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Optimal Carbon Offsets with Heterogeneous Regions

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Abstract

We study optimal climate policy in a global economy where regions differ in wealth and vulnerability to climate change. Carbon emissions from production generate output losses - a negative climate externality - and a technology to absorb and offset these emissions is available to all regions. We investigate how inequality shapes the stance of the global climate policy and the schedule of *net emissions* across regions: emissions net of carbon offsets. We provide an aggregation result that shows that the model with regional heterogeneity can be cast into a representative region world economy with a different discount factor and damage function elasticity to net emissions. We use this result to show that (i) Requiring all regions to contribute equally to carbon offsets exacerbates inequality and, therefore, efficiency calls for a less aggressive climate policy with more emissions and less carbon offsetting than in a representative agent world; (ii) When carbon offsets are allowed to depend on wealth, a more aggressive climate policy is optimal; (iii) Any global net emissions target prescribes positive net emissions for poor regions and negative net emissions for wealthy ones, with the burden on the rich increasing with inequality. These results highlight that carbon offsets play a crucial role in designing global climate policy because they act as a redistribution tool across unequal regions.

Keywords: Optimal Policy; Climate Change; Inequality; Net Emissions; Heterogeneous Agents.

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1 Introduction

Climate change is a global externality problem. The wide consensus around this claim is perhaps the reason why the Paris Agreement set its long-term goals in terms of global objectives (e.g., 1.5º C limit and Net-zero by 2050). Climate change is also, however, a global inequality problem. It affects regions differently, benefiting some and harming others, generating climate-related inequality. In addition, a recurrent difficulty in reaching binding global agreements relies on the historical contribution to the climate problem. The goal of this paper is to study how a climate policy that sets global goals must account for differences across countries both in terms of their wealth and in terms of their vulnerability to climate change. To this end, we explicitly consider atmospheric carbon capture as an activity with global effects that must be financed with the contributions of all countries. This introduces a crucial dimension in the design of climate policy since it must not only set a global goal but also determine which regions must bear the responsibility of cleaning the atmosphere by financing global carbon offsets. Our main interest is in how the stance of climate policy and the schedule of net emissions – emissions net of carbon offsets – respond to differences across countries.

We lay out a neoclassical model with heterogeneous regions and incomplete markets, where carbon emissions from production generate output losses - a negative climate externality - and a technology to absorb and offset these emissions is available to all regions ("carbon capture").[1] In the model, inequality stems from economic and climate factors; regions differ in their initial wealth and in the magnitude of their climate-related output losses. We characterize climate policies by solving the problem of a utilitarian planner that assigns equal weights to all regions and avoids direct transfers of resources across regions,

[1]In the paper, we indistinctly use the words carbon offsets, carbon sequestration, and carbon capture. These terms mean any action that naturally or technologically removes carbon from the atmosphere. According to IPCC (2022), deploying carbon removal to counterbalance hard-to-abate residual emissions is unavoidable to achieve net zero $CO2$ emissions, given the escalating levels of atmospheric carbon. Thus, carbon offsets are crucial in limiting global temperature increases and meeting the Paris Agreement targets.

much in the spirit of the constrained-efficiency literature proposed in Diamond (1967) and Dávila et al. (2012). In our model, however, some redistribution is still embedded in the choice of the schedule of carbon offsets because capturing carbon takes on productive resources. To limit this redistribution, we impose an additional constraint on the planner's problem consisting of a minimum contribution on the amount of carbon capture that she can ask from each region. When this minimum is set to zero, contributions to carbon capture are required to be non-negative, precluding direct transfer of resources among regions. As the minimum increases, the planner is forced to ask positive contributions from all regions in order to finance global carbon capture and, thus, the scope for redistribution is reduced.

The main result of our characterization of constrained-efficient climate policies is an aggregation result. In particular, we show that the solution to the planner's problem in an economy with heterogeneous regions coincides with the planner's solution in a representative region economy in which all the underlying heterogeneity is summarized in two key parameters: the discount factor and the elasticity of the damage function with respect to net emissions. This result is crucial because it lends tractability to the model and allows for a closed-form characterization of the constrained-efficient outcome. We use this aggregation result extensively to understand how the stance of global climate policy depends on inequality and, crucially, how the burden of cleaning up the atmosphere through carbon capture is distributed across regions.

Our analysis provides several important lessons. First, we show that the constrained efficient outcome requires wealthier regions to bear a larger responsibility for cleaning the atmosphere through carbon capture. Moreover, if wealth differences across countries is low enough, the global target in terms of net emissions coincides with that of a representative agent world. This implies that there is a complete separation between climate and redistribution motives in the design of the optimal policy. Low-income regions are not released from their climate action responsibility (cutting emissions and cleaning up the atmosphere), but wealthier regions bear a larger burden of financing carbon offset projects, implying a substantial redistribution of resources.

Second, as wealth differences increase, a tension appears between the motive for redistribution and that of curbing climate change. The planner's choice of carbon offset schedule allows the poorest regions to contribute the minimum towards the global goal of carbon capture. When this minimum is zero, this means that some regions get a "free pass" on global sequestration efforts. Surprisingly, however, in this case constrained-efficiency dictates not a less but a more stringent climate policy in terms of global objectives: more emissions cuts and more carbon capture. In fact, the global economy behaves as a representative agent world with a higher discount factor and a higher climate damage elasticity.

It is important to emphasize that the carbon offset schedule does not imply direct transfers of funds across regions. It is rather a climate financing tool with distributive effects. Our preferred interpretation is that, by taking responsibility for capturing carbon, high-income countries indirectly transfer room in the atmosphere for low-income countries to use during their slower transition to a carbon-free economy. That carbon capture can be a mechanism for addressing differential climate responsibilities and acts as a climate finance tool is a novel and fascinating insight from this paper.

We next show that, perhaps unfortunately, a more stringent climate policy and more redistribution do not always go hand-in-hand. It depends on the planner's constraints in choosing the carbon offset schedule across regions. In the limiting case where all regions must contribute uniformly to carbon capture, efficiency dictates a more lenient climate policy than in a representative agent world. Moreover, the constrained efficient policy compromises on both margins, dictating more emissions and less carbon capture. In this case, the global economy with heterogeneous regions behaves as a representative region world with a lower discount factor and a lower climate damage elasticity. This result provides a rationale for the idea that economic inequality can be a significant obstacle to attaining global climate goals.

Finally, we analyze the effect that differences in climate vulnerability has on the optimal policy design. We distinguish two cases: ex-ante heterogeneity regarding climate-related output losses, and ex-post uncertainty about climate-related shocks. Not surprisingly, climate inequality affects the choice of global net emissions target, but the direction of the effect will depend not only on its source, but also on the degree of economic inequality. In particular, we find that climate heterogeneity calls for more stringent climate policy when inequality is sufficiently high. In turn, climate uncertainty also calls for a more stringent policy, even if it involves more asset accumulation due to precautionary savings. These extra savings actually constitute a blessing because they make easier to finance carbon capture.

From a positive perspective, the results in this paper resemble some of the global climate goals that countries agreed upon in the Paris Agreement. One of the commitments is to reach global net zero emissions by the year 2050. Following the agreement, countries have been evaluated individually regarding their progress towards net zero (see Climate-Tracker. Nevertheless, whether it is optimal to attain the target country by country or in the aggregate remains an open question. In this paper, we show that the constrained-efficient policy in an economy with heterogeneous regions is to reach a homogeneous global net emissions target (i.e., "net-zero by 2050" from the Paris Agreement) with *net negative* emissions in high-income countries and *net positive* emissions in low-income ones.

In the rest of this section we connect the paper with previous literature. We then present the model in Section 2, the laissez-faire economy in Section 3, the constrained-efficient global net emissions in Section 4, and the main theoretical results in Section 5. Section 6 contains a numerical exercise and Section 7 offers some final remarks.

Related Literature This paper contributes to the literature that uses macroeconomic models to study climate policy, following the seminal work of Nordhaus and Boyer (2003). A large part of this literature builds on representative agent models (Golosov et al. (2014), Acemoglu et al. (2012), Barrage (2020), Belfiori (2017) among others), but recent contributions are introducing regional and household heterogeneity to climate economic models. An earlier contribution to this growing literature is Krusell and Smith (2022), who study the distribution of climate impacts around the world, accounting for heterogeneity in income and temperature increases across regions. We share with that paper that we build upon a standard neoclassical growth model augmented with a climate module, and we feature regional heterogeneity in economic and climate outcomes. We differ from them in that we study the optimal climate policy of such an unequal world, while the focus in Krusell and Smith (2022) is to quantify the climate impacts.

The paper more closely related to this one is Hillebrand and Hillebrand (2019), who study the optimal climate policy of a global economy with multiple regions. The focus in Hillebrand and Hillebrand (2019) is on characterizing an optimal climate policy, composed of taxes and transfers, that implements the optimal regional emissions and is also incentive-compatible with the laissez-faire equilibrium. In contrast, this paper studies how alternative sources of inequality (stemming from economic and climate factors) across regions affect the optimal allocation and the stance of global climate policy, defined as a regional net emissions target. Importantly, we specifically rule out transfers across countries as the ones characterized by Hillebrand and Hillebrand (2019). Instead, we introduce a carbon capture technology to highlight its role as a redistribution mechanism across countries.

More broadly, this paper's emphasis on carbon sequestration as a variable critical to the optimal design of global climate policy is novel and differentiates it from previous literature.

This paper also relates to Jacobs and van der Ploeg (2019) for its contribution to optimal climate policy in heterogeneous agents economies. Jacobs and van der Ploeg (2019) studies when the optimal carbon tax differs from the Pigouvian formula to incorporate re-distributive motives. In contrast, this paper studies heterogeneity across regions, not individuals, and characterizes the optimal policy in terms of allocations (i.e., a net emissions target). Also, this paper analyzes how alternative sources of inequality affect the optimal outcome.

The focus on optimal policy design within a heterogeneous agent's neoclassical growth model with climate change is a central aspect of this paper. In this regard, the paper connects to previous general equilibrium literature (especially, Dávila et al. (2012), Park (2018), Aiyagari and McGrattan (1998), and pioneer work by Diamond (1967)). In addition, we feature a climate module in the model. Following this literature, we characterize the optimal policy by looking for the constrained-efficient allocation, which is the best the planner can do, constrained by the fact that she can not overcome the existing inequality and the market incompleteness.

Finally, for its focus on optimal policy, this paper differs from existing literature that works with heterogeneous agents climate-economy models aiming to quantify climate change's consequences. Significant contributions to this literature are Fried et al. (2018) and Fried (2021), with whom we share the neoclassical growth model with heterogeneous agents framework, and Cruz and Rossi-Hansberg (2023) and Conte et al. (2022) add climate heterogeneity into spatial economies. These papers consider the impacts of given carbon taxes while, in contrast, we look for the optimal climate policy. In addition, Fried et al. (2018) and Conte et al. (2022) more broadly belong to a rich literature that studies how the incidence of taxes depends on the government's use of carbon taxation revenue, with recent contributions by Goulder et al. (2019) and van der Ploeg et al. (2022) to this line of research.

2 The Model Economy

The world consists of a unit measure of regions, each inhabited by a representative household and a representative firm. There is a final consumption good produced with capital and labor. Households live for two periods, and all production occurs in the last period.

The use of capital, $\mathbf{K}$, in production releases carbon to the atmosphere, S. A technology, $\mathbf{M}$, that enables the removal of atmospheric carbon through carbon offsets is available to all regions. The amount of carbon in the atmosphere evolves as follows

$$S = S_0 + \Pi(\mathbf{K}, \mathbf{M}), \tag{1}$$

where $\mathbf{S}_0$ is exogenously given, and bold capital letters denote global aggregates. We assume $\partial\Pi/\partial\mathbf{K} > 0$ and $\partial\Pi/\partial\mathbf{M} < 0$.

In the first period, regions differ in their initial wealth, y. Each region's representative household consumes and saves in a risk-free, one-period asset, a. In the second period, the regional-representative firm combines capital and labor to produce the final good according to a constant returns to scale technology. The presence of carbon in the atmosphere creates a negative externality that results in an output loss in the final good sector, which varies across regions, thereby introducing an additional source of inequality into the model. We use z to denote this heterogeneity in regions' climate vulnerability and refer to it as *climate inequality*. Further, regions are subject to idiosyncratic climate shocks. We use ν to denote this source of uncertainty and refer to it as *climate uncertainty*. Thus, the output loss in each region is determined by its total *climate vulnerability*, $\epsilon = z + \nu$. Occasionally, we will abuse notation by using ϵ to denote also the vector $[z, \nu]$; when we do, the appropriate interpretation is always clear from the context.

Households have preferences over consumption given by the utility function

$$u(c_0) + \beta\mathbb{E}\left[u(c_1)\right], \tag{2}$$

where the expectation is taken with respect to the regional climate shock ν. Households are endowed with one unit of labor, which they supply inelastically. Labor markets operate at a regional level - i.e., no migration -, and there is an international asset market. These assumptions imply that households face labor income uncertainty, $w(\epsilon)$, while the return to savings is deterministic as there is no aggregate uncertainty.

The technology to produce carbon offsets uses only capital in a linear fashion and, thus, we use $\mathbf{M}$ to denote both global investment and production. Investment in carbon offsets occurs in the first period and it is financed in the global capital market. This implies that the final consumption good and the carbon offsets compete for global savings as alternative uses of funds. In the second period, regions purchase carbon offsets at the market price q. Under perfect competition, it must hold that $q = R$. Without loss of generality, we assume the decision to purchase carbon offsets is on the household's side. We use $m(y, \epsilon)$ to denote the carbon offset purchased by each region.

Households decide how much to consume and save subject to the following set of budget constraints:

$$c_0 + a \leq y, \tag{3}$$

$$c_1 \leq w(\epsilon) + Ra - qm \tag{4}$$

The problem of each household is to maximize (2), subject to (3) and (4).

In each region, the representative firm solves

$$\max_{K,L} (1 - D(\mathbf{S}, \epsilon)) F(K, L) - w(\epsilon) L - RK, \tag{5}$$

taking factor prices and the amount of carbon in the atmosphere as given. The function $D(\mathbf{S}, \epsilon)$ is the climate damage function. We assume that $\partial D / \partial \mathbf{S} > 0$ and $\partial D / \partial \epsilon > 0$. Firms' optimal behavior implies:

$$
\begin{aligned}
(1 - D(\mathbf{S}, \epsilon)) F_L(K(\epsilon), L) &= w(\epsilon), & (6)\\
(1 - D(\mathbf{S}, \epsilon)) F_K(K(\epsilon), L) &= R, & (7)
\end{aligned}
$$

for all ϵ.

Competitive Equilibrium. We use G to denote the cross-sectional wealth distribution, with density $g(y)$; and H to denote the cross-sectional distribution of exposure across regions, with density $h(\epsilon)$. The function H is generated by the cross-sectional distributions of z and ν, which we denote Φ and Ψ, respectively, with corresponding densities $\phi(z)$ and $\psi(\nu)$. In the first period, market clearing for the asset market is

$$
\int \int a(y, z) g(y) \phi(z) \, \mathrm{d}y \, \mathrm{d}z = \int K(\epsilon) h(\epsilon) \, \mathrm{d}\epsilon + \mathbf{M}. \tag{8}
$$

In the second period, labor markets clear if

$$
L(\epsilon) = 1, \tag{9}
$$

for all ϵ. Market clearing for the carbon offsets is

$$
\int \int m(y, \epsilon) g(y) h(\epsilon) \, \mathrm{d}y \, \mathrm{d}\epsilon = \mathbf{M}, \tag{10}
$$

where the right-hand side is the global amount of carbon removed from the atmosphere, and the left-hand side is the sum of all contributions across regions.

Walras' Law implies that if (8), (9), and (10) are satisfied, then final good market also clears in both periods. For completeness we list here the final good market clearing conditions:

$$
\begin{aligned}
\int \int c_0(y, z) g(y) \phi(z) \, \mathrm{d}y \, \mathrm{d}z &= \int y g(y) \, \mathrm{d}y - \int K(\epsilon) h(\epsilon) \, \mathrm{d}\epsilon - \mathbf{M}, & (11)\\
\int \int c_1(y, \epsilon) g(y) h(\epsilon) \, \mathrm{d}y \, \mathrm{d}\epsilon &= \int (1 - D(\mathbf{S}, \epsilon)) F(K(\epsilon), 1) h(\epsilon) \, \mathrm{d}\epsilon. & (12)\\
& & (13)
\end{aligned}
$$

A *Competitive Equilibrium* consists of households' decision rules $c_0(y,z)$, $a(y,z)$, $c_1(y,\epsilon)$, and $m(y,\epsilon)$; firms' production plan $K(\epsilon)$, $L(\epsilon)$ and $\mathbf{M}$; and prices $w(\epsilon)$ and R such that policies solve individual agents' problems taking prices as given and all markets clear.

Specialization In the following sections, we characterize the equilibrium without intervention and the solution to the planner's problem in the global economy. We provide a closed-form characterization of these allocations building on the following set of assumptions:

1. Preferences: $U(c) = \ln(c)$.

2. Production Function: $F(K,L) = K^{\alpha}L^{1-\alpha}$, $\alpha \in (0,1)$.

3. Damage Function: $D(\mathbf{S},\epsilon) = 1 - \exp(-\gamma(\mathbf{S}+\epsilon))$, with $\gamma > 0$.

4. Carbon Cycle: $S = S_0 + \Pi(\mathbf{K},\mathbf{M}) = \xi_d\mathbf{K} - \xi_g\mathbf{M}$, with $\xi_d > 0$ and $\xi_g > 0$.

5. z and v are independent and normally distributed with standard deviation σ_z and σ_v, and mean μ_z and μ_v, respectively.

6. y is log-normally distributed with mean equal to one and (normal) standard deviation σ_y, and uncorrelated with z and v.

Further, we impose the following condition on parameters.

Assumption 1 $\beta\gamma(\xi_g + \xi_d) > 1 + \beta\alpha + \frac{\xi_d}{\xi_g}$

This assumption is a necessary condition for an interior solution of optimal carbon offsets in a representative region world. In the margin, diverting one unit of capital from the final good sector into carbon capture allows to increase offsets by $\frac{\xi_d}{\xi_g}$ and results in $\gamma(\xi_g + xi_d)$ units of additional output in the second period. Such a decision entails an opportunity cost of one unit of consumption in the first period, and α units of additional output in the second period, since diverted resources could have been used as capital.

In the next section, we characterize the values of gross emissions and carbon offsets ($\xi_d \mathbf{K}$, $\xi_g \mathbf{M}$) in a laissez-faire equilibrium. Our interest throughout the paper is in the equilibrium values of $\mathbf{K}$ and $\mathbf{M}$, which define a global net emissions target (gross emissions net of carbon capture), and to what extent these values depend on the underlying sources of inequality.

3 The Laissez-faire Equilibrium

It is easy to see that carbon offsets are zero in a laissez-faire equilibrium, $\mathbf{M} = 0$. It will prove convenient to reduce the equilibrium to the following three objects: the amount of productive capital $\mathbf{K}$, the distribution of assets holdings across households η, and the distribution of productive capital across regions χ. Productive capital in each region is recovered as $K(\epsilon) = \chi(\epsilon)\mathbf{K}$, household assets as $a(y,z) = \eta(y,z)\mathbf{K}$, and consumption plans from individual budget constraints. We must require $\chi(\epsilon) \geq 0$ for all ϵ, and

$$\int \chi(\epsilon)\, \mathrm{d}\epsilon = 1, \tag{14}$$

$$\int \eta(y,z)\, \mathrm{d}y\, \mathrm{d}z = 1. \tag{15}$$

We characterize the laissez-faire equilibrium using this notation. The following lemma characterizes the distribution of capital and asset holdings. The proof is in Appendix B.

Lemma 1 *In a laissez-faire equilibrium, the distribution of productive capital across regions is given by*

$$\chi(\epsilon) = \exp\left\{-\frac{\gamma}{1-\alpha}\left(\epsilon - \mu_\epsilon + \frac{\gamma}{1-\alpha}\frac{\sigma_\epsilon^2}{2}\right)\right\} \tag{16}$$

for all ϵ, and the distribution of asset holdings across households satisfies

$$\beta\alpha\mathbb{E}\left[\frac{y-\eta(y,z)\mathbf{K}}{(1-\alpha)\chi(\epsilon)\mathbf{K}+\alpha\eta(y,z)\mathbf{K}}\right]=1, \tag{17}$$

for all y and ϵ, where the expectation is taken with respect to the climate shock ν.

Notice that the distribution of productive capital depends only on the stochastic properties of the climate shock, ϵ. Thus, it will be the same in any equilibrium because it does not depend on the allocations. When there is no climate uncertainty, we can solve for the distribution of asset holdings in closed form and use it to obtain the following result.

Lemma 2 *Suppose $\sigma_\nu = 0$. Then, the global emissions in a laissez-faire equilibrium are equal to*

$$\mathbf{K}^{LF}=\frac{\alpha\beta}{1+\alpha\beta}. \tag{18}$$

The proof is in Appendix B. When regions face no climate uncertainty, the laissez-faire global emissions are independent of regions' heterogeneity. In this case, aggregate emissions coincide with those of a representative region economy.

When regions face uncertainty over the climate shocks, it is not possible to obtain an expression for global emissions in closed form but we can establish the following result:

Proposition 1 (Laissez-faire Economy) *Suppose $\sigma_\nu > 0$. Global emissions in a laissez-faire economy, $\mathbf{K}^{LF}$, are increasing in climate uncertainty.*

The intuition for this result is that people tend to save more as a precautionary measure when they face climate uncertainty. This leads to an increase in capital stock and, ultimately, more emissions. It follows that countries' inequality leads to higher global emissions, and exacerbates climate change. The proof of this proposition is in Appendix B.

4 Constrained-Efficient Net Emissions

We investigate what combination of gross emissions, $\mathbf{K}$, and carbon offsets, $\mathbf{M}$, maximizes social welfare from a utilitarian perspective, and how it depends on the underlying heterogeneity. To this end, we focus on constrained efficient allocations: the best the planner can do when she cannot overcome the constraints on private choices imposed by markets. All she can do is to command a different choice to either households or firms, while respecting individual constraint sets and market clearing conditions. Still, two features of this problem make the planner capable of improving on the market allocation. First, she knows that her choices affect equilibrium prices. Second, she is aware of the climate externalities.

4.1 Efficiency and Limits to Redistribution

In this economy, there is a tension between the need to achieve efficiency by fixing the climate externality and considering the existing inequality. To navigate this tension, we follow Dávila et al. (2012) in setting up a utilitarian planner who assigns equal weights to regional welfare and there are no direct transfers of resources across regions available. The focus of global climate policy is, thus, on efficiency.

In our model, however, some redistribution is still embedded in the choice of the carbon offsets schedule because capturing carbon takes on productive resources. To limit this redistribution, we impose an additional constraint on the planner's problem consisting of a minimum contribution on the amount of carbon capture that she can ask from each region. In addition, we preclude the possibility of making the contributions contingent on climate vulnerability. In practice, this is justified on the basis of being a variable difficult to measure. Thus, the following lower bound condition on differential contributions across regions must hold:

$$m(y) \geq \underline{m}, \tag{19}$$

for each y, with $\underline{m} \in [0, \mathbf{M}]$. When this minimum is set to zero, contributions to carbon capture are required to be non-negative, precluding direct transfer of resources among regions. Some countries may efficiently not contribute to removing carbon from the atmosphere, but no country receives a net positive transfer of resources. As the minimum $\underline{m}$ increases, positive contributions are required from all regions to finance the optimal amount of carbon offsets and, thus, the scope for redistribution is reduced.

We will restrict attention to contributions that are functions of initial wealth y.

Definition 1 *A constrained-efficient allocation solves the global planner´s problem, which is*

$$\max_{\substack{c_0(y,z),\, a(y,z),\\ c_1(y,z,\nu),\, m(y),\\ K(\epsilon),\, \mathbf{M}}} \int\int \left\{ u(c_0(y,z)) + \beta \mathbb{E}\left[u(c_1(y,z,\nu))\right] \right\} g(y), \phi(z)\, \mathrm{d}y\, \mathrm{d}z \tag{20}$$

subject to the carbon cycle equation (1), the budget constraints (3) and (4), the market clearing conditions (8)-(10), firm's optimality conditions (6) and (7), as well as the corresponding non-negativity constraints, and the lower bound condition (19).

It is useful to perform a change of variables to characterize the constrained efficient allocation in closed form. Specifically, we characterize the solution to the global social planner´s problem in terms of the following five objects: the stock of capital $\mathbf{K}$, the global amount of carbon offsets $\mathbf{M}$, the distribution of capital across regions χ, the distribution of asset holdings across households η, and the distribution of the financing cost of carbon offsets across regions μ. This last object allows us to recover each region´s contribution to carbon offsetting: $m(y) = \mu(y)\mathbf{M}$. Thus, region y is responsible for financing a fraction

$\mu(y)$ of the global amount of carbon offsets, $\mathbf{M}$. Hence, the region´s contribution to carbon capture is $m(y)$. Consistent with the lower-bound condition (19), we require $\mu(y) \geq 0$ and $\int \mu(y)\,\mathrm{d}y = 1$. A closed form characterization of $\mu(y)$ for all y is in Appendix C.

Using this change of variables, the first order conditions with respect to capital and carbon offsets are:

$$
\begin{aligned}
1 &= \beta R \mathbb{E}_1 \left[\frac{u'(c_1(y,\epsilon))\eta(y,z)}{\mathbb{E}_0\left[u'(c_0(y,z))\eta(y,z)\right]} \right] - \Lambda^{\mathbf{K}}, & (21)\\
1 &\geq \Lambda^{\mathbf{M}} & (22)
\end{aligned}
$$

where the last expression holds with equality if $\mathbf{M} > 0$.

These expressions capture the costs and benefits of carbon emissions and carbon capture at the global level. In both cases, the left-hand side is the cost of capital in terms of the consumption good in the first period; one more unit of capital represents one unit of global consumption foregone. The right-hand side is the benefit of the additional unit net of the externality captured by $\Lambda^{\mathbf{K}}$ and $\Lambda^{\mathbf{M}}$, which can be written:

$$
\begin{aligned}
\Lambda^{\mathbf{K}} \equiv \beta \mathbb{E} \Bigg[\frac{u'(c_1)}{\mathbb{E}\left[u'(c_0)\eta\right]} \Bigg\{ & \frac{\partial R}{\partial \mathbf{K}} \big(\eta \mathbf{K} + (\eta - \mu)\mathbf{M}\big) + \frac{\partial w(\epsilon)}{\partial \mathbf{K}} \\
& + \frac{\partial R}{\partial \mathbf{S}} \frac{\partial \mathbf{S}}{\partial \mathbf{K}} \big(\eta \mathbf{K} + (\eta - \mu)\mathbf{M}\big) + \frac{\partial w(\epsilon)}{\partial \mathbf{S}} \frac{\partial \mathbf{S}}{\partial \mathbf{K}} \Bigg\} \Bigg], \qquad (23)
\end{aligned}
$$

$$
\Lambda^{\mathbf{M}} \equiv \beta \mathbb{E} \left[\frac{u'(c_1)}{\mathbb{E}\left[u'(c_0)\eta\right]} \left\{ \frac{\partial R}{\partial \mathbf{S}} \frac{\partial \mathbf{S}}{\partial \mathbf{M}} \big(\eta \mathbf{K} + (\eta - \mu)\mathbf{M}\big) + \frac{\partial w(\epsilon)}{\partial \mathbf{S}} \frac{\partial \mathbf{S}}{\partial \mathbf{M}} + (\eta - \mu) R \right\} \right]. \qquad (24)
$$

In the case of capital, the overall externality has two components: one associated with a pecuniary externality captured in the terms of the first line and another associated with the climate externality – the terms in the second line. Carbon offsets entail only a climate externality, but the marginal benefit is net of the distributional consequences, if any, of financing them. Specifically, when carbon capture is positive ($\mathbf{M} > 0$), the planner must decide who bears the burden of financing it through the choice of μ.

Importantly, both $\Lambda^{\mathbf{K}}$ and $\Lambda^{\mathbf{M}}$ depend on the underlying heterogeneity, as (23) and (24) make apparent. To highlight this, we exploit the firm's first-order conditions and use the functional form assumptions to rewrite the externality as follows

$$\begin{aligned}
\Lambda^{\mathbf{K}} &\equiv \beta\mathbb{E}\left[\frac{u'(c_1)}{\mathbb{E}\left[u'(c_0)\eta\right]}\left\{\gamma\xi_d c_1(y,\epsilon) + F_{L,K}(1-\chi(\epsilon))\right.\right. \\
&\qquad\qquad \left.\left. + F_{L,K}(\eta(y,z)-1)\frac{\mathbf{K}+\mathbf{M}}{\mathbf{K}} + F_{L,K}(1-\mu(y))\frac{\mathbf{M}}{\mathbf{K}}\right\}\right], \\
\Lambda^{\mathbf{M}} &\equiv \beta\mathbb{E}\left[\frac{u'(c_1)}{\mathbb{E}\left[u'(c_0)\eta\right]}\left\{\gamma\xi_g c_1(y,\epsilon) + (\eta(y,z)-1)R + (1-\mu(y))R\right\}\right].
\end{aligned}$$

Clearly, if the distributions χ, η, and μ were degenerate - as in a representative region economy - $\Lambda^{\mathbf{K}}$ and $\Lambda^{\mathbf{M}}$ only take into account the climate externality. In contrast, in an economy with inequality, the planner chooses optimal net emissions, $K - M$, considering not only the externality but also that her choice necessarily has distributional consequences for societal welfare.

These marginal externality effects capture the social cost of carbon, and climate policy aims at reducing such a cost. The previous expressions suggest that the constrained-efficient climate policy weighs three different concerns. First is efficiency, as there is an externality. Second is redistribution, as there is heterogeneity. Third is insurance, as the choice of climate policy affects the uncertainty faced by each region.

In the next section, we analyze how these three different concerns interact to shape the optimal net emissions of the global economy and its distribution across regions.

5 Analytical Results

5.1 A Representative Region Benchmark

To better grasp the role of inequality in shaping climate policy, it is useful to know what constrained efficiency prescribes in a representative agent economy. In our framework, this corresponds to the case in which the distributions of y and ϵ are degenerate at their mean values. The following result characterizes this benchmark case.

Proposition 2 (Climate Policy in a Representative Agent Economy) *Suppose that* $\sigma_y = \sigma_z = \sigma_\nu = 0$. *The constrained efficient solution of a representative region economy with elasticity of the damage function,* γ*, and discount factor,* β*, is*

$$\mathbf{K}^{RA} = \frac{\alpha}{\gamma(\xi_g + \xi_d)} \tag{25}$$

$$\mathbf{M}^{RA} = 1 - \frac{1 + \alpha\beta + \frac{\xi_d}{\xi_g}}{\beta\gamma(\xi_g + \xi_d)} \tag{26}$$

It is easy to see that Assumption 1 guarantees that $\mathbf{M} > 0$ and $\mathbf{K} < \mathbf{K}^{\text{LF}}$. A representative region incurs in both, emissions reductions (smaller $\mathbf{K}$) and carbon offsetting (larger $\mathbf{M}$) to curb climate change.

5.2 Climate Policy and Inequality

We now turn to study how the net emissions target changes as we introduce different sources of inequality. To this end, we characterize the constrained-efficient allocation as the solution to the global planner's problem in Definition 1. The main outcome of this characterization is an aggregation result. We show that, while heterogeneity certainly matters for climate policy, it is possible to characterize the constrained efficient policy by solving the planner's problem of a representative region world in which two key parameters embed all the underlying heterogeneity: the discount factor and the elasticity of the climate damage function. We state this formally in the following proposition, and we provide its proof in the appendix.

Proposition 3 (Aggregation Result) *Let $\{\mathbf{K}, \mathbf{M}\}$ be part of the solution to the global planner's problem in an economy with wealth and climate heterogeneity. Then, $\{\mathbf{K}, \mathbf{M}\}$ also solve the planner's problem of a representative region economy where the elasticity of the damage function, $\hat{\gamma}$, and the discount factor, $\hat{\beta}$, are given by*

$$
\begin{aligned}
\hat{\gamma} &= \frac{\alpha}{\alpha - \Omega_0(\mathbf{K}, \mathbf{M})}\gamma \\
\hat{\beta} &= \frac{\alpha - \Omega_0(\mathbf{K}, \mathbf{M})}{\alpha\Omega_1(\mathbf{K}, \mathbf{M})}\beta
\end{aligned}
$$

respectively, where

$$
\begin{aligned}
\Omega_0(\mathbf{K}, \mathbf{M}) &= \mathbb{E}\left[\frac{(1-\theta)\alpha(\eta(y,z) - \mu(y))}{(1-\alpha)(1-\theta)\chi(\epsilon) + \alpha(\eta(y,z) - \theta\mu(y))}\right], \\
\Omega_1(\mathbf{K}, \mathbf{M}) &= \mathbb{E}\left[\frac{\Lambda + \frac{\alpha\beta}{1-\theta}\mathbb{E}\left[\frac{(1-\theta)\mu(y)}{(1-\alpha)(1-\theta)\chi(\epsilon) + \alpha(\eta(y,z) - \theta\mu(y))}\right]}{\Lambda + \frac{\alpha\beta}{1-\theta}\mathbb{E}_y\left[\frac{1-\theta}{(1-\alpha)(1-\theta)\chi(\epsilon) + \alpha(\eta(y,z) - \theta\mu(y))}\right]}\right],
\end{aligned}
$$

$\theta = \mathbf{M}/(\mathbf{K} + \mathbf{M})$, *and Λ is the multiplier on the planner's constraint* $\int \eta(y,z)g(y)\phi(z)\,\mathrm{d}y\,\mathrm{d}z \geq 1$.

The proposition provides an explicit way to assess the stance of climate policy in a world with inequality relative to a representative region economy (Proposition 2). In order to do so, we need to know the sign of the function Ω_0 and whether Ω_1 is above or below one when evaluated at the constrained-efficient policy. For instance, if $\Omega_0 > 0$, the planner should act "as if" the elasticity of the climate damage function were larger and, therefore, pursue a more aggressive climate policy (more emission cuts and more carbon offsets). In turn, if $\Omega_1 > 1$, the planner should act "as if" the discount factor were lower and pursue a less aggressive climate policy.

To build intuition about the forces that shape climate policy, in what follows we consider some special cases activating one source of inequality at a time, and then analyzing the interaction between wealth and climate vulnerability.

Wealth Inequality. Consider first the case where there is wealth inequality but no climate inequality. The following result shows that if wealth differences across countries are low enough, the global net emissions coincide with that of a representative agent world. When all regions can contribute positively to carbon offsetting, the planner can equalize consumption in the second period across regions, making Ω_0 equal to zero and Ω_1 equal to one. Thus, we obtain the following result:

Corollary 1 *Suppose $\sigma_z = \sigma_\nu = 0$. Suppose also that condition (19) is not binding with $\underline{m} = 0$. Then $\hat{\gamma} = \gamma$ and $\hat{\beta} = \beta$. Also, $(K, M) = (K^{RA}, M^{RA})$.*

This corollary implies that there is separation between climate policy and inequality when wealth inequality across regions is not substantial. It means that low-income countries are rich enough not to be released from the responsibility of climate action (i.e., reducing emissions and capturing carbon in this economy). Constrained efficiency prescribes as much emissions and carbon offsets as in a representative agent economy and all regions must do some carbon offsetting. Wealthier regions bear a significant burden of financing these offset projects, which results in a substantial redistribution of resources.

The separation between climate policy and inequality breaks down as wealth differences across regions increase, and the poorest regions must get a free pass on climate responsibility. Although this suggests the appearance of an apparent tension between curbing climate change and sustaining regional equity, we obtain the following result:

Corollary 2 *Suppose $\sigma_z = \sigma_\nu = 0$. Suppose also that condition (19) is binding for some regions with $\underline{m} = 0$. Then $\hat{\gamma} > \gamma$ and $\hat{\beta} > \beta$. Also, $K < K^{RA}$ and $M > M^{RA}$*

In this case, the dispersion in second-period consumption among regions and Jensen's inequality imply that Ω_1 is below one. To determine the sign of Ω_0, it is useful to classify regions in two groups: contributing and non-contributing. According to this classification, while the within-group covariance between $\eta(y) - \mu(y)$ and marginal utility is negative, the between-group one is actually positive, as the conditional expectation of $\eta(y) - \mu(y)$ is smaller than zero for contributing regions. In Appendix B we establish that as long as aggregate savings are positive for non-contributing regions, the sign of the between-group covariance dominates and Ω_0 is positive.

The previous result shows that constrained efficiency, surprisingly, dictates not a less but a more stringent climate policy in this case: more emissions cuts and more carbon capture are optimal at the global level. The global economy behaves as a representative agent world with a lower discount rate and a higher climate damage elasticity - both parameters consistent with a more stringent climate policy.

Intuitively, when regions' climate responsibilities (in emission cuts and carbon offsets) are conditional on regions' income, carbon capture effectively acts as a redistribution tool at the global level and is optimally used as such. It is important to emphasize that the carbon offset schedule does not imply direct transfers of funds across regions. It is rather a climate financing tool with distributive effects. Our preferred interpretation is that, by taking responsibility for capturing carbon, high-income countries indirectly transfer room in the atmosphere for low-income countries to use during their slower transition to a carbon-free economy. This result provides a novel insight: carbon capture can be a mechanism for addressing differential climate responsibilities and act as a climate finance tool.

In practice, however, enforcement mechanisms to compel some regions to contribute more than others to the removal of atmospheric carbon may be difficult to implement. If these enforcement mechanisms are completely missing, contributions cannot be conditional on income. Hence, all contributions must be the same and the constraint (19) becomes:

$$m(y) = \mathbf{M}, \tag{27}$$

for every y. Imposing this constraint is akin to requiring uniform contributions from all regions. In this case, $\Omega_0 < 0$ and $\Omega_1 > 1$ and we obtain the following result:

Corollary 3 *Suppose $\sigma_z = \sigma_\nu = 0$. Suppose also that condition (27) holds. Then $\hat{\gamma} < \gamma$ and $\hat{\beta} < \beta$. Also, $K > K^{RA}$ and $M < M^{RA}$.*

In an economy with wealth inequality in which all regions must contribute homogeneously, constrained efficiency prescribes a less aggressive climate policy than in a representative agent world. The economy with heterogeneous regions behaves now as a representative region world but with a higher discount factor and a lower climate damage elasticity - both parameters consistent with a less stringent climate policy.

The intuition behind this result is that a more aggressive climate policy (emissions cuts and carbon offsets) exacerbates inequality in a heterogeneous region's economy, with no redistributive mechanism available to mitigate the policy's effect. This is reminiscent of Dávila et al. (2012), who show that when wealth inequality is the main determinant of consumption dispersion, the planner wants to reduce the relative importance of non-labor income, which requires an increase in productive capital (i.e., increase emissions in this economy) and inducing higher wages. In this economy, such compromise reduces the marginal effect of productive capital, and thus, it also requires a reduction of carbon offsets relative to the representative region benchmark.

When the limits to redistribution lie between constrained differential contributions (condition 19) and homogeneous contributions (condition 27), the optimal net emissions (emissions cuts and carbon offsets) also do. Overall, the results show that the conjunction of existing inequality and the availability of redistributive mechanisms across unequal regions certainly affect the scope of global climate policy. However, it is important to remark that taking into account existing inequality does not necessarily imply a less stringent global climate policy. Instead, it depends on the dispersion of the wealth distribution across regions.

Adding climate heterogeneity. Next, we consider the case in which wealth inequality and climate heterogeneity exist. When all regions contribute to carbon offsets, the only source of dispersion in second-period consumption comes from labor income due, precisely, to heterogeneity in climate vulnerability. To the extent that more vulnerable regions save more but consume less in the second period, η and c_1 are negatively correlated, and Ω_0 is positive. This mechanism carries through the case in which some regions contribute, and others do not. [2] Thus, we obtain the following result:

Corollary 4 *Suppose $\sigma_\nu = 0$. Then $\hat{\gamma} > \gamma$. Also, $K < K^{RA}$.*

Introducing climate heterogeneity breaks down aggregation, even when all regions contribute to carbon offsets. The reason is simple: differences in climate vulnerability translate into differences in labor income and, thus, consumption, which the planner cannot handle because the contributions are not contingent on climate vulnerability. Instead, the planner seeks additional redistribution by reducing the relative importance of labor income. This requires a reduction of productive capital and lower emissions. While it is not possible to characterize the response of carbon offsets analytically, in Section 6, we explore the impact of adding climate heterogeneity on optimal carbon offsets in a numerical example.

Adding climate uncertainty. Finally, we consider the case where regional differences stem from wealth inequality and climate uncertainty. The main difference with the previ-

[2] On one hand, the within-group covariance between $\eta - \mu$ and the marginal utility of consumption increases, as a component of and η now co-varies negatively with consumption. On the other hand, the between-group covariance is unaffected since the new source of variation in marginal utility across groups is unrelated to wealth.

ous two cases is that the precautionary motive to save translates into over-accumulation of capital relative to a representative agent economy. When all regions contribute to carbon offsets, the planner cannot undo consumption dispersion by choosing the carbon offset financing burden. However, any remaining dispersion is unrelated to wealth. This means that Ω_0 equals zero. It is easy to check that this implies Ω_1 equals one. Hence, we obtain the following result:

Corollary 5 *Suppose $\sigma_z = 0$. Suppose also that condition (19) is not binding with $\underline{m} = 0$. Then $\hat{\gamma} = \gamma$ and $\hat{\beta} = \beta$. Also, $(K, M) = (K^{RA}, M^{RA})$.*

This result suggests that capital accumulation due to precautionary savings does not necessarily entail a concern for climate goals. The global planner wants regions to save more to finance climate policy, and precautionary savings are a blessing as long as all regions contribute to carbon capture. When some regions do not contribute to finance carbon offsets (condition 19 binds for some y), determining analytically how Ω_0 and Ω_1 change is not possible. In the next section, we quantitatively explore the implication of adding climate uncertainty when some regions do not contribute to financing carbon offsets.

6 Numerical Example

In this section, we report the results of a simple exercise to illustrate the connection between climate policy and inequality. We first perform comparative statics with respect to wealth inequality as measured by σ_y, and the limits to redistribution faced by the planner captured in $\bar{\mu}$. In each case, we compare the net global emissions under laissez-faire to those corresponding to the constrained efficient outcome. In addition we report the schedule of net emissions across the wealth distribution.

6.1 Parameter choice

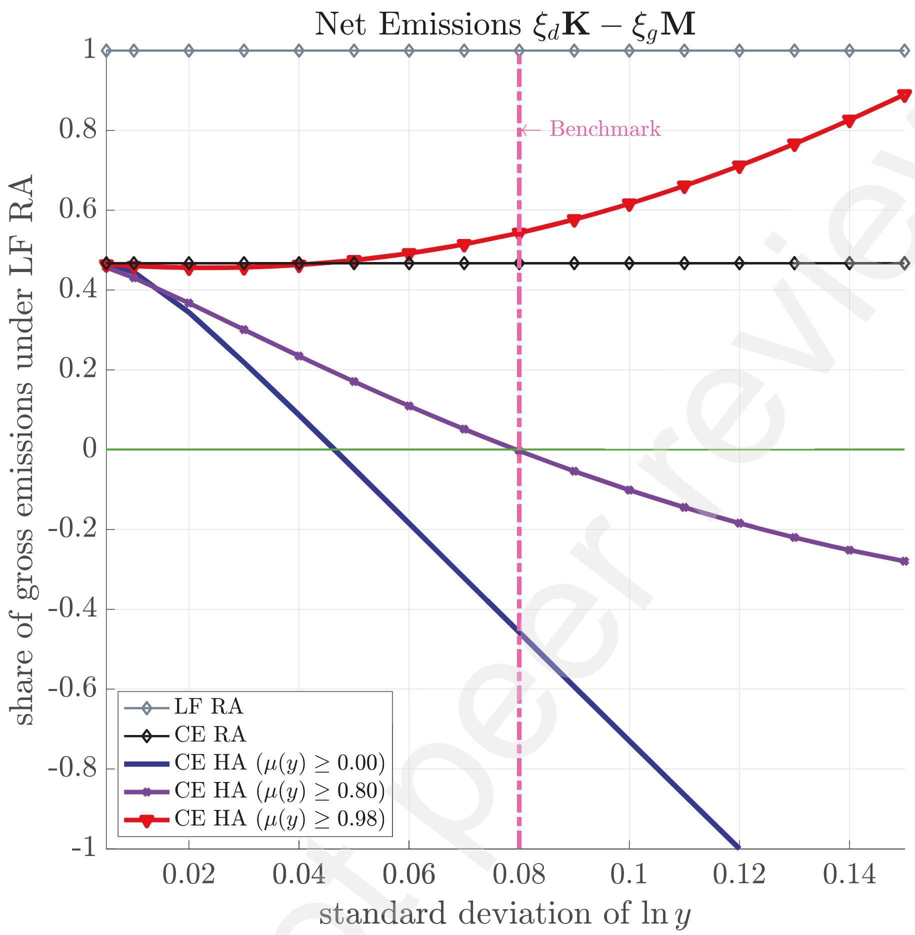
Since the discount factor plays a limited role in the results, we set $\beta = 1$. We take $\alpha = 1/3$, which pins down the share of global income that accrues to owners of capital. For the parameter γ in the damage function we take the average considered in Golosov et al. (2014). Since in their paper γ is associated to a given amount of global damage, we then set the degradation rate ξ_d so as to imply a global damage of 3% of global GDP in a business-as-usual scenario (BAU). To do that, we interpret the laissez-faire equilibrium without heterogeneity as BAU. In the case of the restoration rate ξ_g, we set it to the average between the value that satisfies Assumption 1 with equality and the value that makes zero global net emissions optimal in the representative region economy (see Proposition 2).

We choose the stochastic properties of exposure shocks so that they are TFP-neutral at the global scale, which pins down $\mu_\epsilon = \gamma\sigma_\epsilon^2/2$. The value of σ_ϵ determines how capital is distributed across regions (see equation (16)). We set it equal to 24250, which implies a standard deviation of 0.82 in the distribution of capital $\chi(\epsilon)$. When considering an environment with heterogeneity, we attribute all dispersion to z, whereas in an environment with only exposure uncertainty we attribute it to ν. Finally, for the dispersion in wealth we consider a grid going from 0 to 0.15 and set the benchmark value of σ_y equal to 0.08. Table 1 in the appendix summarizes our parameter choice.

6.2 Results

Let us consider first the effect of varying σ_y, assuming there is no climate inequality, but imposing different limits to redistribution. We report the global net emissions as a share of the gross emissions that occur in the laissez-faire equilibrium of an economy with a representative region. In Figure 1, the diamond-marked horizontal lines correspond to outcomes of a representative region economy. In the laissez-faire outcome (*LF-RA*), there

Figure 1: Global Net Emissions with Wealth Inequality.



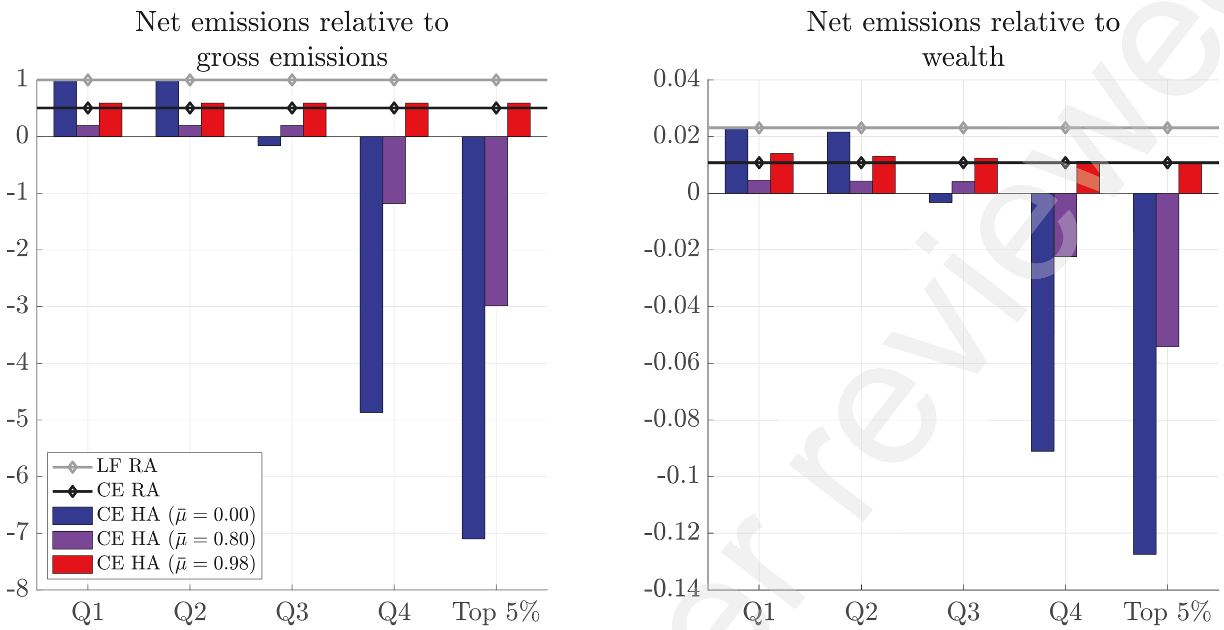
Notes: The horizontal axis measures the standard deviation of log wealth. ***LF RA***: laissez-faire equilibrium with $\sigma_y = \sigma_\epsilon = 0$; ***CE RA***: constrained efficient solution with $\sigma_y = \sigma_\epsilon = 0$; ***CE HA***: constrained efficient solution with $\sigma_\epsilon = 0$ and $\sigma_y > 0$.

is no sequestration and thus net emissions and gross emissions are the same; this corresponds to the line at the top of the figure. In the constrained efficient outcome (*CE-RA*) climate policy calls for reducing global net emissions, in approximately 50%. The fact that net zero is not optimal in this case is not surprising given our choice of the restoration rate ξ_g. Nevertheless, the wedge between these two horizontal lines is a measure of the extent to which mitigation and sequestration are desirable, from a societal perspective, in a representative agent environment.

The figure also depicts the constrained-efficient outcome in a world with different degrees of wealth inequality and in which the global planner faces limits to redistribution (*CE-HA*). We consider two extremes $\mu(y) \geq 0$ and $\mu(y) \geq 0.98$, and the intermediate case $\mu(y) \geq 0.8$ as an illustration; all the other cases must be bracketed by the two extreme cases. We observe that when the limits to redistribution are loose ($\mu(y) \geq 0$), more inequality *always* implies a more stringent climate policy, demanding more emission cuts and carbon offsets relative to the representative region case. Moreover, net zero emerges as a natural global objective when inequality is sufficiently large. This is still the case even if conditioning the contributions to each region's economic background becomes more difficult ($\mu(y) \geq 0.8$). Eventually, however, as the limits to redistribution become tighter ($\mu(y) \geq 0.98$), increasing wealth inequality leads to compromising on global climate goals. In fact, as the figure shows, if inequality is too large carbon capture responsibilities are close to uniform, optimal net emissions do not stray too far from the laissez-faire economy.

We turn next to analyze the optimal net emissions schedule. Our benchmark corresponds to the economy with wealth inequality given by $\sigma_y = 0.08$. Figure 2 displays this schedule when regions are partitioned into quartiles of the wealth distribution, along with the burden corresponding to the richest 5% of regions. In the left hand-side, we express net emissions as a fraction of gross emissions under laissez-faire and in the right hand-side, as a fraction of the average wealth of each group. When limits to redistribution are loose ($\mu(y) \geq 0$), the poorest regions get a "free-pass" on sequestration efforts; their emissions are as large as they would be in the laissez-faire economy. Emissions are still positive for regions in the second quartile, but they engage in some carbon offsetting. Most of the capture is done, however, by regions in the top of the wealth distribution. As the limits to redistribution tighten ($\mu(y) \geq 0.80$), the net emissions schedule rotates counterclockwise which implies increasing the burden of carbon offsets at the bottom of the distribution

Figure 2: Net Emissions Schedule with Wealth Inequality.



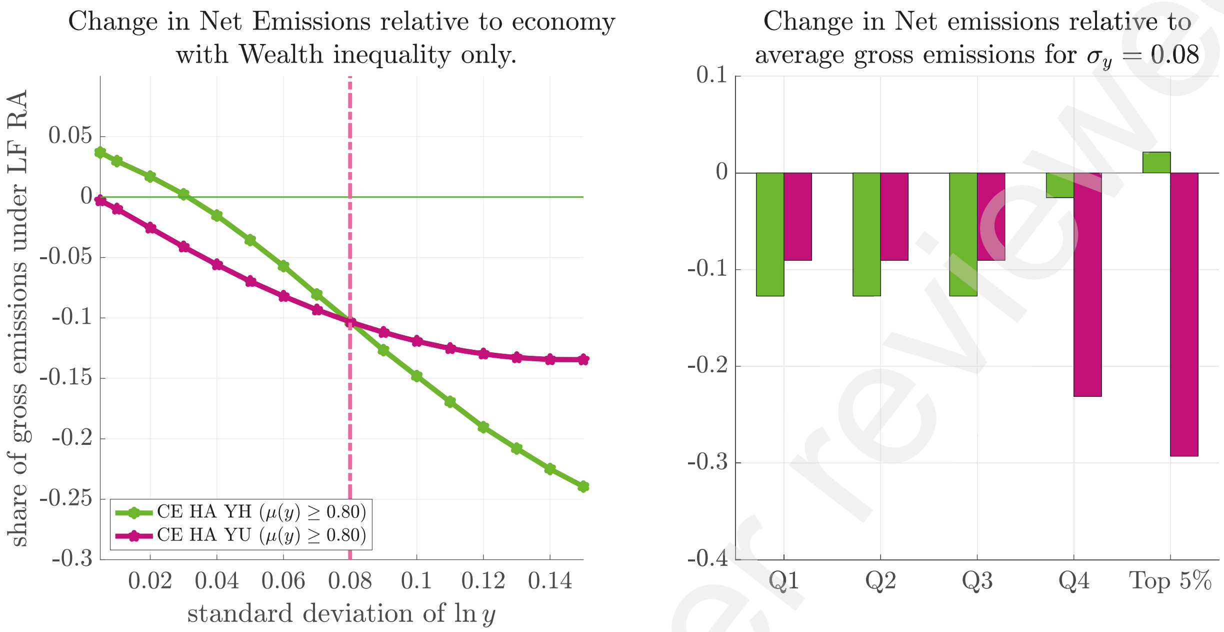
Notes: The horizontal axis measures the implied global damage in production for each value of ξ_d considered. ***LF RA***: laissez-faire equilibrium with $\sigma_z = \sigma_\epsilon$; ***CE RA***: constrained efficient solution with $\sigma_\epsilon = \sigma_y = 0$; ***CE HA***: constrained efficient solution with $\sigma_\epsilon \geq 0$ and $\sigma_y > 0$.

and alleviating it at the top. In the limit ($\mu(y) \geq 0.98$), the burden is almost uniform across regions but, as discussed previously, the global net emissions goal is far from net-zero.

We now introduce climate inequality. We take as the benchmark the economy in which the limits to redistribution are $\mu(y) \geq 0.8$. While this choice is arbitrary, it provides a good benchmark for two reasons: it prescribes global net-zero emissions when wealth inequality is set at the benchmark value of 0.08, and it acknowledges that while the design of climate policy surely faces limits to redistribution, these might not be too extreme so as to require an homogeneous burden in carbon capture.

In the left-hand side of Figure 3, we display the change in global net emissions with respect to an economy with only wealth inequality when we introduce climate inequality. We note that the interaction between climate and economic inequality depends on the nature of differences in climate vulnerability. When these constitute pre-existing differences,

Figure 3: Global Net Emissions with Wealth and Climate Inequality.



Notes: The figure in the left shows the change in global net emissions relative prescribed by an economy with climate inequality relative to an economy without it. The figure in the right shows the change in the net emissions schedule of the policy that corresponds to $\sigma_y = 0.08$, which calls for the same global net emissions target under the two forms of climate inequality. ***CE HA YH***: constrained efficient solution with $\sigma_z > 0$, $\sigma_y > 0$ and $\sigma_\nu = 0$; ***CE HA YU***: constrained efficient solution with $\sigma_z = 0$, $\sigma_y > 0$ and $\sigma_\nu > 0$.

whether the global target of net emissions is higher or lower will depend on the degree of wealth inequality. Specifically, climate policy is more lenient if wealth inequality is low and more stringent if it is high. When the difference in climate vulnerability is due to climate uncertainty, however, the constrained-efficient policy always prescribes a lower objective for global net emissions, and the policy is more stringent the higher the wealth inequality.

At the benchmark level of wealth inequality $\sigma_y = 0.08$, both the economy with climate heterogeneity and the one with climate uncertainty prescribe global net emissions to be reduced by approximately 10%. In the right-hand side of Figure 3, we fix σ_y to its benchmark and examine the change in the net emission schedule relative to the economy with only wealth inequality that prescribes global net zero. A key difference between heterogeneity and uncertainty emerges. In relative terms, the constrained efficient net emission schedule puts the burden of adjustment on the bottom of the distribution when climate inequality comes from pre-existing differences in climate vulnerability. The opposite is true when the climate inequality emerges from uncertain climate shocks. The reason for this difference lies on what is the margin of adjustment to attain the global net emissions goal; in the case of heterogeneity is trough more emission cuts while in the case of uncertainty is through more aggregate carbon offsets.

7 Conclusions

We lay out a model with heterogeneous regions and a carbon-offset technology to study the effect that inequality has on the design of climate policy. Our focus is on the choice of the global net emissions target, and the net emissions schedule across regions. We show that inequality has in fact a non-trivial effect on climate policy, which ultimately depends both on its source and on its magnitude.

We highlight two takeaways from our analysis. First, in an unequal world, the choice of a net emissions schedule across countries can be an effective tool to attain global climate goals. This requires the ability to make the contributions of each country to global carbon-capture conditional on their wealth. Second, if all nations are mandated to contribute uniformly to financing carbon capture, wealth inequality acts as a hindrance to collective climate efforts. In such a scenario, global emissions will remain high, and carbon offsetting will be low.

Our model can be extended to an infinite horizon setup, more suitable to a quantitative exploration that considers the relative importance of different sources of inequality. We leave this extension for future work.

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Appendix

A Modified Planner's Problem

With the change of variables, the utilitarian planner's problem can be written as follows:

$$\max_{\substack{c_0(y,z),\eta(y,z),c_1(y,\epsilon)\\ \mu(y),\chi(\epsilon),\mathbf{K},\mathbf{M}}} \int\int \Big\{u(c_0(y,z)) + \beta\mathbb{E}_\nu\Big[u(c_1(y,\epsilon))\Big]\Big\} g(y)\phi(z)\,\mathrm{d}y\,\mathrm{d}z$$

subject to budget constraints:

$$c_0(y,z) = y - \eta(y,z)(\mathbf{K}+\mathbf{M}) \qquad \forall (y,z), \tag{A.1}$$

$$c_1(y,\epsilon) = w(\epsilon)\theta + R\eta(y,z)(\mathbf{K}+\mathbf{M}) - R\mu(y)\mathbf{M} \qquad \forall (y,\epsilon), \tag{A.2}$$

optimal conditions for regional-representative firms:

$$R = \alpha \exp\Big(-\gamma\,(\mathbf{S}+\epsilon)\Big)\Big(\chi(\epsilon)\mathbf{K}\Big)^{\alpha-1} \qquad \forall \epsilon, \tag{A.3}$$

$$w(\epsilon) = (1-\alpha)\exp\Big(-\gamma\,(\mathbf{S}+\epsilon)\Big)\Big(\chi(\epsilon)\mathbf{K}\Big)^{\alpha} \qquad \forall \epsilon. \tag{A.4}$$

and the constraints on the distributions

$$\int \chi(\epsilon)h(\epsilon)\,\mathrm{d}\epsilon = 1 \tag{A.5}$$

$$\int \eta(y,z)g(y)\phi(z)\,\mathrm{d}y\,\mathrm{d}z = 1 \tag{A.6}$$

$$\int \mu(y)g(y)\,\mathrm{d}y = 1 \tag{A.7}$$

We note that $\chi(\epsilon)$ is pinned down by constraints and it will coincide with the laissez-faire outcome.

The global climate policy must include contributions having a positive lower bound and cannot condition on climate inequality. An interior solution for $\mu(y)$ is not guaranteed. When the zero lower bound binds, it indicates that some regions cannot contribute to carbon offsetting. The planner must set $\mu(y) = 0$ in such a case. As long as these contributions increase with wealth, the choice of μ requires the planner to set a cutoff $\bar{y}$ such that the lower bound constraint on $\mu(y)$ is indeed binding for all regions with $y \leq \bar{y}$. We obtain $\mu(y)$ in closed form in the following

Lemma 3 *Given a climate policy* $\{\mathbf{K}, \mathbf{M}\}$*, the share of carbon offsets done by region* y *is given by*

$$\mu(y) = \begin{cases} \mathcal{H}(y, \bar{y}) & \text{if } y > \bar{y} \\ 0 & \text{if } y \leq \bar{y} \end{cases} \tag{A.8}$$

where $\mathcal{H}(y, \bar{y}) = \dfrac{1 - \underline{\mu} G(\bar{y})}{1 - G(\bar{y})} + \dfrac{1}{\mathbf{M}} \left[y - \mathbb{E}[Y \mid Y \geq \bar{y}] \right]$*, where* $\bar{y}$ *satisfies* $\mathcal{H}(\bar{y}, \bar{y}) = 0$*.*

To understand the circumstances under which the non-negativity constraint on $\mu(y)$ binds, let $\underline{\mu} = 0$ and suppose the planner ignores it so that $\bar{y} = 0$. In such a case, the share of the burden to each region becomes

$$\mu(y) = 1 + \frac{y - 1}{\mathbf{M}},$$

where we are using that $\mathbb{E}[y] = 1$. To the extent that the right-hand side is negative for poorer regions, the planner will not require contributions from them; the financing cost is distributed, instead, among wealthier regions.

The wealth cutoff that determines which regions contribute is determined jointly with **K** and **M** as part of the solution to the global planner's problem.

B Proofs

Proof of Lemma 1

Using the functional forms, first order condition with respect to capital is:

$$R = \alpha \exp(-\gamma(\mathbf{S} + \epsilon))K(\epsilon)^{\alpha-1}L(\epsilon)^{1-\alpha}). \tag{B.1}$$

Since labor market clearing implies that $L(\epsilon) = 1$ in all regions, we can write

$$K(\epsilon) = \left(\frac{\alpha}{R}\right)^{\frac{1}{1-\alpha}} \exp\left(-\frac{\gamma}{1-\alpha}(\mathbf{S} + \epsilon)\right), \tag{B.2}$$

for all ϵ. Integrating across all regions:

$$\mathbf{K} = \left(\frac{\alpha}{R}\right)^{\frac{1}{1-\alpha}} \exp\left(-\frac{\gamma}{1-\alpha}\left(\mathbf{S} + \mu_\epsilon - \frac{\gamma}{1-\alpha}\frac{\sigma_\epsilon^2}{2}\right)\right), \tag{B.3}$$

where we have used the fact that if z and ν are normally distributed and independent, ϵ is also normally distributed. Hence, the share of global capital allocated to each region is

$$\chi(\epsilon) = \exp\left(-\frac{\gamma}{1-\alpha}\left(\epsilon - \mu_\epsilon + \frac{\gamma}{1-\alpha}\frac{\sigma_\epsilon^2}{2}\right)\right), \tag{B.4}$$

for all ϵ. For later use in the numerical example, note also that if $\mu_\epsilon = \gamma\sigma_\epsilon^2/2$, then the share of global capital allocated to each region is

$$\chi(\epsilon) = \exp\left(-\frac{\gamma}{1-\alpha}\left(\epsilon + \gamma\frac{\alpha}{1-\alpha}\frac{\sigma_\epsilon^2}{2}\right)\right), \tag{B.5}$$

for all ϵ.

To obtain the expression that characterizes the distribution of asset holdings, we use the Euler equation of each household in each region

$$\frac{1}{c_0(y,z)} = \beta R \mathbb{E}\left[\frac{1}{c_1(y,z,\nu)}\right], \tag{B.6}$$

for all y, z, and v. Since the public good is not provided in equilibrium, household assets are $a(y,z) = \eta(y,z)\mathbf{K}$. Using this in the first period budget constraint budget constraints yields

$$c_0(y,z) = y - \eta(y,z)\mathbf{K}, \tag{B.7}$$

for all y, and z. Household consumption in the second period can be written as follows

$$c_1(y,\epsilon) = w(\epsilon) + \eta(y,z)R\mathbf{K}, \tag{B.8}$$

and thus

$$\frac{c_1(y,\epsilon)}{R} = \frac{w(\epsilon)}{R} + \eta(y,\epsilon)\mathbf{K}, \tag{B.9}$$

for all y, and ϵ. First order conditions of the final good firm´s problem imply

$$\frac{w(\epsilon)}{R} = \frac{1-\alpha}{\alpha}\chi(\epsilon)\mathbf{K}, \tag{B.10}$$

for all ϵ. Plugging this into (B.9), and using (B.7) and (B.6), yields (17).

Proof of Lemma 2

Without exposure uncertainty, $\sigma_v = 0$ and thus, $\epsilon = z$. Since the expectation operator in (17) becomes redundant because households do not face any uncertainty, we verify that

$$\eta(y,z) = \frac{1}{1+\beta}\left[\beta\frac{y}{\mathbf{K}} - \frac{1-\alpha}{\alpha}\chi(z)\right], \tag{B.11}$$

for all y and z, satisfies the Euler equation of each household. Integrating across all regions and using (14) and (15), we can solve for the global capital stock and obtain (18).

Proof of Proposition 1

Integrating both sides of (B.6) across all regions and using the expressions for household consumption deliver

$$\int \frac{1}{y-\eta(y,z)\mathbf{K}} g(y)\phi(z)\,\mathrm{d}y\,\mathrm{d}z = \beta\alpha \int \mathbb{E}\left[\frac{1}{(1-\alpha)\chi(\epsilon)\mathbf{K}+\alpha\eta(y,z)\mathbf{K}}\right] g(y)\phi(z)\,\mathrm{d}y\,\mathrm{d}z,$$

which determines the global stock of productive capital. The left-hand side increases with $\mathbf{K}$ and the right-hand side does the opposite. A solution is guaranteed because of properties of marginal utility when preferences are logarithmic. The expectation in the right-hand side is taken with respect to ν, which only affects the distribution of productive capital across regions, e.g., $\chi(\epsilon)$. Since marginal utility is convex with respect to $\chi(\epsilon)$, Jensen´s inequality implies that the right-hand side is larger in the presence of uncertainty, for any $\mathbf{K}$. This implies that the solution to the previous expression increases as the variance of σ_ν does.

Proof of Proposition 2

We perform the following change of variables:

$$\begin{aligned}\mathbf{K} &= (1-\theta)\mathbf{A}\\ \mathbf{M} &= \theta\mathbf{A}\end{aligned}$$

First order condition with respect to θ deliver

$$\beta\frac{-R\mathbf{A}-\frac{\partial R}{\partial K}(1-\theta)\mathbf{A}\mathbf{A}-\frac{\partial w}{\partial K}\mathbf{A}+\gamma(\xi_d+\xi_g)\mathbf{A}c_1}{c_1}=0$$

Using the pricing functions, properties of the constant returns to scale production function, and the fact that $\frac{\alpha c_1}{R} = (1-\theta)A$, we write

$$(1-\theta)\mathbf{A} = \frac{\alpha}{\gamma(\xi_d + \xi_g)}. \tag{B.12}$$

First order condition with respect to $\mathbf{A}$ deliver

$$\frac{1}{1-\mathbf{A}} = \beta \frac{(1-\theta)R + \frac{\partial R}{\partial K}(1-\theta)(1-\theta)\mathbf{A} + \frac{\partial w}{\partial K}(1-\theta) - \gamma((1-\theta)\xi_d - \theta\xi_g)c_1}{c_1}$$

Again, we can simplify this to

$$\frac{1}{1-\mathbf{A}} = \beta \frac{\alpha - \gamma\Pi(\mathbf{A},\theta)}{\mathbf{A}}$$

Therefore, aggregate savings are defined implicitly by

$$\mathbf{A} = \frac{\beta(\alpha - \gamma\Pi(\mathbf{A},\theta))}{1 + \beta(\alpha - \gamma\Pi(\mathbf{A},\theta))}$$

Note that we can use (B.12) to write

$$\gamma\Pi(\mathbf{A},\theta) = \gamma\left(\xi_d \frac{\alpha}{\gamma(\xi_d + \xi_g)}) - \xi_g \left(\mathbf{A} - \frac{\alpha}{\gamma(\xi_d + \xi_g)}\right)\right)$$

which implies that

$$\alpha - \gamma\Pi(\mathbf{A},\theta) = \gamma\xi_g\mathbf{A}$$

Thus

$$\mathbf{A} = \frac{\beta\gamma\xi_g - 1}{\beta\gamma\xi_g} \tag{B.13}$$

Using this into the expression for climate capital delivers

$$\theta\mathbf{A} = 1 - \frac{\alpha}{\gamma(\xi_d + \xi_g)} - \frac{1}{\beta\gamma\xi_g}$$

which is positive if

$$\gamma(\xi_d + \xi_g) > \frac{\alpha\beta}{\beta\gamma\xi_g - 1}\gamma\xi_g$$

which holds under assumption 1.

Proof of Proposition 3

First order condition with respect to θ:

$$\beta\mathbb{E}\left[\frac{-R\mu(y)\mathbf{A} - \frac{\partial R}{\partial K}\chi(\epsilon)(\eta(y,z) - \theta\mu(y))\mathbf{AA} - \frac{\partial w}{\partial K}\chi(\epsilon)\mathbf{A} + \gamma(\xi_d + \xi_g)\mathbf{A}c_1(y,\epsilon)}{c_1(y,\epsilon)}\right] = 0$$

where the expectation is taken using the distributions of y, z and v. We simplify this to

$$\mathbb{E}\left[\frac{R\mu(y) + \frac{\partial R}{\partial K}\chi(\epsilon)(\eta(y,z) - \theta\mu(y))\mathbf{A} + \frac{\partial w}{\partial K}\chi(\epsilon) - \gamma(\xi_d + \xi_g)c_1(y,\epsilon)}{c_1(y,\epsilon)}\right] = 0$$

Using the pricing functions and properties of the production function:

$$\mathbb{E}\left[\frac{(1-\theta)R\mu(y) - F_{LK}(\eta(y,z) - \theta\mu(y)) + (1-\theta)F_{LK}\chi(\epsilon) - (1-\theta)\gamma(\xi_d + \xi_g)c_1(y,\epsilon)}{(1-\theta)c_1(y,\epsilon)}\right] = 0$$

Now, we note that we can write:

$$\frac{\alpha c_1(y,\epsilon)}{\mathbf{A}} = (1-\theta)\chi(\epsilon)F_{LK} - F_{LK}(\eta(y,\epsilon) - \theta\mu(y)) + R(\eta(y,\epsilon) - \theta\mu(y))$$

Using this in the previous first order condition yields

$$\frac{\alpha}{(1-\theta)\mathbf{A}} - \mathbb{E}\left[\frac{R(\eta(y,\epsilon)-\mu(y))}{(1-\theta)c_1}\right] = \gamma(\xi_d+\xi_g)$$

Noting again that

$$\frac{\alpha c_1(y,\epsilon)}{R} = \chi(\epsilon)(1-\theta)\mathbf{A} + \alpha(\eta(y,\epsilon)-\chi(\epsilon))\mathbf{A} - \alpha(\mu(y)-\chi(\epsilon))\theta\mathbf{A}$$

We obtain

$$(1-\theta)\mathbf{A} = \frac{\alpha-\Omega}{\gamma(\xi_d+\xi_g)} \tag{B.14}$$

where

$$\Omega_0 = \mathbb{E}\left[\frac{\alpha(1-\theta)(\eta(y,\epsilon)-\mu(y))}{(1-\alpha)\chi(\epsilon)(1-\theta)+\alpha\eta(y,\epsilon)-\alpha\mu(y)\theta}\right]$$

Therefore, productive capital depends purely on the sign of Ω_0.

The first order condition with respect to $\mathbf{A}$ delivers

$$\mathbb{E}\left[\frac{\eta(y,\epsilon)}{y-\eta(y,\epsilon)\mathbf{A}}\right] = \beta\mathbb{E}\left[\frac{R(\eta(y,\epsilon)-\theta\mu(y))-\gamma\frac{\Pi(\mathbf{A},\theta)}{\mathbf{A}}c_1 - F_{LK}(\eta(y,\epsilon)-\theta\mu(y))+F_{LK}\chi(\epsilon)(1-\theta)}{c_1(y,\epsilon)}\right]$$

which once more simplifies to

$$\mathbb{E}\left[\frac{\eta(y,\epsilon)}{y-\eta(y,\epsilon)\mathbf{A}}\right] = \beta\frac{\alpha-\gamma\Pi(\mathbf{A},\theta)}{\mathbf{A}}$$

We define

$$Q \equiv \mathbb{E}\left[\frac{\eta(y,\epsilon)\mathbf{A}}{y-\eta(y,\epsilon)\mathbf{A}}\right]$$

which implies

$$Q \;=\; \beta(\alpha - \gamma\Pi(\mathbf{A}, \theta))$$

We then use (B.14) to write

$$\gamma\Pi(\mathbf{A}, \theta) \;=\; \gamma\left(\xi_d \frac{\alpha - \Omega_0}{\gamma(\xi_d + \xi_g)} - \xi_g\left(\mathbf{A} - \frac{\alpha - \Omega_0}{\gamma(\xi_d + \xi_g)}\right)\right)$$

which delivers

$$\mathbf{A} \;=\; \frac{Q - \beta\Omega}{\beta\gamma\xi_g}$$

We now look now for an expression for $Q - \beta\Omega$. The first order condition with respect to $\eta(y, z)$ is

$$\frac{\mathbf{A}}{y - \eta(y, z)\mathbf{A}} \;=\; \beta\mathbb{E}\left[\frac{R\mathbf{A}}{c_1(y, \epsilon)}\right] + \Lambda$$

where Λ is the Lagrange multiplier on the constraint $\int \eta(y, z)g(y)\phi(z)\, \mathrm{d}y\, \mathrm{d}z \geq 1$, and the expectation in the right-hand side is taken with respect to the exposure shock ν. Multiplying both sides by $\eta(y, z)$ and aggregating across regions delivers

$$\mathbb{E}\left[\frac{\eta(y, z)\mathbf{A}}{y - \eta(y, z)\mathbf{A}}\right] \;=\; \beta\mathbb{E}\left[\frac{R\mathbf{A}\eta(y, z)}{c_1(y, \epsilon)}\right] + \Lambda$$

Using the expression for $c_1(y, \epsilon)/R\mathbf{A}$ allows us to write

$$\begin{aligned} Q &= \Lambda + \frac{\alpha\beta}{1-\theta}\mathbb{E}\left[\frac{(1-\theta)\eta(y, z)}{(1-\alpha)(1-\theta)\chi(\epsilon) + \alpha(\eta(y, z) - \theta\mu(y))}\right] \\ Q - \beta\Omega_0 &= \Lambda + \frac{\alpha\beta}{1-\theta}\mathbb{E}\left[\frac{(1-\theta)\mu(y)}{(1-\alpha)(1-\theta)\chi(\epsilon) + \alpha(\eta(y, z) - \theta\mu(y))}\right] \end{aligned}$$

To save on notation, we let $\tilde{\mathbb{E}}[x]$ stand for

$$\tilde{\mathbb{E}}[x] = \mathbb{E}\left[\frac{(1-\theta)x}{(1-\alpha)(1-\theta)\chi(z) + \alpha(\eta(y,z) - \theta\mu(y))}\right]$$

Plugging these expressions in (B.15) and then in the first order condition with respect to η allows us to write

$$\frac{\Lambda + \frac{\alpha\beta}{1-\theta}\tilde{\mathbb{E}}\left[\mu(y)\right]}{\beta\gamma\xi_g y - \eta(y,z)(Q - \beta\Omega_0)} \quad = \quad \Lambda + \frac{\alpha\beta}{1-\theta}\tilde{\mathbb{E}}_\nu[1]$$

Rearranging and aggregating across regions delivers

$$Q - \beta\Omega_0 \quad = \quad \beta\gamma\xi_g - \mathbb{E}\left[\frac{\Lambda + \frac{\alpha\beta}{1-\theta}\tilde{\mathbb{E}}\left[\mu(y)\right]}{\Lambda + \frac{\alpha\beta}{1-\theta}\tilde{\mathbb{E}}_\nu[1]}\right]$$

We define then

$$\Omega_1 \quad = \quad \mathbb{E}\left[\frac{\Lambda + \frac{\alpha\beta}{1-\theta}\tilde{\mathbb{E}}\left[\mu(y)\right]}{\Lambda + \frac{\alpha\beta}{1-\theta}\tilde{\mathbb{E}}_\nu[1]}\right], \tag{B.15}$$

and note that in the absence of any source of inequality, $\Omega_1 = 1$.

We finally obtain:

$$\mathbf{K} \quad = \quad \frac{\alpha - \Omega_0}{\gamma(\xi_d + \xi_g)} \tag{B.16}$$

$$\mathbf{M} \quad = \quad 1 - \frac{\alpha - \Omega_0}{\gamma(\xi_d + \xi_g)} - \frac{\Omega_1}{\beta\gamma\xi_g} \tag{B.17}$$

$$\mathbf{A} \quad = \quad \frac{\beta\gamma\xi_g - \Omega_1}{\beta\gamma\xi_g} \tag{B.18}$$

Proof of Corollary 2

It is sufficient to show that Ω_0 is positive. Let

$$\begin{aligned}
\mathtt{N}(y) &= \eta(y) - \mu(y) \\
\mathtt{D}(y) &= ((1-\alpha)(1-\theta) + (\eta(y) - \theta\mu(y)))^{-1}
\end{aligned}$$

Then the sign of Ω_0 is the same as the sign of $\mathbb{E}\left[\mathtt{N}(y)\cdot\mathtt{D}(y)\right]$. We can write:

$$\begin{aligned}
\mathbb{E}\left[\mathtt{N}(y)\cdot\mathtt{D}(y)\right] &= \mathbb{E}\left[\mathtt{Cov}\left(\mathtt{N}(y),\mathtt{D}(y)\mid y\right)\right] + \mathtt{Cov}\left(\mathbb{E}\left[\mathtt{N}(y)\mid y\right],\mathbb{E}\left[\mathtt{D}(y)\mid y\right]\right) \\
&= \mathtt{Cov}\left(\mathtt{N}(y),\mathtt{D}(y)\mid y\right)\Pr[y\geq\bar{y}] + \mathtt{Cov}\left(\mathtt{N}(y),\mathtt{D}(y)\mid y\right)\Pr(y<\bar{y}) \\
&\quad +\mathtt{Cov}\left(\mathbb{E}\left[\mathtt{N}(y)\mid y\right],\mathbb{E}\left[\mathtt{D}(y)\mid y\right]\right)
\end{aligned}$$

Since $\mathtt{D}(y)$ is constant across contributing regions, the first term in the right-hand side equals zero. Using properties of the covariance, we use:

$$\mathtt{Cov}\left(\mathtt{N}(y),\mathtt{D}(y)\mid y<\bar{y}\right) = \mathtt{Cov}\left(\mathtt{N}(y),\mathtt{D}(y)-\mathbb{E}[\mathtt{D}(y)]\mid y<\bar{y}\right)$$

and then write

$$\begin{aligned}
\mathbb{E}\left[\mathtt{N}(y)\cdot\mathtt{D}(y)\right] &= \mathtt{Cov}\left(\mathtt{N}(y),\mathtt{D}(y)-\mathbb{E}[\mathtt{D}(y)]\mid y<\bar{y}\right)\Pr(y<\bar{y}) \\
&\quad +\left(\mathbb{E}\left[\mathtt{N}(y)\mid y\geq\bar{y}\right]-\mathbb{E}[\mathtt{N}(y)]\right)\left(\mathbb{E}\left[\mathtt{D}(y)\mid y\geq\bar{y}\right]-\mathbb{E}[\mathtt{D}(y)]\right)\Pr(y\geq\bar{y}) \\
&\quad +\left(\mathbb{E}\left[\mathtt{N}(y)\mid y<\bar{y}\right]-\mathbb{E}[\mathtt{N}(y)]\right)\left(\mathbb{E}\left[\mathtt{D}(y)\mid y<\bar{y}\right]-\mathbb{E}[\mathtt{D}(y)]\right)\Pr(y<\bar{y})
\end{aligned}$$

Using the fact that $\mathbb{E}[\mathtt{N}(y)] = 0$ and since

$$\begin{aligned}
\mathtt{Cov}\left(\mathtt{N}(y),\mathtt{D}(y)-\mathbb{E}[\mathtt{D}(y)]\mid y<\bar{y}\right) &= \mathbb{E}\left[\mathtt{N}(y)(\mathtt{D}(y)-\mathbb{E}[\mathtt{D}(y)])\mid y<\bar{y}\right] \\
&- \mathbb{E}\left[\mathtt{N}(y)\mid y<\bar{y}\right]\mathbb{E}\left[(\mathtt{D}(y)-\mathbb{E}[\mathtt{D}(y)])\mid y<\bar{y}\right]
\end{aligned}$$

we obtain

$$\begin{aligned}\mathbb{E}\left[\mathtt{N}(y)\cdot\mathtt{D}(y)\right] &= \mathbb{E}\left[\mathtt{N}(y)(\mathtt{D}(y)-\mathbb{E}[\mathtt{D}(y)])\mid y<\bar{y}\right]\mathbf{Pr}(y<\bar{y})\\ &\quad +\mathbb{E}\left[\mathtt{N}(y)\mid y\geq\bar{y}\right]\left(\mathbb{E}\left[\mathtt{D}(y)\mid y\geq\bar{y}\right]-\mathbb{E}[\mathtt{D}(y)]\right)\mathbf{Pr}(y\geq\bar{y})\end{aligned}$$

Using again $\mathbb{E}[\mathtt{N}(y)]=0$ we rewrite

$$\begin{aligned}\mathbb{E}\left[\mathtt{N}(y)\cdot\mathtt{D}(y)\right] &= \mathbb{E}\left[\mathtt{N}(y)(\mathtt{D}(y)-\mathbb{E}[\mathtt{D}(y)])\mid y<\bar{y}\right]\mathbf{Pr}(y<\bar{y})\\ &\quad -\mathbb{E}\left[\mathtt{N}(y)\mid y<\bar{y}\right]\left(\mathbb{E}\left[\mathtt{D}(y)\mid y\geq\bar{y}\right]-\mathbb{E}[\mathtt{D}(y)]\right)\mathbf{Pr}(y<\bar{y})\end{aligned}$$

We operate in the first term in the right hand side without altering the equation

$$\begin{aligned}\mathbb{E}\left[\mathtt{N}(y)\cdot\mathtt{D}(y)\right] &= \mathbb{E}\left[\mathtt{N}(y)\mid y<\bar{y}\right]\mathbb{E}\left[\frac{\mathtt{N}(y)}{\mathbb{E}\left[\mathtt{N}(y)\mid y<\bar{y}\right]}(\mathtt{D}(y)-\mathbb{E}[\mathtt{D}(y)])\mid y<\bar{y}\right]\mathbf{Pr}(y<\bar{y})\\ &\quad -\mathbb{E}\left[\mathtt{N}(y)\mid y<\bar{y}\right]\left(\mathbb{E}\left[\mathtt{D}(y)\mid y\geq\bar{y}\right]-\mathbb{E}[\mathtt{D}(y)]\right)\mathbf{Pr}(y<\bar{y})\end{aligned}$$

And thus

$$\begin{aligned}\mathbb{E}\left[\mathtt{N}(y)\cdot\mathtt{D}(y)\right] &= \mathbb{E}\left[\mathtt{N}(y)\mid y<\bar{y}\right]\left(\mathbb{E}\left[\frac{\mathtt{N}(y)}{\mathbb{E}\left[\mathtt{N}(y)\mid y<\bar{y}\right]}(\mathtt{D}(y))\mid y<\bar{y}\right]-\mathbb{E}[\mathtt{D}(y)]\right)\mathbf{Pr}(y<\bar{y})\\ &\quad -\mathbb{E}\left[\mathtt{N}(y)\mid y<\bar{y}\right]\left(\mathbb{E}\left[\mathtt{D}(y)\mid y\geq\bar{y}\right]-\mathbb{E}[\mathtt{D}(y)]\right)\mathbf{Pr}(y<\bar{y})\end{aligned}$$

Which implies:

$$\mathbb{E}\left[\mathtt{N}(y)\cdot\mathtt{D}(y)\right] = \mathbb{E}\left[\mathtt{N}(y)\mid y<\bar{y}\right]\left(\mathbb{E}\left[\frac{\mathtt{N}(y)}{\mathbb{E}\left[\mathtt{N}(y)\mid y<\bar{y}\right]}\mathtt{D}(y)\mid y<\bar{y}\right]-\mathbb{E}\left[\mathtt{D}(y)\mid y\geq\bar{y}\right]\right)\mathbf{Pr}(y<\bar{y})$$

To the extent taht $\mathbb{E}\left[\mathtt{N}(y) \mid y < \bar{y}\right]$ is positive for non-contributing regions, the sign of Ω is determined by whether

$$\mathbb{E}\left[\frac{\mathtt{N}(y)}{\mathbb{E}\left[\mathtt{N}(y) \mid y < \bar{y}\right]}\mathtt{D}(y) \mid y < \bar{y}\right] - \mathbb{E}\left[\mathtt{D}(y) \mid y \geq \bar{y}\right]$$

is positive or negative. Since $\mathtt{D}(y)$ is basically the marginal utility of each region, the first term is the weighted average marginal utility of non-contributing regions, while the second is the average marginal utility of contributing regions. We know that the former is larger than the latter if it weren't by the fact that it has weights. However, we should note that the marginal utility of *each* non-contributing regions is larger than that of a contributing region, which means that the weighting is innocuous. Therefore, Ω_0 is positive.

C Savings and Carbon Offset Contributions in Special Cases

C.1 Wealth inequality only

In this case, the first order condition with respect to μ implies that

$$\mu(y) = \frac{1}{1 - G(\bar{y})} + \frac{\eta(y) - \mathbb{E}\left[\eta(y) \mid y \geq \bar{y}\right]}{\theta} \tag{C.1}$$

This implies that for any contributing region, we must have that

$$\theta\mu(y) = \begin{cases} \eta(y) - (1-\theta) \\ \eta(y) - \left(\mathbb{E}\left[\eta(y) \mid y \geq \bar{y}\right] - \frac{\theta}{1-G(\bar{y})}\right) \end{cases}$$

We define

$$Q(\bar{y}) \equiv \left(\mathbb{E}\left[\eta(y) \mid y \geq \bar{y}\right] - \frac{\theta}{1 - G(\bar{y})}\right)$$

Note that $Q(\bar{y}) = 1 - \theta$ when all regions contribute. We must have:

$$\frac{1}{y - \eta(y)A} = \begin{cases} \dfrac{\alpha\beta}{(1-\alpha)(1-\theta)A + \alpha Q(\bar{y})A} + \Lambda & \text{if } y \geq \bar{y} \\ \dfrac{\alpha\beta}{(1-\alpha)(1-\theta)A + \alpha\eta(y)A} + \Lambda & \text{if } y < \bar{y} \end{cases} \tag{C.2}$$

Note that for a contributing region, the right-hand side does not depend on y. Therefore

$$\frac{1}{\mathbb{E}[y \mid y \geq \bar{y}] - \mathbb{E}[\eta(y) \mid y \geq \bar{y}]A} = \frac{\alpha\beta}{(1-\alpha)(1-\theta)A + \alpha Q(\bar{y})A} + \Lambda$$

This expression defines a value for Λ given $\bar{y}$. Therefore, we must have that that for a contributing region

$$\begin{aligned} \eta(y) &= \frac{y - \mathbb{E}[y \mid y \geq \bar{y}]}{A} + \mathbb{E}[\eta(y) \mid y \geq \bar{y}] \\ \mu(y) &= \frac{1}{1 - G(\bar{y})} + \frac{y - \mathbb{E}\left[y \mid y \geq \bar{y}\right]}{\theta A} \end{aligned}$$

and for a non-contributing region $\mu(y) = 0$ and $\eta(y)$ is defined implicitly in (C.2). In the special case in which all regions contribute. we can solve for Λ, and use (B.18) to obtain

$$\Lambda = \frac{\beta\gamma\xi_g}{\Omega_1} - \frac{\alpha\beta\gamma(\xi_g + \xi_d)}{\alpha - \Omega_0}$$

Using this expression we obtain

$$\begin{aligned} \mu(y) &= 1 + \frac{y-1}{\theta \mathbf{A}} \\ \eta(y) &= \frac{\beta\gamma\xi_g y - \Omega_1}{\beta\gamma\xi_g - \Omega_1} \end{aligned}$$

for each y.

C.2 Adding exposure uncertainty

In this case, it is easy to verify that the first order condition with respect to μ still implies:

$$\mu(y) = \frac{1}{1-G(\bar{y})} + \frac{\eta(y) - \mathbb{E}\left[\eta(y) \mid y \geq \bar{y}\right]}{\theta} \tag{C.3}$$

Therefore, for any contributing region we must have that

$$\theta\mu(y) = \begin{cases} \eta(y) - (1-\theta) \\ \eta(y) - \left(\mathbb{E}\left[\eta(y) \mid y \geq \bar{y}\right] - \frac{\theta}{1-G(\bar{y})}\right) \end{cases}$$

We define

$$Q(\bar{y}) \equiv \left(\mathbb{E}\left[\eta(y) \mid y \geq \bar{y}\right] - \frac{\theta}{1-G(\bar{y})}\right)$$

and note that $Q(\bar{y}) = 1-\theta$ when all regions contribute. The difference with the previous case is in the first order condition with respec to $\eta(y)$. Now, we must have:

$$\frac{1}{y-\eta(y)A} = \begin{cases} \mathbb{E}\left[\dfrac{\alpha\beta}{(1-\alpha)\chi(\nu)(1-\theta)A + \alpha Q(\bar{y})A}\right] + \Lambda & \text{if } y \geq \bar{y} \\ \mathbb{E}\left[\dfrac{\alpha\beta}{(1-\alpha)\chi(\nu)(1-\theta)A + \alpha\eta(y)A}\right] + \Lambda & \text{if } y < \bar{y} \end{cases} \tag{C.4}$$

where the expectation is taken with respect to the exposure shock ν. Note that for a contributing region, the right-hand side does not depend on y. Therefore

$$\Lambda = \frac{1}{\mathbb{E}[y \mid y \geq \bar{y}] - \mathbb{E}[\eta(y) \mid y \geq \bar{y}]A} - \frac{\alpha\beta}{(1-\alpha)(1-\theta)A + \alpha Q(\bar{y})A}$$

We have an expression for Λ given $\bar{y}$ and $\eta(y)$. Hence, for a contributing region we still must have

$$\begin{aligned}
\eta(y) &= \frac{y - \mathbb{E}[y \mid y \geq \bar{y}]}{A} + \mathbb{E}[\eta(y) \mid y \geq \bar{y}] \\
\mu(y) &= \frac{1}{1 - G(\bar{y})} + \frac{y - \mathbb{E}\left[y \mid y \geq \bar{y}\right]}{\theta A}
\end{aligned}$$

and for a non-contributing region $\mu(y) = 0$ and $\eta(y)$ is defined implicitly in (C.4). Note that the value of eta must be larger than without uncertainty due to Jensen's inequality. The difference is precautionary savings. Notice also that the magnitude of these savings is constant among contributing regions and increasing in y for those regions that do not contribute.

C.3 Adding Exposure heterogeneity

We guess, and later verify, that for any contributing region

$$\eta(y,z)A = (\tilde{\eta}(y) + \tilde{\eta}(z))A,$$

This implies again that

$$\mu(y) = \frac{1}{1 - G(\bar{y})} + \frac{\tilde{\eta}(y) - \mathbb{E}\left[\tilde{\eta}(y) \mid y \geq \bar{y}\right]}{\theta} \tag{C.5}$$

Therefore, for any contributing region we must have that

$$\theta\mu(y) = \begin{cases} \tilde{\eta}(y) - (1 - \theta) \\ \tilde{\eta}(y) - \left(\mathbb{E}\left[\tilde{\eta}(y) \mid y \geq \bar{y}\right] - \frac{\theta}{1 - G(\bar{y})}\right) \end{cases}$$

We define

$$Q(\bar{y}) \equiv \left(\mathbb{E}\left[\tilde{\eta}(y) \mid y \geq \bar{y}\right] - \frac{\theta}{1 - G(\bar{y})} \right)$$

First order condition with respect to η deliver for contributing and non contributing regions respectively

$$\frac{1}{y - (\tilde{\eta}(y) + \tilde{\eta}(z))A} = \frac{\alpha\beta}{(1-\alpha)\chi(z)(1-\theta)A + \alpha\tilde{\eta}(z)A + \alpha Q(\bar{y})A} + \Lambda \tag{C.6a}$$

$$\frac{1}{y - \eta(y,z)A} = \frac{\alpha\beta}{(1-\alpha)\chi(z)(1-\theta)A + \alpha\eta(y,z)A} + \Lambda \tag{C.6b}$$

Note that for a contributing region, the right-hand side depends on z but not on y. Then we must have again that for those regions

$$\begin{aligned}
\tilde{\eta}(y) &= \frac{y - \mathbb{E}[y \mid y \geq \bar{y}]}{A} + \mathbb{E}[\tilde{\eta}(y) \mid y \geq \bar{y}] \\
\mu(y) &= \frac{1}{1 - G(\bar{y})} + \frac{y - \mathbb{E}\left[y \mid y \geq \bar{y}\right]}{\theta A}
\end{aligned}$$

and $\tilde{\eta}(z)$ is defined implicitly in (C.6) for contributing regions. For a non-contributing region $\mu(y) = 0$ and $\eta(y,z)$ is defined implicitly in (C.6).

D Algorithm for Computation

1. Guess **K** and **M**, and translate it into θ and **A**. A good initial guess is the representative agent solution.

2. Check if all regions contribute for this initial guess. If not, find the last contributing region. This sets $\bar{y}$

3. Solve for $\{\Lambda, \eta, \mu\}$ as follows:

(a) Guess Λ and use this guess to obtain η from Euler equation both for contributing and non contributing regions.

(b) Obtain μ give $\bar{y}$.

(c) Check if η adds up to one. If not, adjust Λ

(d) Iterate until convergence.

4. Use $\{\Lambda, \eta, \mu\}$ to obtain $\{\Omega_0, \Omega_1\}$ according to the expressions of Proposition 3.

5. Compute new values for **K** and **M** using Proposition 3.

6. Iterate until convergence

E Parameter values

Table 1: Parameter values

Parameter	Symbol	Benchmark	Comment
Discount factor	β	1	
Capital intensity	α	0.33	
Damage Function			
Elasticity	γ	2.4e-05	$\diamond$
Degradation rate	ξ_d	5.1e+02	†
Restoration rate	ξ_g	5.6e+04	‡
Dispersion			
Wealth	σ_y	0.08	$\circledast$
Exposure	σ_ϵ	24250	

Notes: ($\diamond$) average reported in Golosov et al. (2014); (†) implied global production damage of 3%; (‡) guarantees slackness of Assumption 1 and positive global net emissions in the representative region case; ($\circledast$) sensitivity performed with a grid going from 0.01 to 0.15.