\frac{1}{1-\alpha}} & \textit{with probability } \mu \\ 0 & \textit{with probability } 1-\mu \end{cases}$$

With probability μ, in equilibrium, $X_t^d = \gamma A_{t-1} L (\frac{\alpha}{p_t})^{\frac{1}{1-\alpha}} = X_t^s = \gamma A_{t-1} L (\frac{\alpha^2}{r_t})^{\frac{1}{1-\alpha}}$

So the monopolist chooses a price p_t such that

$$p_t = \frac{r_t}{\alpha} \quad \text{(I.3)}$$

Note that $p_t > r_t$ for every period t

The supply of capital K_t^s is determined by the motion law of capital.

With probability μ, in equilibrium, $K_t^d = \gamma A_{t-1} L (\frac{\alpha^2}{r_t})^{\frac{1}{1-\alpha}} = K_t^s$

r_t adjusts to clear the market of capital. It can be rewritten as

$$r_t = \alpha^2 (\frac{\gamma A_{t-1} L}{K_t^s})^{1-\alpha}$$

And therefore p_t is

$$p_t = \alpha (\frac{A_{t-1} L}{K_t^s})^{1-\alpha}$$

From equations (I.2) and (I.3) and using that $X_t = K_t$,

$$r_t = \alpha^2 (\gamma A_{t-1} L)^{1-\alpha} K_t^{\alpha-1} \quad \text{(I.4)}$$

$$p_t = \alpha (A_{t-1} L)^{1-\alpha} K_t^{\alpha-1}$$

Combining equations (I.1) and (I.4) yields

$$\pi_t = \mu\alpha(1-\alpha)(\gamma A_{t-1} L)^{1-\alpha} K_t^{\alpha} - R$$

Annex 2

In this annex are presented the steady states for both types of economies.

II.1 Stagnated Economy ($K^{innovation} > K^{ss}$)

From the motion law of capital,

$$K_{t+1} = I_t + (1-\delta)K_t = sY_t + (1-\delta)K_t$$

where $Y_t = (A_t L)^{1-\alpha}(K_t)^{\alpha}$

$$K_{t+1} = s(A_t L)^{1-\alpha}(K_t)^{\alpha} + (1-\delta)K_t$$

$$\frac{K_{t+1} - K_t}{K_t} = s(A_t L)^{1-\alpha}(K_t)^{\alpha-1} - \delta$$

In a steady state, the term in the left has to equal zero

$$0 = s(A_t L)^{1-\alpha}(K_t)^{\alpha-1} - \delta$$

$$K^{ss} = A_t L (\tfrac{s}{\delta})^{\frac{1}{1-\alpha}}$$

II.2 Innovative Economy ($K^{innovation} < K^{ss}$)

Let $\kappa_t = \frac{K_t}{A_t L}$ denote the effective capital stock and $g = E(g_t)$.

$$\frac{K_{t+1}}{A_{t+1}L} = \frac{s(A_t L)^{1-\alpha}(K_t)^{\alpha} - sR}{A_t(1+g)L} + \frac{(1-\delta)K_t}{A_t(1+g)L}$$

$$\frac{K_{t+1}}{A_{t+1}L} = \frac{s\kappa_t^{\alpha}}{(1+g)} - \frac{sR}{A_{t+1}L} + \frac{(1-\delta)\kappa_t}{(1+g)}$$

$$\kappa_{t+1}(1+g) = s\kappa_t^{\alpha} - \frac{sR}{A_{t+1}L} + (1-\delta)\kappa_t$$

Assuming that ng is small and that the time period between t and $t+1$ is also small

$$\kappa_{t+1} = s\kappa_t^{\alpha} - \frac{sR}{A_{t+1}L} + (1-\delta-g)\kappa_t$$

$$\kappa_{t+1} - \kappa_t = s\kappa_t^{\alpha} - \frac{sR}{A_{t+1}L} - (\delta+g)\kappa_t$$

In a steady state, the first term has to equal zero and $\frac{sR}{A_{t+1}L}$ tends to zero as A_{t+1} grows

$$0 = s\kappa_t^{\alpha} - (\delta+g)\kappa_t$$

$$\kappa^{ss} = (\frac{s}{\delta + g})^{\frac{1}{1-\alpha}}$$

Y_t can be written as $A_t L \kappa^{\alpha}$. Let λ_t be the growth rate of effective capital. Its expected growth rate is

$$\begin{cases} (\frac{\kappa_{t+1}}{\kappa_t})^{\alpha} = {\lambda_t}^{\alpha} & if\ t < t^{inn} \\ (\frac{A_{t+1}}{A_t})(\frac{\kappa_{t+1}}{\kappa_t})^{\alpha} = \{\mu(\gamma - 1) + 1\}{\lambda_t}^{\alpha} & if\ t^{inn} < t < t^{ss} \\ (\frac{A_{t+1}}{A_t}) = \mu(\gamma - 1) + 1 & if\ t^{ss} < t \end{cases}$$

This means that after the steady state, the growth rate of capital equals the growth rate of productivity.