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RECURRENT HYPERINFLATIONS AND LEARNING

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Abstract: This paper uses a model of boundedly rational learning to account for the observations of recurrent hyperinflations in the last decade. We show that, in a standard monetary model, when the full rational expectations assumption is replaced by a pseudo rational learning, the model replicates some stylized facts observed during the recurrent hyperinflations observed in some countries in the 80's much better than pre-existing models. We argue that this departure of rational expectations does not preclude falsifiability of the model and it does not violate reasonable rationality requirements.

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1 INTRODUCTION

The recent literature on boundedly rational learning in macroeconomics has centered, almost exclusively, on the issue of convergence to rational expectations (RE). This literature did not pay attention to the behavior of the models *during the transition* to the rational expectations equilibria. It is commonly believed that, using models of learning to explain empirical observations would entail problems similar to those found in models of adaptive expectations, namely, that there are too many degrees of freedom available to the economist so that the models are not falsifiable, and that expectations are inconsistent with the model. On the other hand, the RE hypothesis places very strong requirements on agents' knowledge about the economy, and it seems important to study the effect of small deviations from full rationality, specially in very unstable environments like the hyperinflation episodes we want to study..

The purpose of this paper is to show that a model of learning can explain the observations of recurrent hyperinflations in many economies during the 80's. In order to avoid the two criticisms mentioned above, we restrict our study to learning mechanisms that produce good forecasts within the model; therefore, our choice of learning mechanism is restricted and the model is falsifiable; also, since the resulting equilibria reinforce the use of the learning mechanism (because good forecasts are generated along the equilibrium), agents expectations are not inconsistent with the model.

The observation of recurrent hyperinflations in many economies during the 80's is quite striking. In several countries, inflationary peaks occurred in succession, with periods of fairly low inflation in between. These peaks appear to be independent from any strong movement in fundamental variables and, in particular, there seems to be some consensus that the peaks in inflation were not caused by peaks in seigniorage[1]. Nevertheless, it is observed that countries with average high seigniorage tend to have large inflation. Also, it is observed that exchange rate controls are able to reduce inflation temporarily. The transition of the model under learning reproduces these stylized facts, much better than any of the alternative explanations available in the literature based on RE. In addition, our model does not rely on agents'

[1] For details, see Bruno et al.(1988) and (1991), Sargent and Wallace (1987) or Zvi Eckstein (1987). The numbers reported are from Kiguel and Liviatan (1991).

having a perfect knowledge about the economy; this is a good thing to have in most applications, but specially if one wants to explain observations on recurrent hyperinflations where, apparently, the behavior of the economy was quite difficult to discern.

The economic fundamentals of the model we present are standard: a money demand function that depends on inflation expectations and a government budget constraint that sustains government's exogenous seigniorage. A fixed exchange rate rule (ERR) is established if inflation goes beyond a certain high level. We depart from the usual assumption of RE, and assume that agents form their expectations by learning about the model as the economy goes along. We show that the model accounts for the facts just described.

Our learning rules are chosen to satisfy some lower bounds on rationality and, in this sense, we call our learning rules pseudo-rational. If inflation is stable, agents incorporate new information slowly into their beliefs: in the language of stochastic approximation, they use 'decreasing gains'. But if agents detect instability (a burst in inflation), they quickly incorporate new information; in the language of stochastic approximation, they use a 'constant gain' algorithm. By combining decreasing and constant gains, the same learning mechanism produces good forecasts in periods of relatively stable inflation and during hyperinflations. The sensitivity of the learning mechanism is what generates good forecasts within the model, and it is also a crucial ingredient in generating the hyperinflationary episodes in the model, since it makes it more likely that agents' expectations land in the unstable region of the dynamic system of the economy, where a hyperinflation occurs. In this sense, the use of a sensitive learning mechanism is justified by the outcome of the economy and, therefore, agents are likely to stay with this mechanism. We provide a formalization of this intuition in defining some *lower bounds* on rationality that the learning mechanism has to satisfy.

The next section provides a summary of the stylized facts during hyperinflations, together with some evidence of four Latin-American countries during the eighties. Section 3 describes the sense in which we require pseudo-rationality. Section 4 presents the model, section 5 characterizes equilibria, section 6 discusses numerical solutions and section 7 discusses the literature. We end with the conclusions. An appendix contains the calculations of the rational expectations equilibrium and a proof of local convergence of the learning mechanism.

2 Recurrent Hyperinflations

A number of countries, including Argentina, Bolivia, Brasil, Perú and Israel experienced during the eighties the highest average inflation rates of their history. Stopping inflation was then, almost the only item in the policy agenda of these countries. While the duration and severity of the hyperinflations and the policy experiments differ substantially, there are several stylized facts that are common to those experiences and, to some extent, common to the experiences of some European countries after the first world war and to the experiences of East European countries after the end of the cold war. These stylized facts are

1. Recurrence of hyperinflationary episodes. Time series show relatively long periods of moderate and steady inflation, and a few short periods of extremely high inflation rates.

2. Bursts in inflation are often stopped by establishing exchange rate rules (ERR). In many circumstances, these plans only lower inflation temporarily, and new hyperinflations occur eventually.

3. For a given country, there is no clear positive contemporaneous correlation across time between the size of the seigniorage and the inflation rate.

4. Across countries there is a clear relation between the size of inflation and seigniorage: hyperinflations only occur in countries where inflation rate is high on average.

Points 2 and 4 can be combined to state the following observation on monetary policy: stabilization plans that do *not* make a permanent fiscal effort (i.e., that do not reduce the average deficit and average seigniorage) may be successful in substantially reducing the inflation rate *only* in the short run. Stabilization attempts that focused only on fixing the exchange rate, sometimes with additional price controls, are called *"heterodox"* plans; when the focus is on the fiscal adjustment required to reduce government deficit, they are called *"orthodox"* plans. Most stabilization plans that were successful in reducing inflation substantially and permanently, relied on the fixing of the exchange rate but they also made a severe fiscal adjustment to permanently having a perfect knowledge about the economy; this is a good thing to have in most applications, but specially if one wants to explain observations on recurrent hyperinflations where, apparently, the behavior of the economy was quite difficult to discern.

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Our summary of stylized facts should be uncontroversial[2], but first-hand evidence to support them is provided in figures 1 to 5, which present data on the recent inflationary experiences of Argentina, Bolivia, Brasil and Peru.

Inflation rates for selected periods were computed from IFS consumer price indexes. These periods have been selected so as to show the main stabilization efforts carried out by each country and the effect they had on the evolution of inflation. Periods when an explicit fixed exchange rate rule was in place are indicated by shaded areas; the end of the shading indicates the date in which convertibility was explicitly abandoned. Figures 1 to 4 illustrate quite clearly stylized facts 1 and 2.

Figure 5 depicts the evolution of the quarterly[3] inflation rate for Argentina together with the evolution of the seigniorage for the period 1983 to 1990. The left hand side vertical axis measures seigniorage as a percentage of GNP, while inflation, measured as the log (P_t/P_{t-1}), is measured on the right hand side vertical axis. The figure clearly states the lack of (contemporaneous) correlation across time between the two variables (fact 3), specially when hyperinflations are occurring in certain periods of rapidly increasing inflation, seigniorage goes down, and vice versa; also, the level of seigniorage that led the spectacular hyperinflation of the second quarter of 1989 is the same as the one of the first quarter of 1984, with subsequent inflation rates that were below 80%.

In this paper we limit our study to some very specific stylized facts. A closer look at Figure 5, however, points to some interesting facts that merit a more careful empirical investigation. Note, in particular, that seigniorage appears to lead the hyperinflationary bursts. Also, there is some correlation between inflation and seigniorage in the sub samples periods when inflation was not too high; for example, in the periods 80.I-82.IV and 86.II-88.IV. Both of these features are consistent with our model but they are not studied carefully in this version of the paper.

[2]For instance, see Bruno et al. (1988) and (1991).

[3]The data were taken from Ahumada, Canavese, Sanguinetti y Sosa Escudero (1993). We use quarterly data for this Figure because the seignorage is typically expressed as a share of GNP.

3 Learning and Lower Bounds on Rationality

Until the mid-seventies, economic agents' expectations were specified according to ad-hoc assumptions; the most popular alternative was 'adaptive expectations'. This was criticized because: *i)* it introduced too many degrees of freedom in the specification of expectations so it made the models less falsifiable and, *ii)* agents' expectations were inconsistent with the model; hence, rational agents would be likely to abandon their adaptive expectations after a while, and the predictions of the model would be invalid. The first criticism is hyperbolized by the sentence: 'any economic model can match any observation by choosing expectations appropriately'; the second criticism is typified by the sentence 'economic agents do not make systematic mistakes'. Indeed, it is a much documented and well accepted fact that 'economic agents do not make systematic mistakes'.

The rational expectations hypothesis is, nowadays, the most commonly used paradigm in macroeconomics, mainly, because it solved these two issues: under RE, expectations are determined by the model; after some time agents will just realize that they are doing the right thing, and they will never abandon their rational expectations.

A questionable feature of RE is that, if interpreted literally, it assumes too much knowledge about the structure of the economy on the part of agents. The recent literature on learning in macroeconomics finds conditions under which a simple learning mechanisms converge to RE. In the many cases where convergence obtains, the use of RE is reinforced.[4] In this paper we will show, however, that by introducing boundedly rational learning in a very simple model, one can match the stylized facts described in the last section much better that with the existing alternative RE models available in the literature. One could simply argue that hyperinflations are such confusing events that it is reasonable to assume non-RE behavior, but a natural question comes to mind: are we slipping into a use of learning models that is as objectionable as adaptive expectations?.

The term *boundedly rational learning* (which, in this paper, we use as

[4]For example, Bray (1982), Marcet and Sargent (1989a, 1989b), Evans and Honkapohja (1995) or Woodford (1990). For extensive reviews, see the book by Sargent (1993) and the survey by Marimon (1995).

synonymous with the term *learning*) is used to refer to learning mechanisms that place *upper* bounds on rationality; for example, agents are assumed not to know the exact economic model or to have bounded memory. Macroeconomists have been averse to the use of learning models in order to explain empirical observations, probably, because this would be subject to the same criticisms as adaptive expectations (non-falsifiability and inconsistency of expectations). This is why research on learning has concentrated, almost exclusively, on the issue of convergence to RE or on the issue of selecting among a multiplicity of RE equilibria[5].

The dilemma is the following: on the one hand, RE makes unrealistic demands on agents' rationality; on the other hand, it seems that by moving away from RE we will only fall back into old mistakes and the 'jungle of irrationality'. Bayesian learning is not a way out of this dilemma, since it requires that agents know part of the model in order to form the likelihood function; which simply begs the question of 'how did agents learn the likelihood function?'. Furthermore, in models with endogenous state variables such as the one we lay out in section 4 (where money, or past inflation, is a state variable), Bayesian learning requires agents to use a complicated state space and, in principle, the law of motion changes from period to period; agents often need to remember the whole past, and it is hard to justify how agents could learn a law of motion that changes every period. Finally, the literature has also accumulated a number of paradoxes generated by Bayesian learning, among them, that small mistakes in the formulation of the prior will cause agents to make very bad predictions, since errors accumulate over time[6].

In this section we want to set up criteria for specifying learning mechanisms that are immune to the two criticisms leveled to adaptive expectations. Our strategy will be to allow for only *small* deviations from rationality both along the transition and asymptotically; this solves the issue of falsifiability and it does not violate reasonable definitions of rationality (or pseudo-rationality). In other words, given an economic model and some empirical observations, we look for learning mechanisms that satisfy certain *lower* bounds on rationality and that match the observations. In later sections we

[5]A careful justification of this position can be found in the conclusion of Sargent (1993).

[6]See, for example, Bolton and Rustichini (1995) and Marimon (1995) for descriptions of such paradoxes.

will show this small departure from rationality generates equilibria that are often quite different from RE, precisely in the direction of matching empirical observations much better, even if we consider countries that were following different policies.

Let us now be precise about the lower bounds that we place on rationality. Assume that the relevant expectation that agents have to formulate at time t is the forecast of the variable x_{t+1}, and that the economic model determines that this variable obeys

$$x_t = g(x_{t-1}, x_{t+1}^e, \epsilon_t, \eta) \tag{1}$$

where g is determined by market equilibrium and agents' behavior, and η is a vector of parameters in the economy including, for example, parameters of government policy. Agents summarize past information in certain statistics $\beta_t(\mu)$, generated by a learning mechanism f and the learning parameters μ that satisfy

$$\beta_t(\mu) = f(\beta_{t-1}(\mu), x_t, \mu). \tag{2}$$

and they set their expectations as a function of these statistics, so that $x_{t+1}^e = h(\beta_t(\mu), x_t)$. For now, (f, μ) are unrelated to the model (g, η). The learning mechanism f says how new information is incorporated into the new statistics, while the learning parameters μ govern, for example, the weight that is given to recent information. In the next section we provide a concrete example for the model g and we discuss several alternatives for f.

Equations (1) and (2) determine the equilibrium sequence under learning for given parameters. Obviously, since the process for x_t is self-referential, it depends on the parameter μ but this dependence will be left implicit in most of the paper.

Let $\pi^{\epsilon,T}$ be the probability that the perceived errors in a sample of T periods, will be within $\epsilon > 0$ of the conditional expectation error.

$$\pi^{\epsilon,T} \equiv P\left(\frac{1}{T}\sum_{t=1}^{T}\left[x_{t+1} - x_{t+1}^e\right]^2 < \frac{1}{T}\sum_{t=1}^{T}\left[x_{t+1} - E_t^{\mu}(x_{t+1})\right]^2 + \epsilon\right) \tag{3}$$

where $E_t^{\mu}(x_{t+1})$ is the true conditional expectation when agents use the parameter values μ.

We dub these lower bounds *LB1, LB2, ...* (these names were not precisely found in a strike of creativity). The first lower bound on rationality we propose is:

Definition 1 Asymptotic Rationality: *(f, μ, g, η) satisfy LB1 if $\pi^{\epsilon,T} \to 1$ as $T \to \infty$ for all ϵ.*

This requires that the perceived forecast has to be at least as good as the forecast with the conditional expectation asymptotically. In this case, agents would not have any incentive to change their learning scheme after they have been using it for an arbitrarily long time.

This seems like a minimal requirement; it is similar in spirit to the of rational belief equilibria of Kurz (1994). It rules out adaptive expectations for most stochastic models, or learning models where agents use the wrong state variables to forecast in h. It does not rule out models of least squares learning that converge to RE[7].

Even though concepts similar to *LB1* can be found in the literature, our claim is that this is not enough to generate reasonable applications of learning models for empirical purposes. The reason is that mechanisms that satisfy *LB1* can generate very bad forecasts along the transition, and it would be unlikely that rational agents kept using learning schemes with such bad forecasts. For example, we will see that, in our model, least squares learning would generate very bad forecasts along a hyperinflation; this is despite the fact that least squares satisfies *LB1*.

For this reason, we will study learning schemes that imposes additional restrictions. The second lower bound we consider is

Definition 2 $(\epsilon - \delta)$ Consistency: *(f, μ, g, η) satisfy LB2 at T if $\pi^{\epsilon,T} \leq \delta$.*

If *LB2* is satisfied for most periods T, agents are unlikely to switch to another learning scheme, even if they were told the whole truth. Clearly, it only makes some sense to study this probability for T moderately high, to give a chance to the sample mean of the prediction error to settle down[8].

LB1 is unambiguously satisfied (there is a yes or no answer), but the second requirement can only be satisfied in a quantitative way, for certain ϵ and δ; the researcher is supposed to report to the reader the probabilities $\pi^{\epsilon,T}$

[7]This requirement was implicitely imposed in the literature on stability of RE under learning. For example, Marcet and Sargent (1989a) point out that, in the limit, least squares learning is optimal in the model at hand.

[8]This is related to the $(\epsilon - \delta)$ consistency requirement of Fudenberg and Levine (1995), although they looked at agents who learnt to maximize their utility.

for a given model and, hopefully, convince the reader that these probabilities are 'sufficiently' high. *LB2* is quite stringent; we will see, however, that it is satisfied by our model for certain parameter values even for very strict ϵ and δ.

The last bound on rationality requires the agent to use values of μ that are nearly optimal within the learning mechanism f. Denote by $\beta_t(m, \mu)$ the forecast produced by the learning parameter m when all agents are using the parameter value μ :

$$\beta_t(\eta, \mu) = f\ (\beta_{t-1}(m, \mu),\ x_t^{\mu},\ m),$$

Definition 3 Internal Consistency: (f, μ, g, η) *satisfy LB3 for* T *and* ϵ *if*

$$E\left(\frac{1}{T}\sum_{t=1}^{T}\left(x_{t+1}^{\mu} - x_{t+2}^{e}\right)^2\right) \leq \min_{\eta}\ E\left(\frac{1}{T}\sum_{t=1}^{T}(x_{t+1}^{\mu} - h(\beta_t(m, \mu), x_t^{\mu})^2\right) + \epsilon. \tag{4}$$

Thus, if the mechanism satisfies this bound, agents do not perceive on average alternative μ's as being much better than the one they have been using for T periods[9]. Notice that *LB1* implies that *LB3* holds for all $\epsilon > 0$ and any m for T high enough; hence, once *LB1* has been imposed, it only makes sense to study *LB3* in the context of 'moderately high' T.

The first two bounds compare the performance of the consumer that is learning relative to an external agent who knows (f, μ, g, η), the right model, the probability distributions and, in addition, the learning mechanism that all other agents are using, and is able to calculate the conditional expectation. The bound *LB3*, instead, compares the consumer that is learning, with other agents that are forced to use the same family of mechanisms f in their forecasts, but are allowed to pick alternative parameter values μ. This last bound replicates the intuition of rational expectations, in the sense of looking for an approximate fixed point, in which the equilibrium expectations that the consumers are using, minimize the errors *within the mechanism* f. These criteria could be readily generalized to more complicated models or to objective functions other than the average prediction error.

Rational expectations can be interpreted as imposing extreme versions of the second and third bounds: RE satisfies *LB1;* it also satisfies *LB2* for

[9] Evans and Honkapohja (1993) propose to use a related criterion.

$\pi^{\epsilon,T} = 1$ for all ϵ and T large enough. Also, if h uses the right state variables, if f is a dense class of functions (for example, polynomials) and we impose *LB3* for any ϵ, T, we are left with rational expectations. In this sense, a learning mechanisms that satisfies all the above bounds can be interpreted as a small deviation from full rationality.

In solving the model of this paper, we will be using *LB1* and *LB3* as our main criteria. We are currently solving the model with *LB2* and the preliminary results look promising. We expect to include these results in future versions of the paper.

4 THE MODEL

4.1 Economic Fundamentals.

The assumptions in this subsection are standard. The model consists of a portfolio equation for the demand of real money balances, a budget constraint equation relating seigniorage, money creation, and changes in reserves, and a rule for establishing fixed exchange rates.

Money demand

The demand for real balances is given by

$$P_t = \gamma M_t^d + (1/\phi) P_{t+1}^e \tag{5}$$

where γ and ϕ are parameters, P_t, M_t^d are price level and nominal demand of money; P_{t+1}^e is the price level that agents expect for next period. As is well known, this equation is consistent with utility maximization and general equilibrium in the context of an overlapping generations model.

Money supply

We assume government policy rules that mimic those used by governments with hyperinflationary experiences in the last decade. Seigniorage is specified exogenously, and money creation is driven by the need to finance seigniorage; on the other hand, government's concern about current levels of inflation prompts the government to establish a fixed exchange rate rule (ERR) when inflation gets out of hand. Seigniorage is given by an exogenous i.i.d. stochastic process $\{d_t\}_{t=0}^{\infty}$ with mean $\overline{d}$ and variance σ_d^2, and it is the only source of uncertainty in the model.

In periods with no ERR, the government budget constraint is given by

$$M_t = M_{t-1} + d_t P_t \tag{6}$$

which determines money supply M_t.

Exchange Rate Rules

In periods of ERR, the government pegs the nominal exchange rate by buying or selling foreign reserves at an exchange rate e_t satisfying

$$\frac{P_t^f}{P_{t-1}^f} \frac{e_t}{e_{t-1}} = \bar{\beta},$$

where $\bar{\beta}$ is the targeted inflation rate, and P_t^f is the price level abroad. Arbitrage in the international currency market implies that

$$\frac{P_t}{P_{t-1}} = \bar{\beta} \tag{7}$$

and that the targeted inflation rate is achieved. In order to implement this policy, the government only needs to know past values of exchange rate and foreign price levels. In the case that targeted inflation $\overline{\beta}$ is the same as foreign inflation, the government announces a fixed exchange rate; otherwise, a crawling peg is followed.

Under ERR, equilibrium price level is determined by (7). This price level and equation (5) determine the demand for nominal money. In general, this money demand will not match money supply as determined by (6), so that some variable needs to be introduced in order to satisfy the government budget constraint: the stock of international reserves is the variable that makes the adjustment, and the government will enforce the ERR by decreasing its reserves. Therefore, the following equation holds in periods of ERR:

$$M_t = M_{t-1} + d_t \ P_t + \ e_t \ (R_t - R_{t-1}), \tag{8}$$

where R_t denotes the level of international reserves.

Clearly, convertibility can only be maintained as long as the stock of international reserves is nonnegative. Thus, there cannot be a systematic unbalance between the crawling peg and the long run inflation rate. In order to take care of this issue, we assume that the crawling peg implies a long run inflation rate equal to the low steady state inflation rate of the model, $\overline{\beta}$.

However, some reserves may be lost during the beginning of the ERR. This would only

However, some reserves may be lost during the beginning of the ERR. This would only be maintained with additional policies of accumulating reserves during periods of low inflation for instance. This can be achieved by maintaining the ERR while the real value of the money stock is increased after the stabilization. Alternatively, one could interpret a situation were the government runs out of reserves as a case in which a reduction on the seigniorage is the only way to restore the equilibrium. None of this alternatives would change the results of the paper in a substantive way. Note, however, that the policy of the government in the model is to establish an ERR after a hyperinflation, precisely when the real value of the money stock is very low, and thus, the reserves required to back it are lower. In fact, one way to justify delayed intervention with our model is that the government is letting the hyperinflation erode the real value of the money stock to the point where it can easily back it with the available stock of reserves[10].

One could also argue that a more reasonable policy is to have a permanent ERR, so that equation (7) determines the inflation rate and there can never be hyperinflations. This is not quite right, because then the shocks will affect the stock of international reserves, at a point in which the value of the real money stock is high. This can create a balance of payment crisis, the ERR should be abandoned, and a hyperinflation could start. But once the real value of the money stock is low enough, a new ERR could be establish to stop the hyperinflation. Thus, the qualitative nature of the equilibrium would be very similar with this alternative policy[11].

Then, we impose the rule that government policy acts to satisfy

$$\frac{P_t}{P_{t-1}} < \beta^U, \tag{9}$$

where β^U is the maximum inflation tolerated. The ERR is *only* imposed

[10] This interpretation would suggest that the burst in inflation at the begining of 1991 in Argentina was crucial for the succes of the Convertibility Plan launched in April of the same year, because it substantially reduced the value of the money stock to a point were, at a price equal to one, the government could back the whole money stock.

[11] In fact, some of the episodes could be described with this balance of payment-devaluation-hyperinflation cycle. For an early explanation along this lines, see Rodriguez (1980).

in periods when inflation would otherwise violate this bound or in periods where no price level clears the market[12].

In effect, fixing the exchange rate acts to reduce the seigniorage and the money supply in the economy. In principle, any reduction in the government deficit of $e_t\ (R_t - R_{t-1})$ units would also keep inflation below the bound and fix the inflation to $\overline{\beta}$ in periods of ERR. In fact, the reduction in seigniorage that is needed to achieve an inflation equal to $\overline{\beta}$ is often quite moderate, which raises the issue of why have governments used ERR instead of lowering seigniorage sufficiently. One possible answer is that lowering seigniorage by the exact amount requires much more information: it can only be implemented when the government knows exactly the model and all the parameter values, including those that determine the (boundedly rational) expectations P^e_{t+1}, and all the shocks. By contrast, an ERR can be implemented only with knowledge of $\bar{\beta}$ and β^U.

The fact that ERR seems to have been the choice of governments under hyperinflationary experiences is further evidence that governments live in a world where agents' expectations and the model generating inflation are not easily determined. The second advantage of ERR for real governments would be that, for institutional reasons, it can be implemented quickly, while lowering government expenses or increasing taxes may take a long time.

In summary, the government in our model sets money supply to finance seigniorage; if inflation is too high, the government establishes ERR. The parameters determining government policy are $\overline{\beta}$, β^U and the process for d_t.

4.2 Pseudo-Rational learning mechanism.

Agents are assumed to form their expectations using boundedly rational learning mechanisms in line with our discussion in section 3.

Letting perceived inflation for next period be β_t we have

$$P^e_{t+1} = \beta_t P_t \tag{10}$$

We assume that the learning mechanism is given by

[12]Since both the demand and supply of money depend positively on the price level, it can be shown that no equilibrium price exists for high enough β_t. See Marcet and Sargent (1989b) for a detailed description.

$$\beta_t = \beta_{t-1} + \frac{1}{\alpha_t}\left(\frac{P_{t-1}}{P_{t-2}} - \beta_{t-1}\right) \tag{11}$$

That is, perceived inflation is updated by a term that depends on the last prediction error[13]. The prediction error is weighted by $1/\alpha_t$. This is a simple version of stochastic approximation algorithms, where the weights are often denoted the 'gain' sequence. The right side of (11) determines the learning mechanism f in equation (2), together with the evolution of the gain sequence.

In stochastic approximation[14], the gain sequence is often specified exogenously. For example, consider the law of motion

$$\alpha_t = \alpha_{t-1} + 1 \tag{12}$$

which is consistent with (2). Simple algebra shows that, in this case, $\alpha_t = t$ and

$$\beta_t = \frac{1}{t}\sum_{i=1}^{t} \frac{P_i}{P_{i-1}} \tag{13}$$

so that perceived inflation is just equal to the sample mean of past inflations or, equivalently, it is the result of a least squares regression of inflation on a constant.

Another exogenous gain sequences is the so-called 'tracking' algorithms, also known as 'constant gain' algorithms. These set $\alpha_t = \tilde{\alpha} > 1$. Here, perceived inflation satisfies

$$\beta_t = \frac{1}{\tilde{\alpha} - 1}\sum_{i=1}^{t} \left(1 - \frac{1}{\tilde{\alpha}}\right)^i \frac{P_{t-i}}{P_{t-i-1}} \tag{14}$$

[13]This formula implies that agents do not use today's inflation in order to formulate their expected inflation; the last observed inflation used to formulate expectations at t is inflation at $t-1$. This assumption is made purely for convenience; it simplifies solving the model, since it avoids simulateneity in the determination of perceived inflation and inflation. It would probably be more desirable to incorporate today's inflation in β_t, since information about prices is revealed very quickly and, in a hyperinflationary world, inflation may change strongly from one period to the next. Furthermore, incorporating today's inflation is likely to improve the empirical fit of the model under learning, since *LB2* and *LB3* are even more likely to be satisfied. We are planning to introduce this case in the final version of this paper.

[14]See, for example, Robbins and Monro (1951) and Ljung and Söderström (1983).

so that past information is now a weighted average of past inflations, where the past is discounted at a geometric rate.

Notice that least squares (13) gives equal weight to all past observations, while tracking (14) gives more importance to recent events. Which alternative algorithm generates better forecasts depends on whether the system generating inflation is stable or not; the 'tracking' system is designed to adapt more quickly to a change in the environment, while least squares has good chances of being superior when the environment is stable.

Unfortunately, neither of these alternatives has a good chance of satisfying the lower bounds on rationality. This will be clear from our calculations in the next section, but the main intuition can be provided now. Tracking (14) performs poorly in stable periods, because tracking algorithms do not converge to a constant, since the prediction errors always affect the perceptions; in fact, it does not even satisfy *LB1* since, under RE, perceptions are a constant (see appendix 1). On the other hand, if our model has any success at replicating the observations on recurrent hyperinflations in Figures 1 to 5, least squares does not have a chance of generating 'good' forecasts because, along a hyperinflation, formula (13) will be extremely slow in adapting during the bursts in inflation. In those periods, 'tracking' will be a better idea, so that least squares does not satisfy *LB2* or *LB3*.

Since the phenomenon of recurrent hyperinflations seems to have both stable and unstable periods, we will specify a learning mechanism that uses OLS in stable periods and it switches to 'tracking' when some instability is detected. This amounts to assuming that agents use an endogenous gain sequence such that, as long as agents don't make large prediction errors, the weights $1/\alpha_t$ decrease over time at the same rate as in least squares, but in periods where a large prediction error is detected, the weight is increased to a fixed value $1/\bar{\alpha}$, mimicking the 'tracking' algorithms. Formally, the gain sequence follows[15]

$$\begin{aligned} \alpha_t &= \alpha_{t-1} + 1 && \text{if } \left| \frac{\frac{P_{t-1}}{P_{t-2}} - \beta_{t-1}}{\beta_{t-1}} \right| < \nu \\ &= \bar{\alpha} && \text{otherwise} \end{aligned} \tag{15}$$

Thus, the expectation formation mechanism is the same whether or not

[15] In this version, we specify the maximum acceptable error ν as a given parameter. An interesting extension would be to relate that value to the perceived value of the standard deviation of the variables.

ERR is enforced. The conventional wisdom that the importance of an ERR is the effect it has on expectations is consistent with the model, since the exchange rate rule has an impact on expectations by its effect on the current price level and by setting the gain factor to its base value $\bar{\alpha}$.[16]

In summary, we assume that the gain sequence of the learning mechanism is updated according to OLS in periods of stability, but it uses constant gain (or tracking) in periods of instability. The learning mechanism is fully described by equations (11) and (15), h is equal to the perceived inflation, and the learning parameters μ are given by $\nu, \bar{\alpha}$.

5 Equilibrium under learning.

The system of variables that we need to solve for is $\left\{\frac{P_t}{P_{t-1}}, \beta_t, \alpha_t\right\}$. We first describe how to solve the model given some learning parameters. Using simple algebra it is easy to show that equilibrium inflation satisfies

$$\begin{aligned} \frac{P_t}{P_{t-1}} &= \frac{1-\lambda\beta_{t-1}}{1-\lambda\beta_t-\gamma d_t} && \text{if } 0 < \frac{1-\lambda\beta_{t-1}}{1-\lambda\beta_t-\gamma d_t} < \beta^U \\ &= \bar{\beta} && \text{otherwise,} \end{aligned} \tag{16}$$

while β_t and α_t are determined from (11) and (15). This defines a system of stochastic, second-order difference equations; characterizing the solution analytically is unfeasible due to the fact that the system is highly non-linear. Fortunately, solving the model numerically is extremely simple

Even though the solutions will be analyzed by numerical simulations described in the next section, some intuition for the behavior of the model can be provided at this point. Define the function

$$\begin{aligned} h(\beta, d) &= \frac{1-\lambda\beta}{1-\lambda\beta-\gamma d} && \text{if } 0 < \frac{1-\lambda\beta}{1-\lambda\beta-\gamma d} < \beta^U \\ &= \bar{\beta} && \text{otherwise,} \end{aligned} \tag{17}$$

If $\beta_t \simeq \beta_{t-1}$, then $P_t/P_{t-1} \simeq h(\beta_t, d_t)$; therefore, the graph of $h(\cdot, d)$ in Figure 6 can be interpreted as providing an approximation of the actual inflation

[16] In the final version of the paper we also want to explore a learning mechanism where, in periods where the government establishes ERR, agents set their believed inflation equal to the target inflation so that $\beta_t = \bar{\beta}$. This would correspond to a case where the government's plans about targeted inflation are fully credible. Preliminary calculations indicate that the behavior of the model does not change qualitatively.

rate as a function of perceived inflation. The first graph corresponds to a low average level of seigniorage $E(d_t) = \overline{d}$; the dotted lines contain the values of the function $h(\cdot, d_t)$ as it shifts due to shocks to seigniorage. The limiting rational expectations equilibrium β^1_{RE} is close to the lower fixed point of $h(\cdot, \overline{d})$ (see appendix 1).

On average, if $\beta_t \in S$, inflation is closer to β^1_{RE} than perceived inflation; this pushes perceived inflation, in average in the direction of β^1_{RE}. Roughly speaking, S is the stability set of perceived inflation. On the other hand, if perceived inflation is in U, actual inflation is always higher than β_t, so that a hyperinflation will occur until the upper bound β^U is reached; then, a fixed exchange rule will be established, and this will bring back the economy into the stable set. The economy may end up in the set U due to a number of reasons: a few high shocks to seigniorage when α_t is not yet close to zero, initially high perceived inflation, the second-order dynamics which add momentum to increasing inflation, etc.

Notice that the economy is likely to end up in U if the gain $\overline{\alpha}$ is large since, in that case, perceived inflation is more heavily influenced by shocks to actual inflation; if α_t is arbitrarily close to zero and initial inflation starts out in S, hyperinflations are impossible. But if hyperinflations occur, agents will set the weight $\alpha_t = \overline{\alpha}$, so that the presence of hyperinflations prompts agents to pay more attention to recent observations which, in turn, makes it more likely that hyperinflations occur.

This intuition tells us that the model is consistent with stylized fact 1, since a number of hyperinflations may occur in the economy before it settles down. Also, it is clear that an ERR will end each hyperinflation, so that fact 2 is found in this model. Also, once β_t is in the set U, inflation will grow on average even if seigniorage stays roughly constant, which is consistent with fact 3.

To analyze fact 4 , consider the second graph of Figure 6, which corresponds to a high average level of seigniorage. Now, the stable set S is much smaller; furthermore, the hyperinflationary set U is dangerously close to the rational expectations equilibrium, where the economy tends to live; it is more likely for the model to end up in the set U and a hyperinflation to occur. Thus, the model is well endowed to match the high cross-country correlation of average seigniorage and the occurrence of hyperinflationary episodes, and fact 4 is consistent with the model.

Finally, we want to restrict our study to those learning mechanisms that are pseudo-rational in the sense of satisfying *LB1* and *LB3*. We will also study *LB2* as an afterproduct.

We discussed previously why least squares or tracking were unlikely to satisfy the lower bounds on rationality specified in section 3. From that discussion, it is clear that the learning mechanism proposed here has a chance of satisfying those criteria for positive $\overline{\alpha}$'s: the algorithm can be shown to converge to the rational expectations equilibrium (see appendix 1), so that *LB1* is satisfied. From our previous intuition, when $\overline{\alpha}$ is high, hyperinflations are likely to occur, so that setting $\alpha_t = \overline{\alpha}$ is likely to generate good forecasts within the model, so that *LB3* is likely to be satisfied for $\overline{\alpha}$'s that generate hyperinflations.

We are now ready to define our equilibrium concept. The variables we have to determine are the sequences of inflation, expected inflation and nominal balances, together with the parameter $\overline{\alpha}$.

Definition 4 *A sequence* $\left\{\frac{P_t}{P_{t-1}}, \beta_t, M_t\right\}$ *is an* ϵ, T *equilibrium if:*

1. *Given* $\overline{\alpha}, \left\{\frac{P_t}{P_{t-1}}, \beta_t, M_t\right\}$ *satisfy (16), (11), (15) at all periods.*

2. *Given* $\left\{\frac{P_t}{P_{t-1}}, \beta_t, M_t\right\}$, $\overline{\alpha}$ *satisfies*

$$E\left(\frac{1}{T}\sum_{t=1}^{T}\left(\frac{P_{t+1}}{P_t} - \beta_t\right)^2\right) < \min_m \; E\left(\frac{1}{T}\sum_{t=1}^{T}\left(\frac{P_{t+1}}{P_t} - \beta_t(\bar{\alpha}, m)\right)^2\right) + \epsilon$$

where $\beta_t(\bar{\alpha}, m)$ *is the forecast of inflation obtained if* m *replaces* $\overline{\alpha}$ *in equations (11), (15).*

6 Characterization of the solution by simulation.

To generate simulations we must assign values for the parameters of the economic fundamentals, the money demand equation and the government policy. For the money demand equation, we have to determine the two parameters in the linear functional form 5. It is well known, though, that the linear functional form does not perform very well empirically. However, departing from linearity would make the analytics of the model impossible to deal with.

While we do maintain linearity, we want to use parameter values that are not clearly at odds with the observations. Since we are interested in the public finance aspect of inflation, we use observations from empirical Laffer curves to calibrate the two parameters. In particular, as one empirical implication of our model is that "high" average deficits increase the probability of a hyperinflation, we need to have a benchmark to discuss what high means. Thus, a natural restriction to impose to our numbers is that the implied maximum deficit is close to what casual observation of the data suggest. And we also want the inflation rate that maximizes seigniorage in our model to be consistent with the observations.

In Figure 6 we plot quarterly data on inflation rates and seigniorage as a share of GNP for Argentina[17] from 1980 to 1990 from Ahumada, Canavese, Sanguinetti y Sosa (1993). While there is a lot of dispersion, it seems that the maximum feasible seigniorage is around 5% of GNP, and the inflation rate that maximizes seigniorage is close to 60%. These figures are consistent with the findings in Fernandez and Mantel (1989), Kieguel and Newmayer (1992) and Rodriguez (1991).

The parameters of the money demand γ and ϕ, are uniquely determined by the two numbers above. Note that the money demand function 5 implies a stationary Laffer curve equal to

$$\frac{\pi}{1+\pi}m = \frac{\pi}{1+\pi}\frac{1}{\gamma}\left(1 - \frac{1}{\phi}(1+\pi)\right) \tag{18}$$

where m is the real quantity of money and π is the inflation rate. Thus, the inflation rate that maximizes seigniorage is

$$\pi * = \sqrt{\phi} - 1$$

which, setting $\pi * = 60\%$, implies $\phi = 2.56$. Using this figure in 18, and making the maximum revenue equal to 0.05, implies $\gamma = 2.7$. The Laffer curve implied by our parameters is also plotted in Figure 6. A decent fit is obtained.

[17] The choice of country is arbritrary. We chose Argentina because we were more familiar with the data.

For the standard deviation of the deficit we used 0.01[18]. The parameter ν, which measures the error level at which the learning rule sets alpha equal to the base value was set equal to 10%.

Finally, we assumed that the government established ERR whenever expectations were such that inflation rates would be above 5000%, so that we set $\beta^U = 50$.

As we mentioned above, the maximum level of average seigniorage in the model is 0.05. In order to quantify the relevance of the average seigniorage (which determines fact 4), we performed our calculations for four different values of the deficit, 0.049, 0.047, 0.045 and 0.043.

First of all, we describe the typical behavior of the model. A particular realization is presented in Figure 7. That realization was obtained with a deficit equal to 0.049, a standard deviation equal to 0.01, and the initial alpha $\overline{\alpha} = 0.2$. These values correspond to a particular equilibrium we describe below, except that this particular simulation is larger than the ones described below. This graph should not be taken as representative in any way. Its only role is to show the potential of the model to generate enormous inflation rates. In the same graph, we also plotted a horizontal line at the two stationary rational expectation equilibria, to show how the model can generate inflation rates that are way higher than them. The economy starts close to the low stationary equilibrium. When a large shock occurs, it drives perceived inflation into the unstable region U. Then a hyperinflation episode starts. Eventually, ERR is established and the economy is brought back into the stable region. If no large shocks occurred for a long while, the model would converge to the rational expectations equilibrium; however, since average seigniorage is so high for this simulation, it is likely that a new large shock will put the economy back into the unstable region and a new burst in inflation will occur. Clearly, we have recurrent hyperinflations, stopped by ERR, without serial correlation between seigniorage and inflation (facts 1, 2 and 3). In order to reduce (or eventually eliminate) the chances of having a new burst, the government must reduce the amount of seigniorage collected and increase the size of the stable set (an "orthodox" stabilization plan); this would separate the two horizontal lines and it would stabilize the economy permanently.

[18] We also used a value for sigma equal to 0.005. The results were similar except that, as expected, the probabilities of hyperinflations were lower.

An important aspect of the parameter selection is the choice of learning parameters. We look for $\overline{\alpha}$ that satisfy the lower bound criterion *LB3*. for (ϵ, T)=(.01,120). Tables 1, 3, 5 and 7 report the mean square errors in the right side of (4); the mean square errors are calculated by Monte-Carlo integration[19]. Each column reports the MSE for a different $\bar{\alpha}$ actually used by agents, each row refers to the forecasting error that would be made with alternative learning parameters m. We included 13 points between 1.2 and 0 for $\bar{\alpha}$ and the alternative learning parameters m. In accordance with *LB3*, those alternative learning parameters that generate a mean square error within $\epsilon = .01$ of the minimum in each column are displayed in boldface. Thus, a bold number ***in the diagonal*** indicates a value for alpha that satisfies our *LB3*.

Tables 2, 4, 6, 8 report the probabilities of having n hyperinflations in 10 years for those values of $\overline{\alpha}$ that satisfy the *LB3* criterion.

For values of alpha lower than 0.043, the best alternative alpha is always zero, and there are no hyperinflations in equilibrium.

Table 1 presents the results for a low value $\bar{d} = 0.043$. In this case, only $\bar{\alpha} = 0$, and 0.1 satisfy the pseudo-rationality requirement. Table 2 shows that for none of the two values the economy exhibits hyperinflations. This table shows that the only learning parameters that are pseudo-rational are those that preclude hyperinflations from happening: when $\bar{\alpha}$ is low, hyperinflations do not occur, and giving too much importance to recent observations does not generate good forecasts.

Table 3 shows the results of increasing average seigniorage to 0.045. In this case the criterion is satisfied for all values of alpha between 0.5 and zero. As indicated by Table 4, there are equilibria in which the probability of experiencing recurrent hyperinflations is high, so that higher alternative α's generate good forecasts, and the hyperinflationary behavior is reinforced. Tables 5 to 8 show that, as the mean of seigniorage increases, it is still the case that pseudo-rational learning is consistent with the observation of hyperinflations.

This exercise formalizes the sense in which the equilibria with a given learning mechanism reinforces the use of the mechanism. Note for instance

[19] This is the only feasible integration procedure to compute the expectations in (4), since that expectation involves 120 random variables. We use 1000 realizations of the shocks.

in Table 7, that when $\overline{\alpha} = 0.2$, an agent using an alternative alpha equal to zero, which is the collective behavior that replicates the REE, will make larger MSE than the agent using $\overline{\alpha} = 0.2$. The reason is that in equilibrium there are many hyperinflations, and the agent that expects the REE will not make good forecasts. Incidentally, note that if agents use alpha equal to zero, the alpha that minimizes the MSE is also zero. This is the REE.

Whenever there exist equilibria with hyperinflations, there is multiplicity of equilibria. The REE is always an equilibrium, and in general, there is more than one alpha that satisfies our criteria. At this level, we cannot say much about the multiplicity problem, but if the initial conditions are far from the good stationary equilibrium, as they would be after a sudden change in policy, the REE may no longer be an equilibrium[20].

The numerical solutions show that the chances of facing a hyperinflation during the transition to the rational expectations equilibrium, depend on both the sensitivity of the learning rule with respect to changes in prices and on the size of the deficit. The lower the deficit, the lower the chances of experiencing a hyperinflation. In our model, the sensitivity of the learning rule depends on the size of the deficit. The larger the deficit, the larger will be the optimal sensitivity of the learning rule, which increases the chances of having a hyperinflation.

7 Related literature

Hyperinflations have been widely analyzed in the literature. Much progress has been made, and our work draws heavily from this literature. Somewhat unfairly, we concentrate the discussion in this section to describing aspects of the observed hyperinflations that are not well matched by the existing literature.

A seigniorage model like the one of this paper, but with rational expectations and no ERR was developed by Sargent and Wallace (1987). Rational expectations requires that $\beta_t = E_t\left(\frac{P_{t+1}}{P_t}\right)$; as long as d is below a certain maximal level, this model has two equilibria with constant inflation levels denoted $\bar{\beta} < \bar{\beta}$ (respectively called low- and high-inflation equilibria) and a

[20]We replicated the exercise for initial beliefs thaht were far away from the REE, and the REE was not a learning equilibrium.

continuum of bubble equilibria that converge to the high-inflation equilibrium $\bar{\beta}$. This case is discussed in the appendix. Under these assumptions, hyperinflations can only be observed as bubble equilibria. Bubble equilibria agree with fact 3 qualitatively, and this is the main motivation behind the work of Sargent and Wallace. The original model contradicts fact 1, since either the economy is in a hyperinflation (bubble) or it is not. The recent work by Funke et al. (1994) shows that a sunspot can be introduced that sets hyperinflations on and off, which would match the recurrence of hyperinflations (fact 1), even though at the cost of having a sunspot that coordinates the start and the end of the hyperinflations. Fact 4 is contradicted since it predicts that hyperinflations are *less* severe in countries with high seigniorage, since in those countries $\bar{\beta}_2$ is lower. Finally, fact 1 is not matched quantitatively: for reasonable parameter values, the magnitude of the hyperinflations that can be generated with this model is very small; for example, for the parameter values used in Figure 7, the hyperinflations can never go beyond the higher horizontal line, while with our learning model the hyperinflations can get arbitrarily high. A wide empirical literature tested the existence of a speculative component in the German hyperinflation of the twenties. A short summary of the literature can be found in Imrohoroglu (1992).

Fact 2 is not even addressed in the papers discussed in the previous paragraph. Obstfeld and Rogoff(1983) and Nicolini(1993) provide a model where the effects of convertibility can be studied, since they introduce ERR that goes into effect if inflation goes beyond a certain level. Their results show that the threat of convertibility eliminates bubble equilibria. Thus, once ERR is introduced, the model is inconsistent with the existence of hyperinflations and, presumably, with all the observed facts.

It seems particularly important to study the stability of bubble rational expectations equilibria under learning, since the dynamics of these equilibria are very complicated. Marcet and Sargent (1989b) studied stability of rational expectations equilibria under least squares learning[21]. They found that, if the deficit is low enough, the low-inflation equilibrium is *locally* stable; the high-inflation equilibrium is always unstable. Taken literally, these

[21] Marcet and Sargent (1989b) is a special case of the present paper when uncertainty is eliminated, β^U is arbitrarily high, and we use $\alpha_t = t$. Other differences are that MS only studied local stability and the learning mechanism was slightly different, since they assumed that agents ran a regression of P_i on P_{i-1}. This last factor has some effect on the stability conditions M&S but does not change the main conclusions.

results would say that bubble equilibria can not be learnt by agents; hence, if we limit our attention to rational expectations equilibria that can be learnt, bubble equilibria become an even worse justification for the presence of hyperinflations. Therefore, if learning is taken seriously as a stability criterion, then the model of Sargent and Wallace does not have hyperinflations and, again, none of the above facts is appropriately matched. Evans, Honkapohja and Marimon (1995) discuss a learning model where seigniorage is lowered to maintain inflation if this is high enough; their model amounts to setting an h mapping in figure 6 where the horizontal part is continuous with respect to the rest of the mapping; they show that the low stationary inflation equilibrium is globally stable in this case.

The issue of recurrent hyperinflations has also been studied by Zarazaga (1995) in a very interesting model with rational expectations and private information. The key ingredient of his model is the existence of a federal system of fiscal authorities that have access to the monetary authority but with private information regarding the own spending. The hyperinflations are interpreted as punishment periods that prevent each single fiscal authority from over spending. There are many differences between his work and our paper. His paper can definitely match fact 1 and can do reasonably well on fact 3. However, exchange rate policies like the one we study in our paper, and that very closely resemble the policies actually carried out by most governments to stop severe inflation rates (fact 3) play no role in his model. There is a sense in which Zarazaga's paper point toward the forces behind the seigniorage, while in our paper the seigniorage is exogenous.

Eckstein and Leiderman (1992) can generate large increases in inflation with moderate increases in seigniorage by assuming a monotonically increasing Laffer Curve that converges to a positive value. Their aim is explaining why average inflation in Israel in 1985 was 20 times higher than in 1978, while seigniorage was almost the same. However, they only compare steady states, so it is not obvious that their model could explain the recurrent hyperinflations as we do in this paper.

Recently, some papers have argued that models of learning can be used for something more than a stability criterion. Bolton and Rustichini (1995) and Marimon (1995) are some examples.

8 CONCLUSION

There is some agreement by now that the hyperinflations of the 80's were caused by the high levels of seigniorage in those countries, and that the cure for these hyperinflations is fiscal discipline and abstinence from seigniorage. The IMF is currently imposing tight controls on the previously hyperinflationary countries that are consistent with this view. Nevertheless, no currently available model justified this view and was consistent with some basic facts of hyperinflations; for example, the fact that seigniorage has gone down during some hyperinflations and inflation continued to grow makes it more difficult for the IMF to argue in favor of these controls. Furthermore, some Eastern European economies are now engaging in hyperinflationary episodes similar to those of the 80's, and it seems important to have a solid model that can help judging the reasonability of the IMF recommendations.

Our model, is consistent with the main stylized facts of recurrent hyperinflations. The policy recommendations that come out of the model are in agreement with the views we discussed at the beginning of this conclusion: an ERR will temporarily stop a hyperinflation, but to eliminate hyperinflations average seigniorage must be lowered.

The economic fundamentals of the model are perfectly standard except for the use of a boundedly rational learning rule instead of rational expectations. We show that the learning rule is pseudo-rational in a sense that is made precise in the body of the paper; despite abandoning RE, we maintain falsifiability of the model, and we restrict the deviation from rationality to be small. This deviation from rational expectations is attractive in itself, because it avoids the strong requirements on rationality placed by RE, and because the fit of the model improves dramatically despite the *small* deviation from rationality.

The conclusion that learning is consistent with the observations on hyperinflations is quite robust. It happens under most parameter settings and for most learning schemes that satisfy our lower bounds on rationality.

On the practical side, this paper shows that hyperinflations can be stopped with a combination of heterodox and orthodox policies. The methodological contribution of the paper is to show that, as long as we carry along adequate equipment for orientation and survival, an expedition into the "jungle of irrationality" can be quite a safe and enjoyable experience.

APPENDIX 1

The following characterizes the behavior of the model with uncertainty under rational expectations. For each β, d, let us define

$$h(\beta, d) \equiv \frac{1 - \lambda\beta}{1 - \lambda\beta - \gamma d}.$$

Notice that $h(\beta, \overline{d})$ is the S mapping corresponding to the deterministic case.

Proposition 1 *Assume that there is a $K < \infty$ such that $P(d_t \leq K) = 1$ and such a K is the lowest almost sure bound on d_t.*

1. *Assume that expectations about inflation are given by*

$$P^e_{t+1} = \beta P_t \tag{19}$$

   If $1 - \lambda\beta > \gamma K$, expected inflation conditional on today's information is given by $E_t(P_{t+1}) = E\left(h(\beta, d_{t+1})\right) \equiv S(\beta)$.

2. *The set of REE of the form (19) coincides with the set of fixed points of the mapping $S : [0, (1 - \gamma K)/\lambda) \rightarrow R_+$*

3. *Assume that $P(d_t \geq 0) = 1$ and that either d_t has a point mass at K or that $F'(K) > 0$, where F represents the distribution of d_t. Then, S has the following properties:*

   - *S is increasing, concave, and it asymptotes to infinite as $\beta \rightarrow (1 - \gamma K)/\lambda$.*
   - *S has at most two fixed points denoted $\bar{\beta}_1 < \bar{\beta}_2$. For $\overline{d}$, σ^2_ϵ and K low enough, two fixed points exist; for $\overline{d}$, σ^2_ϵ and K high enough no fixed point exists.*
   - *$S(\beta) > h(\beta, \overline{d})$, so that for higher variances of seigniorage a fixed point may not exist, even if the mean could be financed with a deterministic seigniorage.*

Proof

1. Simple algebra shows that, under (19), $P_t = h(\beta, d_t)\ P_{t-1}$ and, since d_t is i.i.d., this implies that $E_t(P_{t+1}) = P_t\ E[\ h(\beta, d_{t+1})\ ] \equiv P_t\ S(\beta)$.

2. Follows by definition of S.

3. Using the definition of S we have

$$S'(\beta) = E\left(\frac{\partial\ h(\beta, d_t)}{\partial\ \beta}\right) = E\left(\frac{\lambda\gamma d_t}{(1-\lambda\beta-\gamma d_t)^2}\right) \qquad \text{and}$$

$$S''(\beta) = E\left(2\lambda\frac{\lambda\gamma d_t}{(1-\lambda\beta-\gamma d_t)^3}\right)\ ;$$

since the expression inside the expectation is non-negative, this proves that $S', S'' > 0$.

To prove the existence of an asymptote; note that the derivative at the upper limit $\beta = (1-\gamma K)/\lambda$ is given by

$$S'((1-\gamma K)/\lambda) = \int_0^K \frac{K}{K-d_t}\ \mathrm{d}\ F(d_t)\ . \tag{20}$$

If F has a point mass at K, since the integrand is infinite at $d_t = K$, the result is obvious. If $F'(K) > 0$, there exist positive, finite constants η, C such that for all $\widetilde{K}$ such that $K - \eta < \widetilde{K} < K$, $F'(\widetilde{K}) > C$. Then we can write:

$$S'((1-\gamma K)/\lambda) > \int_{K-\eta}^{K} \frac{K}{K-d_t}\ \mathrm{d}\ F(d_t) = \int_{K-\eta}^{K} \frac{K}{K-d_t}\ F'(d_t)\mathrm{d}\ d_t$$

$$> C\int_{1-\eta/K}^{1} \frac{1}{1-x}\ \mathrm{d}\ x = C\int_0^{\eta/K} \frac{1}{x}\ \mathrm{d}\ x$$

where for the first inequality we have used (20) and additivity of integrals, the next equality follows from differentiability of F' at K the next inequality follows from our choice of C, η and a change of variables, and the last equality is another simple change of variables. Now, since $\int_0^1 \frac{1}{x} dx$ is infinite, we have that $S'((1-\gamma K)/\lambda) = \infty$.

Then we have that either two or no fixed points exist, just as in the deterministic case.

Also, we have that

$$\frac{\partial^2 h(\beta, d)}{\partial d^2} = 2\gamma \frac{1 - \lambda\beta}{(1 - \lambda\beta - \gamma d)^3} > 0 ,$$

so that $h(\beta, \cdot)$ is convex. Therefore, using Young's inequality, we conclude that $S(\beta) > h(\beta, \overline{d})$.□

One difference with the deterministic case is that, as the variance of seigniorage increases the mapping S moves upward, so that the unstable region also shrinks with a higher σ_d^2. This implies that, if the variability of seigniorage is high, this increases the probability of hyperinflations for two reasons: *i)* given a value for today's beliefs, it is more likely to obtain a large enough shock that will send the next beliefs to the unstable region, *ii)* the unstable region shrinks.

The following proposition characterizes the equilibrium under least squares learning.

Proposition 2 *Assume that agents use the following least squares learning algorithm*

$$\beta_t = \beta_0 + (1/t) \sum_{i=0}^{t-1} P_i / P_{i-1}$$

to formulate their perceived rate of inflation, then $\bar{\beta}_1$ is locally stable with probability one, and $\bar{\beta}_2$ is locally unstable.

Proof:

We apply the framework in Marcet and Sargent (1989), (MS). We rewrite the law of motion for prices so that it is linear with respect to the exogenous shock. This can be simply accomplished by setting:

$$\frac{P_{t-1}}{P_t} = \frac{1 - \lambda\beta_t}{1 - \lambda\beta_t^{(1)}} - \frac{\gamma d_t}{1 - \lambda\beta_t^{(1)}}$$

which is a special case of equation (9.c) in MS, and rewriting the learning scheme to obtain

$$\beta_t = \beta_{t-1} + (1/t)(P_{t-1}/P_{t-2} - \beta_{t-1})$$

$$\beta_t^{(1)} = \beta_{t-1}^{(1)} + (1/t)(P_{t-2}/P_{t-3} - \beta_{t-1}^{(1)}),$$

which is a special case of equation (10) in MS taking $z_t = (P_t/P_{t-1}, d_t)$ and, using their propositions 1 and 2 we conclude that the algorithm can only converge to fixed points of S such that $S'(\beta) < 1$ and that it converges locally with probability one to such fixed points. Since $S'(0) < 1$, convexity of S and the existence of an asymptote implies that $S'(\overline{\beta}_1) < 1$ and $S'(\overline{\beta}_2) < 1$, and we have proved the proposition.□

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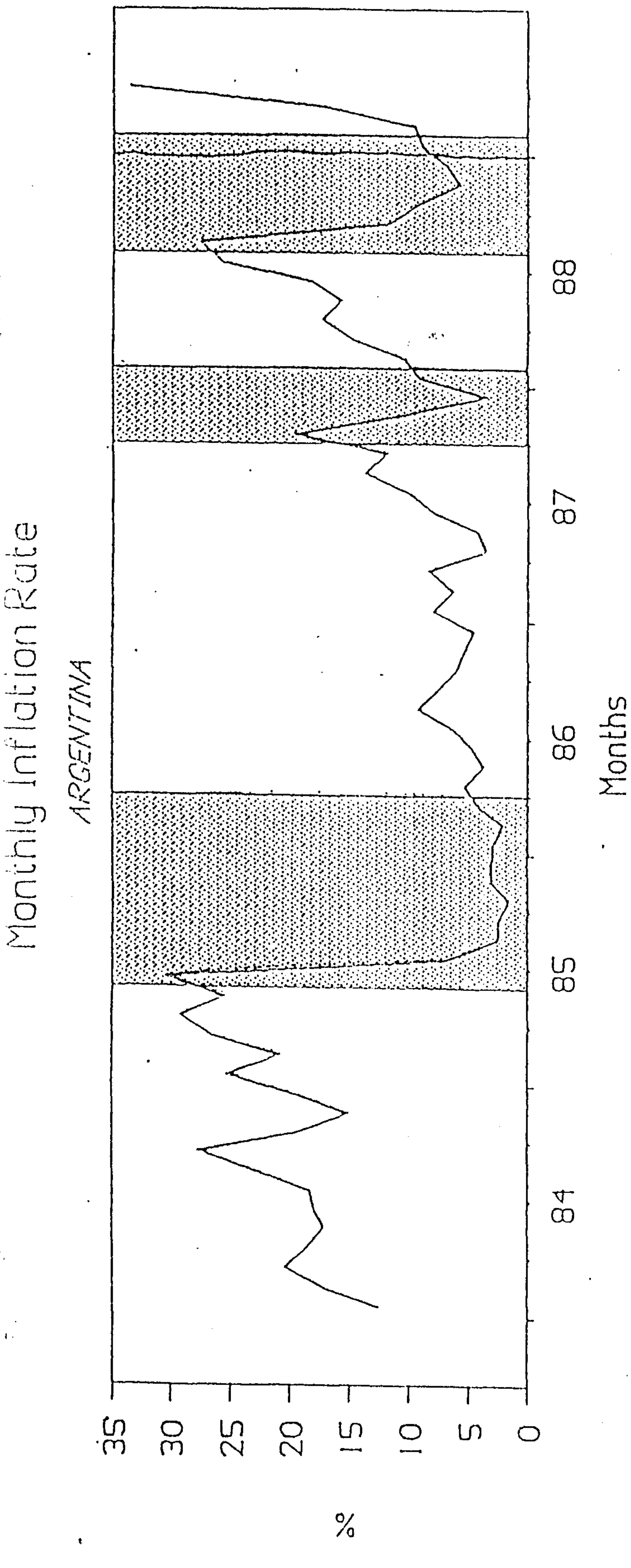
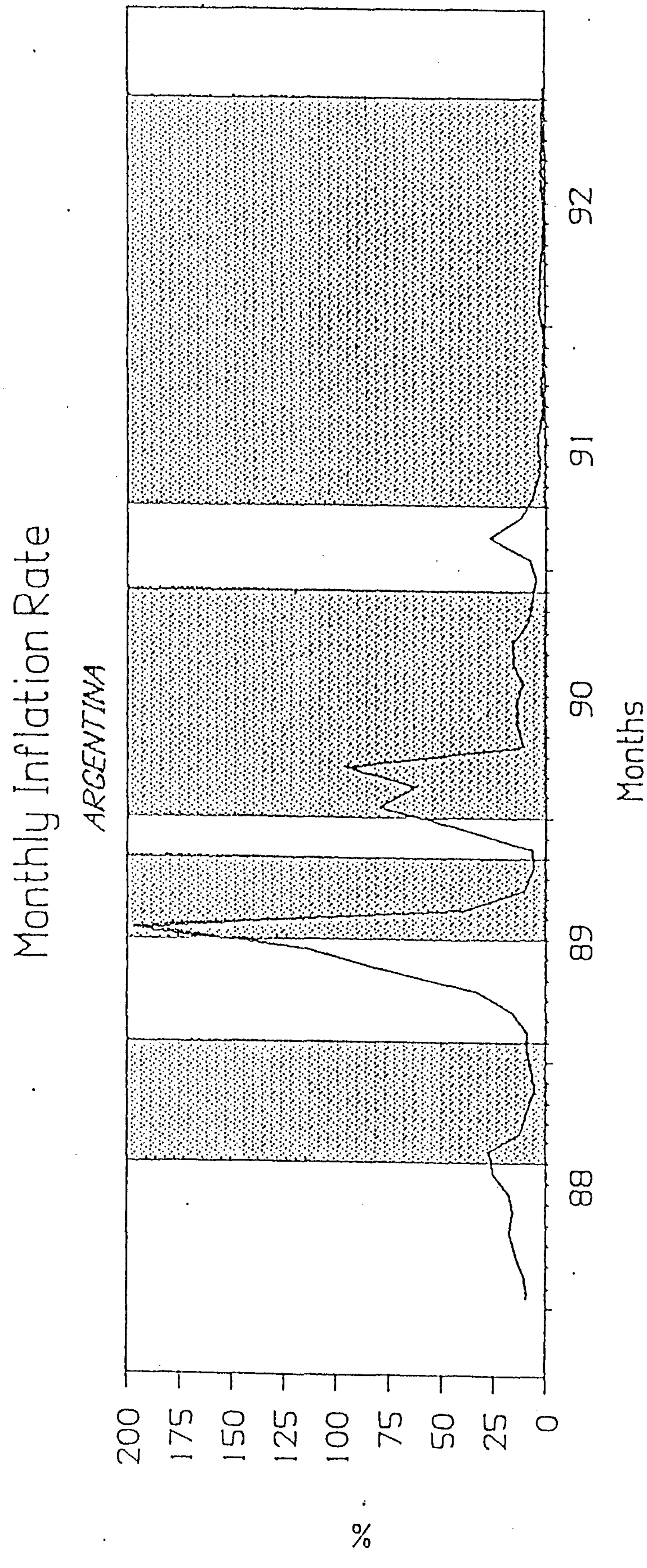
FIGURE 1



Monthly Inflation Rate
ARGENTINA
Months
%
35 30 25 20 15 10 5 0
84 85 86 87 88



Monthly Inflation Rate
ARGENTINA
Months
%
200 175 150 125 100 75 50 25 0
88 89 90 91 92

FIGURE 2

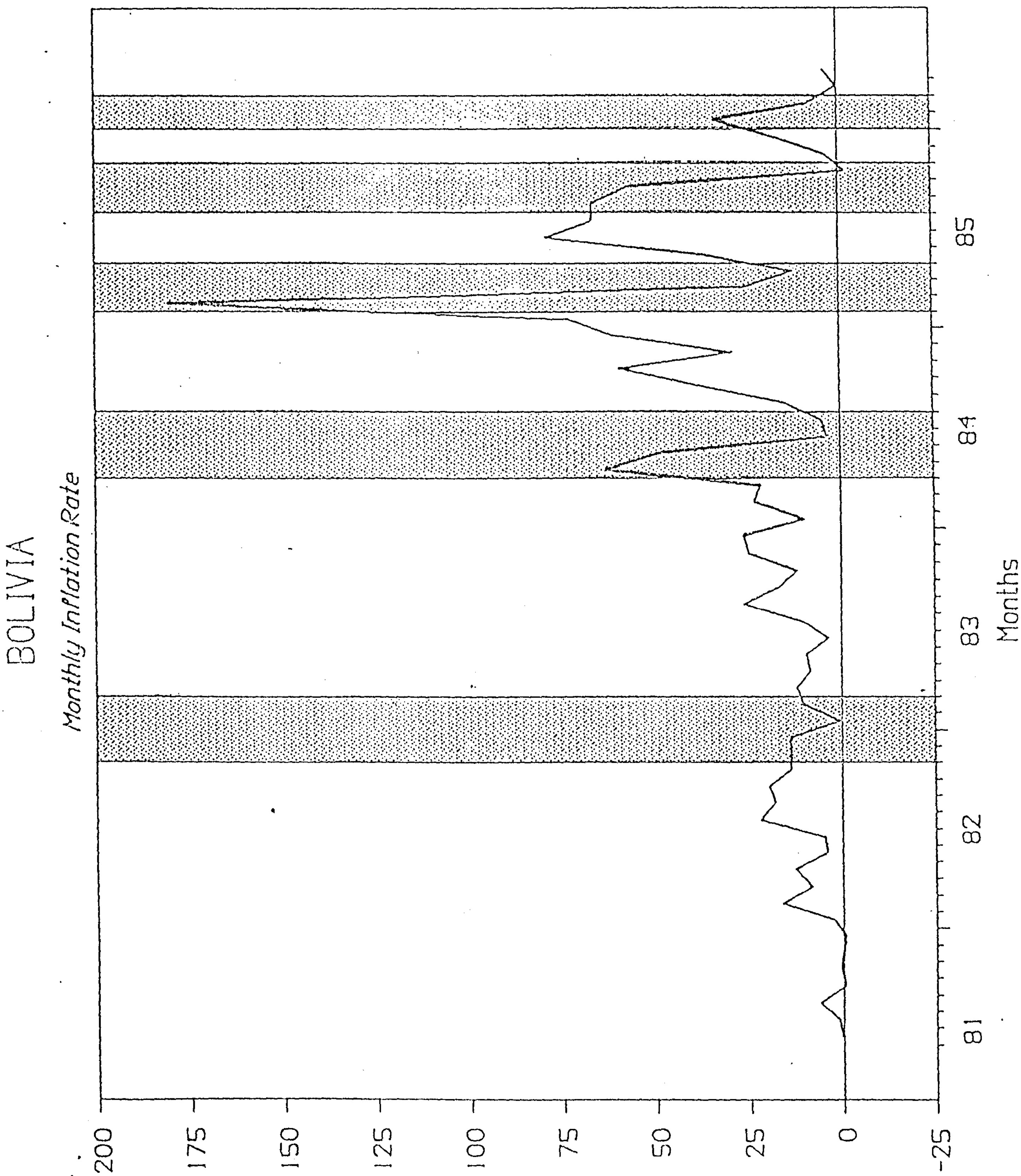
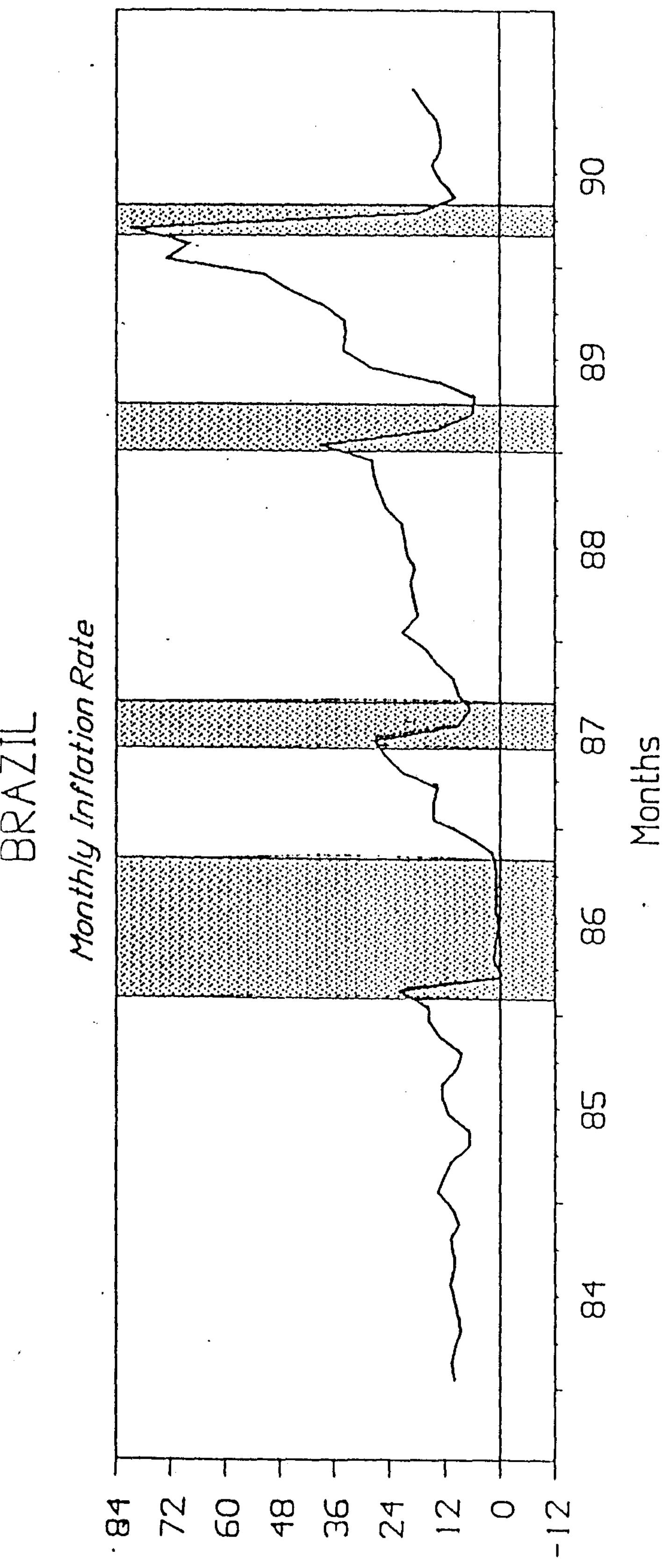
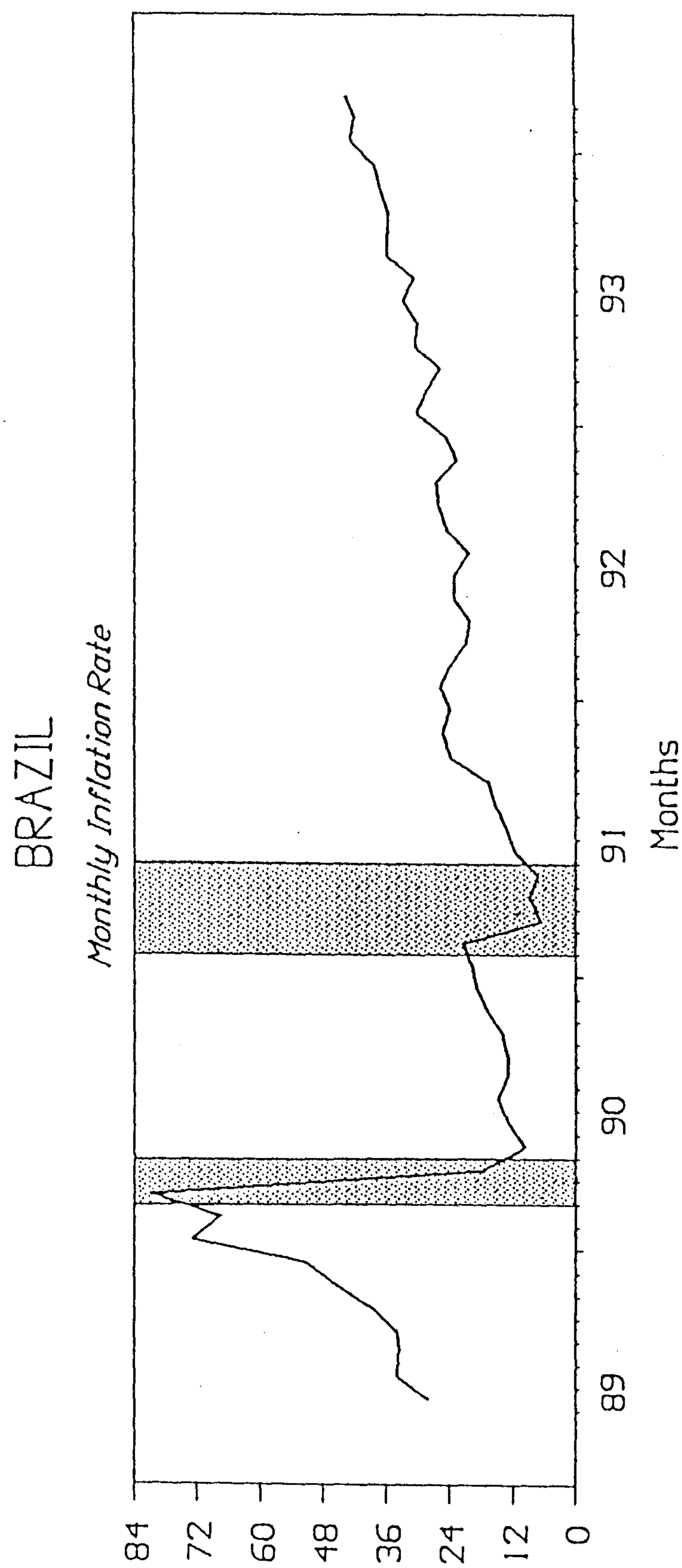

BOLIVIA
Monthly Inflation Rate
Months
81
82
83
84
85
200
175
150
125
100
75
50
25
0
-25

FIGURE 3



BRAZIL
Monthly Inflation Rate
Months
84
72
60
48
36
24
12
0
-12
84
85
86
87
88
89
90



BRAZIL
Monthly Inflation Rate
Months
84
72
60
48
36
24
12
0
89
90
91
92
93

FIGURE 4

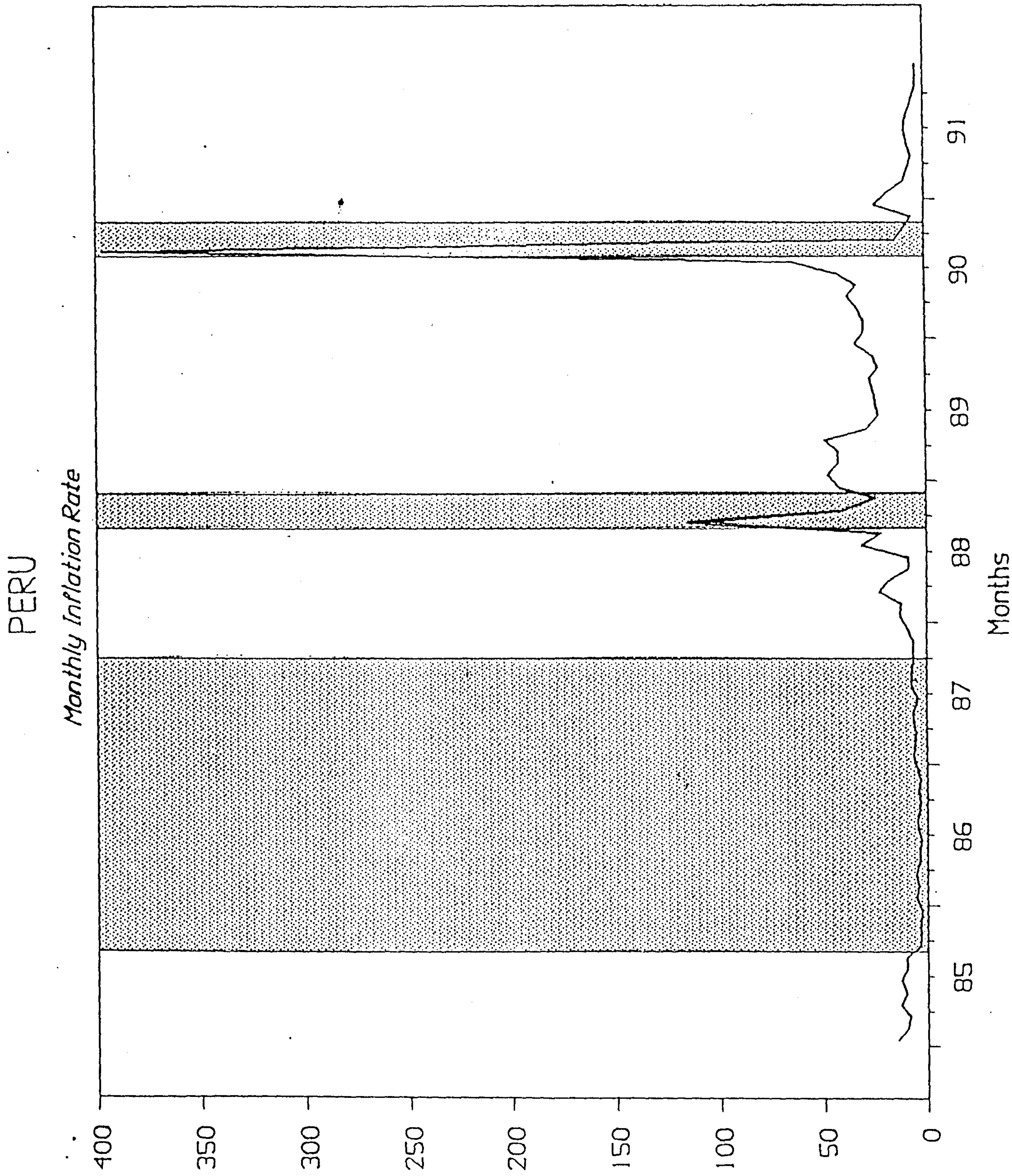
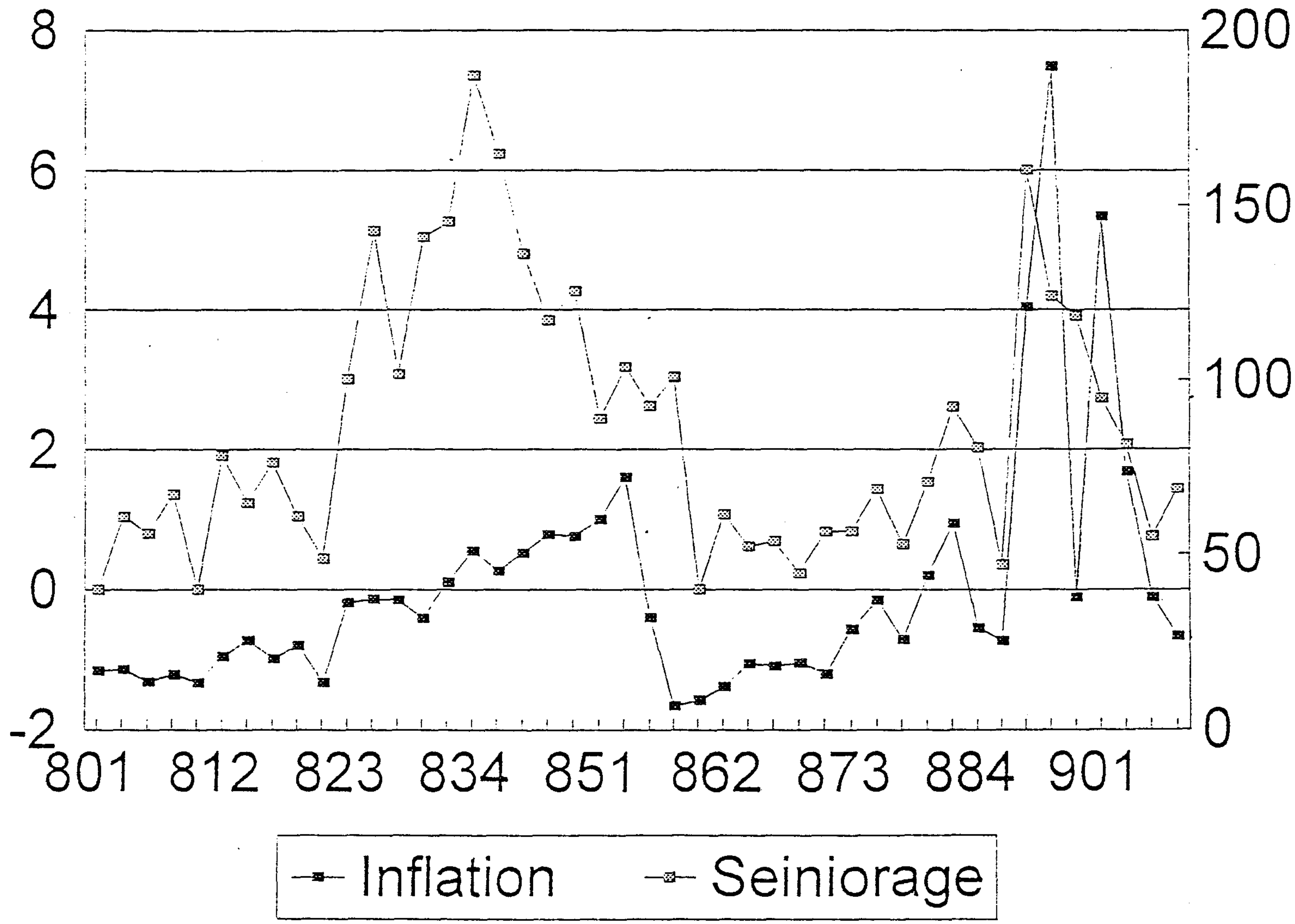

PERU
Monthly Inflation Rate
400
350
300
250
200
150
100
50
0
85
86
87
88
89
90
91
Months

FIGURE 5


8
6
4
2
0
-2
200
150
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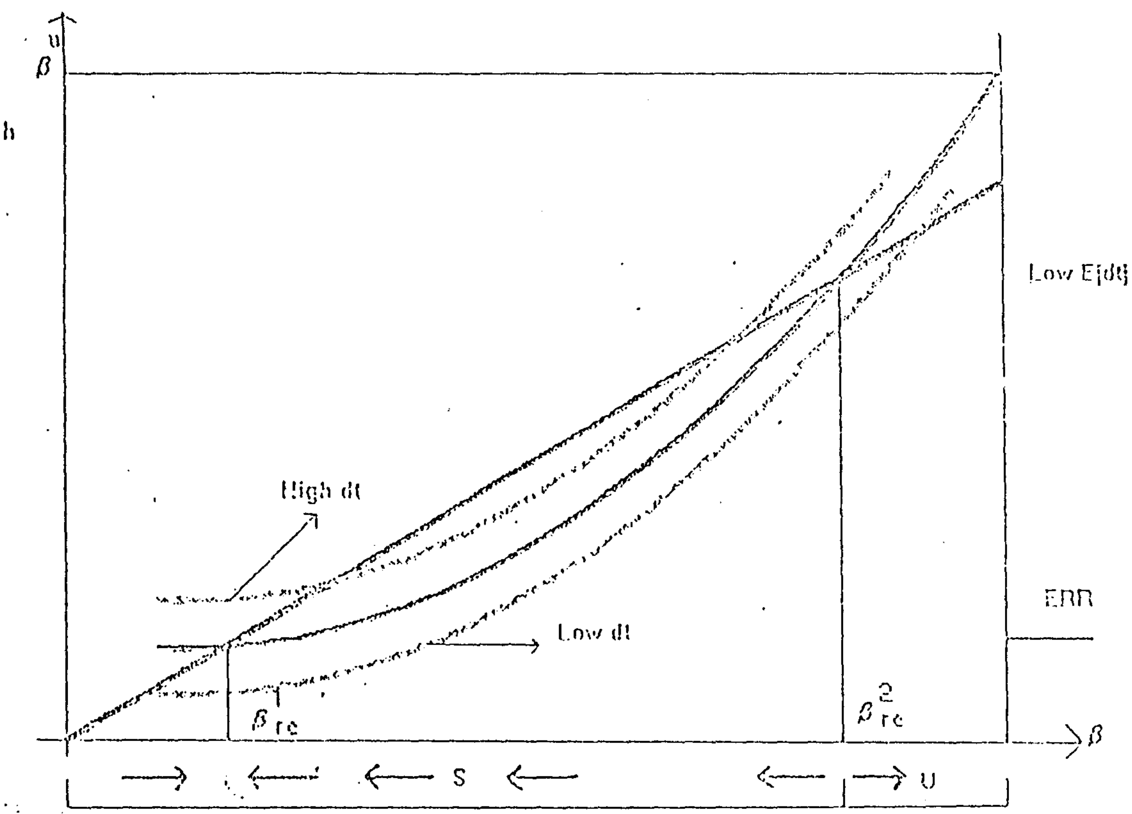
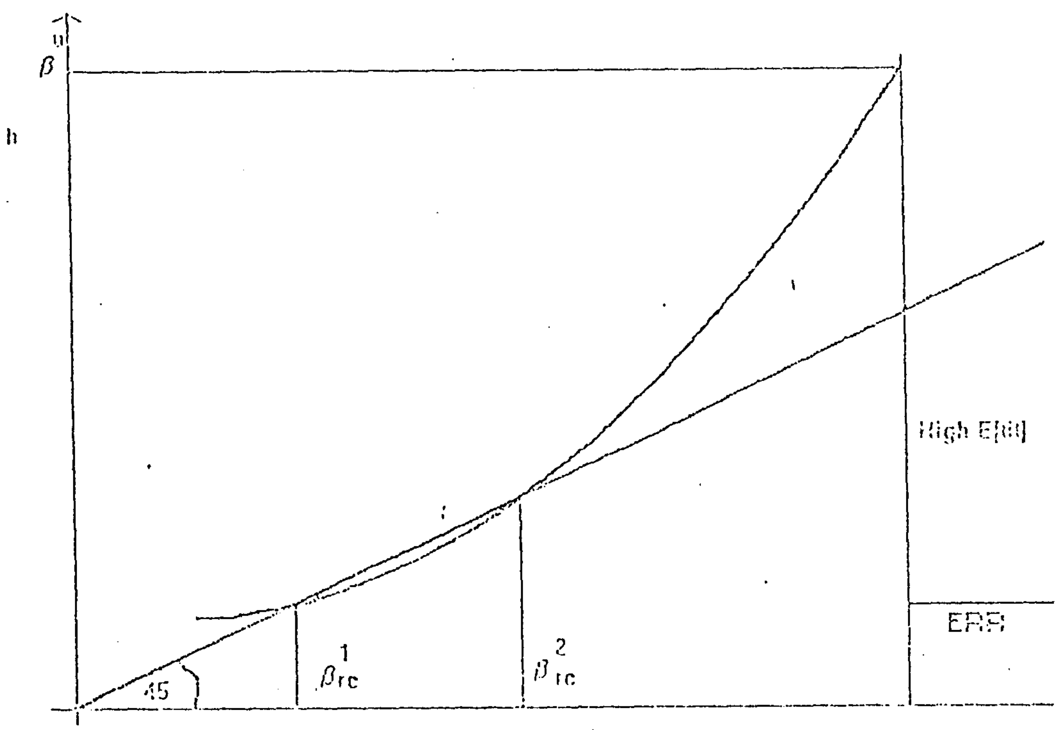
50
0
801
812
823
834
851
862
873
884
901
Inflation
Seiniorage

FIGURE 6



β^u
h
Low E[dt]
High dt
Low dt
ERR
β^1_{rc}
β^2_{rc}
β
S
U



β^u
h
High E[dt]
ERR
45
β^1_{rc}
β^2_{rc}
S
U

Figure 7

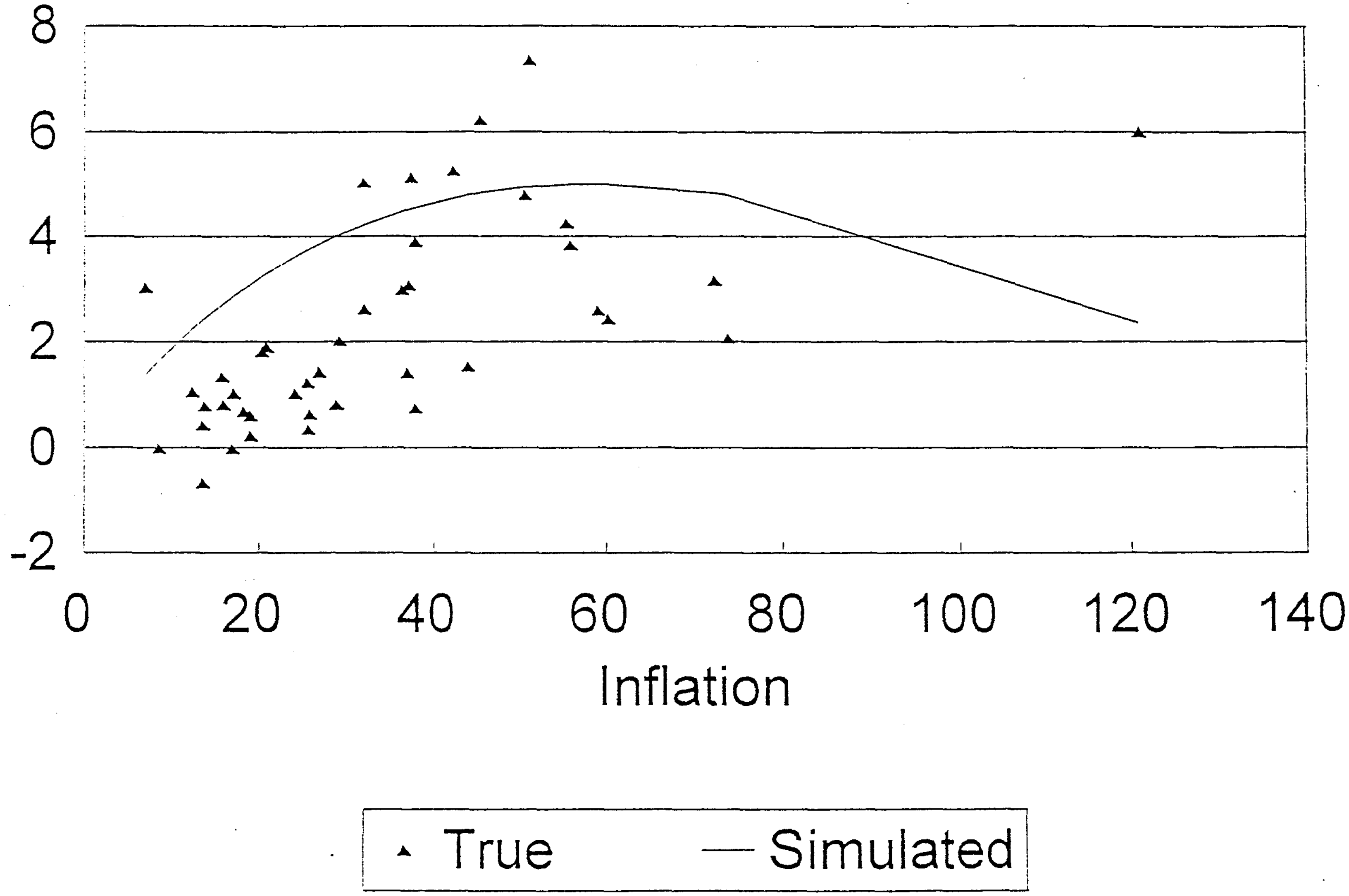
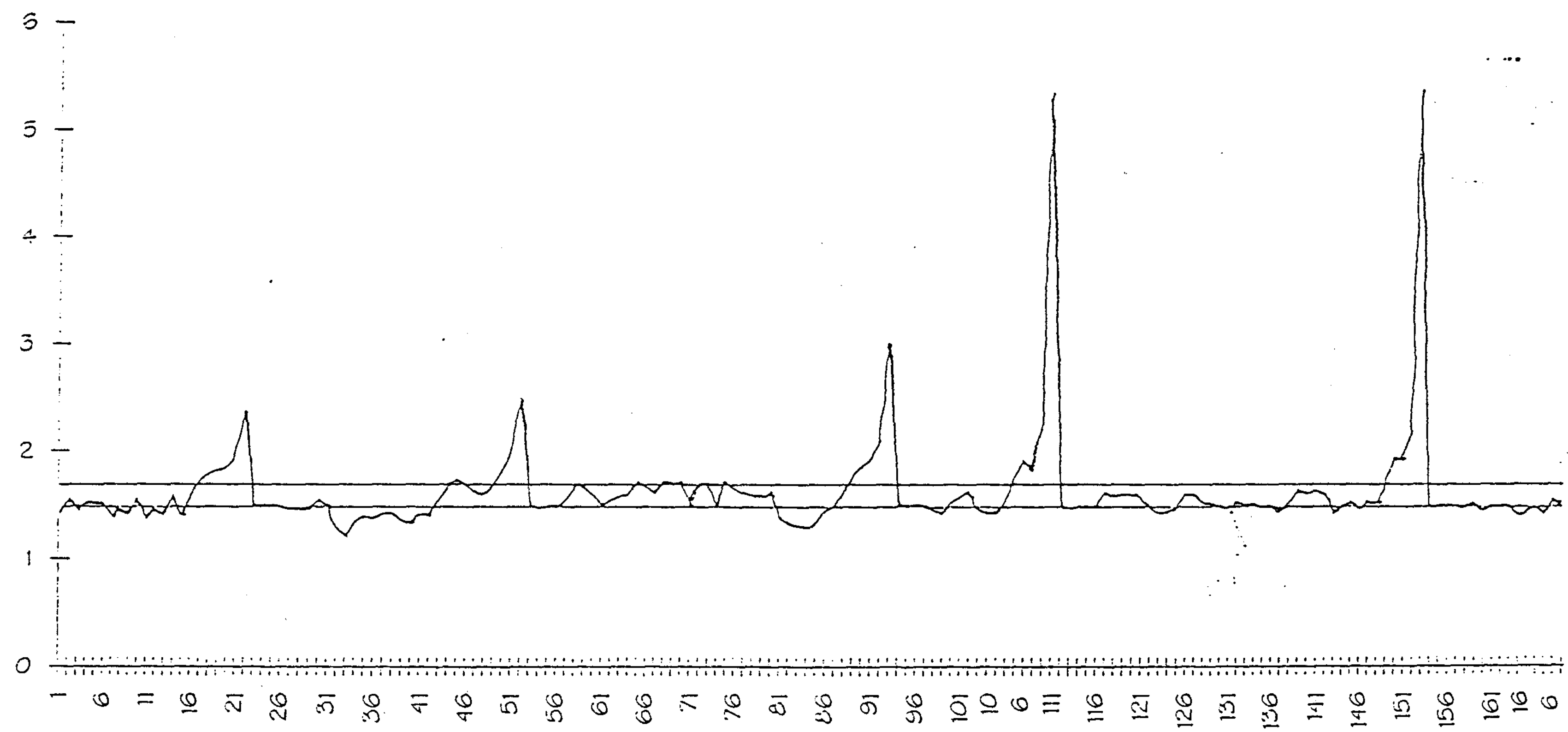


FIGURE 8



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6

Table 1

		1.2	1.1	1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0
	1.2	397.3315	67.3504	62.5505	91.5152	104.8549	150.7581	27,159	1.4806	0.5064	0.4222	0.4126	0.4159	0.4332
	1.1	393.5888	66.6049	62.3065	91.6847	104.6040	149.6575	27.339	1.4818	0.4999	0.4102	0.3966	0.3958	0.4087
	1	387.2765	65.3180	61.6237	90.6678	103.6917	146.5668	27.311	1.4833	0.4946	0.4000	0.3827	0.3785	0.3881
	0.9	378.0127	63.4524	60.4065	88.2307	101.9401	141.1047	26.980	1.4851	0.4901	0.3910	0.3704	0.3634	0.3702
Alternartive	0.8	365.4666	60.9808	58.5721	84.1878	99.1605	132.9421	26.240	1.4872	0.4860	0.3829	0.3594	0.3500	0.3547
	0.7	349.4583	57.9420	56.0829	78.4402	95.0608	121.8445	24.989	1.4898	0.4818	0.3752	0.3493	0.3379	0.3409
	0.6	330.0602	54.4418	52.9633	71.0054	89.5549	107.7698	23.148	1.4929	0.4774	0.3678	0.3399	0.3269	0.3286
	0.5	307.7048	50.5788	49.3810	62.0689	82.5249	91.0392	20.749	1.4968	0.4722	0.3604	0.3310	0.3167	0.3176
	0.4	282.4528	46.5261	45.4751	52.0354	74.0188	72.6269	17.975	1.5018	0.4661	0.3527	0.3224	0.3072	0.3075
	0.3	257.8454	42.6611	41.7964	41.3795	64.2910	54.1260	15.234	1.5080	0.4589	0.3445	0.3139	0.2982	0.2984
	0.2	234.3314	39.4299	38.3427	32.0172	53.9364	40.2515	13.377	1.5161	0.4506	0.3355	0.3057	0.2899	0.2900
	0.1	214.4042	37.4498	34.8755	26.1373	43.8559	34.1691	12.650	1.5266	0.4413	0.3259	0.2978	0.2823	0.2822
	0	3.4069	37.7642	30.7061	31.8042	35.8994	41.8663	12.383	1.5401	0.4317	0.3170	0.2930	0.2791	0.2760

Deficit= 4.3%, st. dev. =0.005 & n= 120

Table 2

		Prob. of Hiperinflations		
		0	1	2 or more
Alpha	0.1	100%	0%	0%
	0	100%	0%	0%

Table 3

		1.2	1.1	1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0
	1.2	52.1257	236.9282	40.0857	42.1290	28.8243	7.6121	2.3029	1.1013	0.7586	0.4643	0.4027	0.4073	0.4258
	1.1	51.9800	230.6242	39.1320	39.6452	28.7532	7.5966	2.3334	1.1017	0.755	0.4548	0.3874	0.3878	0.4018
	1	51.7339	222.1368	37.8044	36.4787	28.3419	7.5623	2.3452	1.1026	0.7532	0.4468	0.3742	0.3710	0.3814
	0.9	51.3543	211.3050	36.1278	32.6810	27.5205	7.4379	2.3427	1.1037	0.7515	0.4400	0.3626	0.3564	0.3640
Alternartive	0.8	50.8560	197.9533	34.1316	28.2481	26.2148	7.1996	2.3188	1.1050	0.7502	0.4339	0.3523	0.3434	0.3486
	0.7	50.0805	182.2076	31.8693	23.3960	24.3635	6.8198	2.2656	1.1062	0.7492	0.4284	0.3428	0.3316	0.3351
	0.6	48.9684	164.1133	29.3622	18.4047	21.8863	6.2891	2.1794	1.1071	0.7482	0.4232	0.3340	0.3209	0.3230
	0.5	47.4289	143.8367	26.8475	13.8612	18.8006	5.6043	2.0588	1.1076	0.7472	0.4183	0.3257	0.3110	0.3122
	0.4	45.5149	121.5240	24.4650	10.8556	15.2616	4.7847	1.9139	1.1079	0.7462	0.4134	0.3177	0.3018	0.3023
	0.3	42.9003	97.7183	22.5229	10.5789	11.5885	3.8975	1.7727	1.1083	0.7454	0.4086	0.3100	0.2931	0.2933
	0.2	39.6324	74.1336	21.0678	12.3188	8.2635	3.0663	1.6806	1.1095	0.7450	0.4040	0.3027	0.2850	0.2850
	0.1	35.9737	54.7830	19.7967	14.1271	5.9704	2.5271	1.6624	1.1119	0.7458	0.4003	0.2964	0.2779	0.2774
	0	32.3859	52.9770	18.2174	14.5542	5.1640	2.2991	1.6744	1.1172	0.7501	0.4004	0.2953	0.2769	0.2714

Deficit= 4.5%, st. dev. =0.01 & n= 120

Table 4

		Prob. of Hiperinflations						
		0	1	2	3	4	5	6 or more
	0.5	16%	34%	28%	16%	5%	1%	0%
	0.4	55%	34%	9%	1%	0%	0%	0%
Alpha	0.3	90%	10%	0%	0%	0%	0%	0%
	0.2	99%	1%	0%	0%	0%	0%	0%
	0.1	100%	0%	0%	0%	0%	0%	0%
	0	100%	0%	0%	0%	0%	0%	0%

Table 5

		1.2	1.1	1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0
	1.2	214.5975	53.9121	307.7863	19.3317	17.0984	5.2139	2.1089	1.5525	1.1587	0.7484	0.5782	0.4034	0.4223
	1.1	210.7428	53.6996	310.0670	19.1759	17.1172	5.1780	2.1120	1.5538	1.1578	0.7438	0.5685	0.3843	0.3984
	1	204.9639	53.1771	310.3358	18.8630	17.0033	5.0906	2.1155	1.5556	1.1576	0.7401	0.5606	0.3680	0.3783
	0.9	197.0979	52.2864	307.2750	18.3517	16.7381	4.9365	2.1189	1.5580	1.1578	0.7372	0.5538	0.3538	0.3610
Alternartive	0.8	186.9736	50.9650	300.7709	17.5863	16.2545	4.7028	2.1221	1.5607	1.1587	0.7349	0.5480	0.3411	0.3458
	0.7	174.4743	49.2026	290.5053	16.5220	15.5111	4.3800	2.1249	1.5638	1.1600	0.7332	0.5429	0.3297	0.3324
	0.6	159.4845	46.9890	276.3116	15.1235	14.4691	3.9683	2.1272	1.5674	1.1618	0.7321	0.5383	0.3193	0.3204
	0.5	141.9837	44.3115	257.6812	13.3997	13.0893	3.4907	2.1293	1.5713	1.1642	0.7315	0.5344	0.3096	0.3096
	0.4	122.0231	41.2591	235.7340	11.4893	11.4489	2.9983	2.1313	1.5759	1.1674	0.7316	0.5310	0.3006	0.2998
	0.3	99.8790	37.8966	211.0646	9.5721	9.7199	2.6235	2.1342	1.5812	1.1718	0.7328	0.5283	0.2922	0.2909
	0.2	76.4313	34.2836	185.0648	7.9853	8.1444	2.4585	2.1384	1.5878	1.1778	0.7357	0.5268	0.2846	0.2827
	0.1	54.6647	30.4820	159.5901	6.9787	6.8619	2.4340	2.1440	1.5959	1.1865	0.7422	0.5280	0.2782	0.2752
	0	45.1459	26.9213	137.7320	6.3051	5.5312	2.4173	2.1491	1.6061	1.2005	0.7588	0.5419	0.2830	0.2692

Deficit= 4.7%, st. dev. =0.01 & n= 120

Table 6

		Prob. of Hiperinflations						
		0	1	2	3	4	5	6 or more
	0.4	9%	26%	30%	22%	7%	3%	3%
	0.3	45%	37%	15%	3%	0%	0%	0%
Alpha	0.2	82%	14%	4%	0%	0%	0%	0%
	0.1	100%	0%	0%	0%	0%	0%	0%
	0	100%	0%	0%	0%	0%	0%	0%

Table 7

		1.2	1.1	1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0
	1.2	142.5925	63.8465	15.2672	10.2173	12.5694	7.5765	2.4780	2.1444	1.8291	1.4010	0.9838	0.6198	0.4267
	1.1	138.2079	60.7906	15.2897	10.0161	12.8318	7.5885	2.4814	2.1467	1.8300	1.3998	0.9794	0.6076	0.4026
	1	132.6909	57.1160	15.2319	9.7286	13.0415	7.6294	2.4852	2.1494	1.8315	1.3993	0.9761	0.5975	0.3823
	0.9	126.0383	52.8342	15.0712	9.3357	13.1774	7.5323	2.4894	2.1528	1.8335	1.3995	0.9735	0.5891	0.3647
Alternartive	0.8	118.2478	47.9836	14.7827	8.8189	13.1931	7.3226	2.4939	2.1566	1.8361	1.4003	0.9716	0.5819	0.3494
	0.7	109.3231	42.6532	14.3411	8.1681	13.0355	6.9740	2.4988	2.1610	1.8394	1.4016	0.9704	0.5757	0.3358
	0.6	99.2720	37.0072	13.7223	7.3892	12.6467	6.4626	2.5039	2.1661	1.8434	1.4037	0.9699	0.5704	0.3237
	0.5	88.1110	31.4845	12.9113	6.5077	11.9728	5.7756	2.5094	2.1720	1.8483	1.4068	0.9702	0.5661	0.3128
	0.4	75.8690	26.8647	11.9054	5.6185	10.9797	4.9344	2.5152	2.1788	1.8545	1.4110	0.9715	0.5625	0.3029
	0.3	62.6062	24.5814	10.7275	4.8504	9.7311	4.0266	2.5210	2.1866	1.8623	1.4171	0.9744	0.5601	0.2939
	0.2	48.4346	26.2007	9.4343	4.4089	8.3862	3.2897	2.5268	2.1953	1.8721	1.4259	0.9799	0.5594	0.2857
	0.1	33.6136	31.9293	8.1072	4.3423	7.2036	2.8797	2.5318	2.2046	1.8844	1.4392	0.9912	0.5627	0.2780
	0	19.2097	40.7016	6.9339	4.4096	6.1868	2.7441	2.5336	2.2121	1.8988	1.4609	1.0217	0.5959	0.2723

Deficit= 4.9%, st. dev. =0.01 & n= 120

Table 8

		Prob. of Hiperinflations						
		0	1	2	3	4	5	6 or more
Alpha	0.2	23%	40%	27%	9%	2%	0%	0%
	0.1	73%	26%	1%	0%	0%	0%	0%
	0	100%	0%	0%	0%	0%	0%	0%