HETEROGENOUS PRODUCTIVITIES AND INTEREST RATE SHOCKS: A TWO SECTOR MODEL FOR ARGENTINA

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Understanding economic cycles has been one of the most challenging items in the macroeconomist research agenda for decades. This study aims to build a dynamic stochastic general equilibrium model capable of reproducing the empirical regularities of economic cycles in Argentina. Two main features are highlighted. First, the recognition of heterogenous productivity levels across the country´s productive sectors, captured by a tradable sector and a non tradable sector. Second, the importance of shocks to international interest rates in explaining numerous episodes of recessions in the country economic history. In addition, local interest rates are impacted by the country risk premium, which in turn is elastic to the expected total factor productivity of the tradable sector. Such elasticity is found to be key for the potential of the model to satisfactorily explain business cycles in Argentina.

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Understanding economic cycles has been one of the most challenging items in the macroeconomist research agenda for decades. During the twentieth century, the debate was largely dominated by schools of thought that focused their attention on nominal factors, such as interest rates and price rigidities. However, empirical and theoretical shortcomings of both keynesianism and monetarism, especially from the 1970 decade on, encouraged the surge of new theories that gave real factors greater explanatory potencial (Kydland and Zarazaga, 1997). The basic idea of the Real Business Cycle (RBC) theory, first attributed to Finn Kydland and Edward Prescott (1982), was that a standard neoclassical growth model, with shocks to the total factors productivity, captured satisfactorily the main features of observed economic fluctuations, at least in developed countries.

The models built within the RBC framework were intended to replicate three main empirical regularities of industrialized economies. First, the procyclical nature of consumption, investment and employment; second, the higher volatility of investment relative to output and consumption and, third, the positive persistance of the main macroeconomic aggregates (Medonza, 1991). Emerging market economies, however, posed new challenges to researchers, as additional aspects particular to those economies were identified in observational data. In concrete, emerging countries, vis-a-vis their developed counterparts, are characterized by a higher variability of consumption relative to output and a strong countercyclicality of the trade balance. As a consequence, the focus of the literature on emerging economies has been put in the development of models which could potentially approximate such empirical regularities.

Mendoza´s (1991) model of a small open economy has been the benchmark for the emerging market literature. The model environment features production with endogenous capital and labor and costs associated with adjusting capital. Also, the representative agent can borrow and lend in international capital markets, the asset markets of which are incomplete as the only financial instrument available is a one period non contingent bond that pays an interest rate set exogenously. Despite satisfactorily explaining the observed persistence and variability of output fluctuations as well as the countercyclicality of the trade balance for industrialized economies, the model requires further inputs to capture the dynamics of emerging market economies.

While acyclical in industrial countries, interest rates in emerging markets have been documented to have a strong negative correlation with overall economic activity. Such observation is present in Neumeyer and Perri (2005) and in Uribe and Yue (2006), who combine shocks to the interest rate with working capital constraints in order to match the empirical regularities of emerging market economies. In these settings, when the total factor productivity is low, interest rate is likely to be high, making it more difficult for agents to borrow from the rest of the world and smooth consumption. On the other hand, as firms need to pay a fraction of the production factors before production takes place, labor demand falls, leading to a fall in wages and putting further pressure on consumption. Even though the authors find that interest rates and working capital constraints capture well the regularities of fluctuations in emerging countries, observational data do not appear to support the existence of very tight working capital constraints (Mendoza and Yue, 2012).

Fernandez-Villaverde et al (2011) model interest rates and total factor productivity as two independent autoregressive processes, where interest rates face separate volatility shocks. Even when the interest rate remains constant, an increase in its volatility is thought to distort decision rules, lowering consumption, investment and, eventually, output. Thus, volatility shocks could help increase the variability of consumption significantly. However, in spite of the fact that volatility shocks help explain the higher variability of consumption relative to output, they seem to be unable to account for the strong countercyclicality of the trade balance, which is either modestly countercyclical or acyclical, depending on the model specification.

Collateral constraints and leverage are introduced by Mendoza (2010), whose object of interest are sudden stops. Mendoza's model stipulates shocks to the price of imported intermediate goods and a working capital constraint, which is set much less tightly than in earlier studies. The collateral constraint implies that total debt, which includes a one period bond and working capital loans, cannot exceed a fraction of the value of capital. Relative to a no collateral constraint environment, considering collateral constraints reduces the variability of consumption while increasing the countercyclicality of the trade balance. The effect of the collateral constraint on consumption is understood by the idea that those constraints lead to an increase in buffer stock of savings, which, in turn, increases the ability of agents to smooth consumption.

Using Mexican data to estimate model parameters, Aguiar and Gopinath (2007) find that a small open economy model with trend growth shocks could generate higher variability of consumption relative to output and strong countercyclicality of the trade balance. Under trend growth shocks, whenever output increases, future output is expected to be even higher, which induces the agent to optimally increase consumption more than the increase in current output. On the other hand, using much longer time series, Garcia-Cicco, Pancrazzi and Uribe (2010) estimate a RBC model, very similar to that of Aguiar and Gopinath's, and find that it does a poor job at explaining the observed regularities in Argentina and Mexico. More specifically, the authors conclude that the model predicts a consumption path less volatile than output and a weakly countercyclical trade balance. Then, they move on to incorporate to the previous model preference shocks, country premium shocks and a realistic debt elasticity of the country premium, and find that the augmented model mimics remarkably well the observed business cycles in Argentina over the period 1900–2005.

In Boz, Daude and Durdu (2011), agents observe total factor productivity shocks but not its decomposition into transitory and permanent shocks. Thus, under imperfect information, they learn and make optimal plans based on their beliefs about these shocks. The authors´ motivation relies on the uncertainty surrounding the duration of structural changes in emerging economies and the existence of more severe informational frictions. They find that the introduction of informational frictions to the Aguiar and Gopinath structure is successful in capturing the empirical regularities observed in emerging countries.

The impact of introducing labor market frictions to an environment governed by shocks to both total factor productivity and interest rates is studied in Boz, Durdu and Li (2015). The modeling of the labor market conditions follows that of Mortensen and Pissarides (1994). There, wages are determined by Nash bargaining and job matches depend on the unemployment rate and the vacancies posted by the firms. The interaction of shocks to both total factor productivity and countercyclical interest rates with search-matching frictions, the study concludes, can help account for the joint behavior of consumption, the external sector and wages observed in emerging countries.

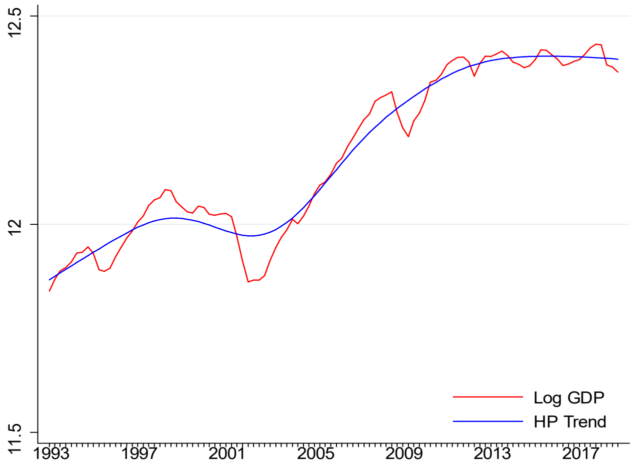
There are some other relevant features present in the emerging market business cycles literature. To start with, in general, agents preferences are represented by the utility function first modeled by Greenwood, Hercowitz, and Huffman (1988). The high degree of substitutability between leisure and consumption in the GHH utility function, which eliminates the income effect on labor supply, generates large responses of consumption and labor to productivity shocks. On the other hand, among others, two strategies have been followed to close the model, that is, to guarantee the existence of a unique stable solution: either an endogenous discounting factor similar to Uzawa-type preferences or a debt-elastic interest rate. In the present study, we follow the latter.

Large economic fluctuations are typical of Argentina. The country has suffered from recurrent recessions, high and persistent inflation rates and profound reversals in the trade balance for decades, even up to the days in which this study was conducted. The difficulty to stabilize output fluctuations has encouraged the emergency of numerous theoretical frameworks, of varied nature and formality levels. This study aims to build a dynamic stochastic general equilibrium RBC model capable of reproducing the empirical regularities of economic cycles in Argentina. Two main features are highlighted. First, the recognition of heterogeneous productivity levels across country sectors, summarized in a tradable sector and a non tradable sector. Second, the importance of shocks to international interest rates in explaining numerous episodes of recessions in the country economic history.

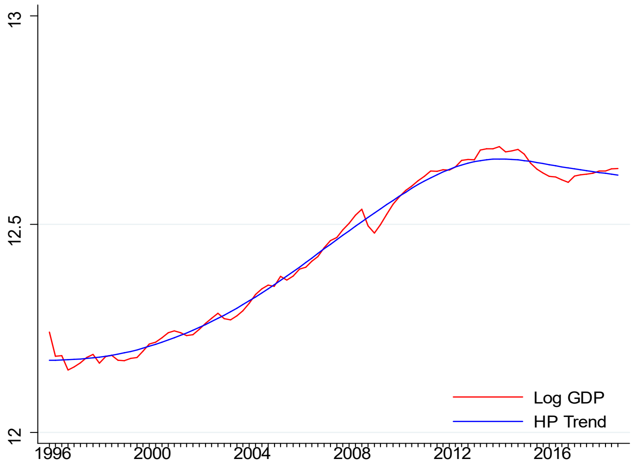
The present study is structured as follows. Section I analyzes empirical regularities of business cycles in three South American countries: Argentina, Brazil and Colombia. Section II outlines the model of an economy that presents two sectors, a tradable and a non tradable sector, and includes a capital adjustment cost, a debt-elastic local interest rate, an endogenous country spread and shocks to the international interest rate. Using data of Argentina from 1993 to 2018, Section III describes the calibration of parameters in the model. Section IV studies the impulse response to shocks to the exogenous processes and evaluates whether the main empirical regularities for emerging economies are matched by the model. Section V concludes.

I. EMPIRICAL REGULARITIES

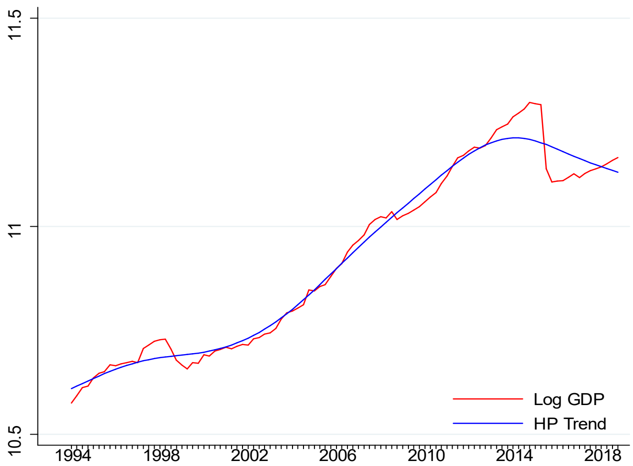
The empirical regularities of emerging economies stated in the introduction seem to be more prominent in Argentina, even when compared to countries of similar income level in the region. Figure 1 depicts the evolution of real output (red line) and its trend component (blue line) for three South American countries: Argentina (Panel A), Brazil (Panel B) and Colombia



(A) ARGENTINA (1993-2018)



(B) BRAZIL (1996-2018)



(C) COLOMBIA (1994-2018)

FIGURE 1: OUTPUT PATHS AROUND TREND

(Panel C)[1]. Data sources are presented in Appendix I. Output is measured in natural logarithmic units and the trend was obtained through the application of the Hodrick-Prescott (HP) filter with a smoothing parameter of 1600. Even though Colombia has recently evidenced some volatility, the panels appear to show much smother paths of economic activity for Brazil and Colombia than for Argentina. In fact, from the mid-nineties up to 2018, while the standard deviation (in percentage points) of the cyclical component of output was 3.91 for Argentina, it was 1.60 for Brazil and 1.62 for Colombia. In other words, the economic cycle in Argentina was approximately 2.4 times more volatile than its regional neighbours Brazil and Colombia during the referenced period of time.

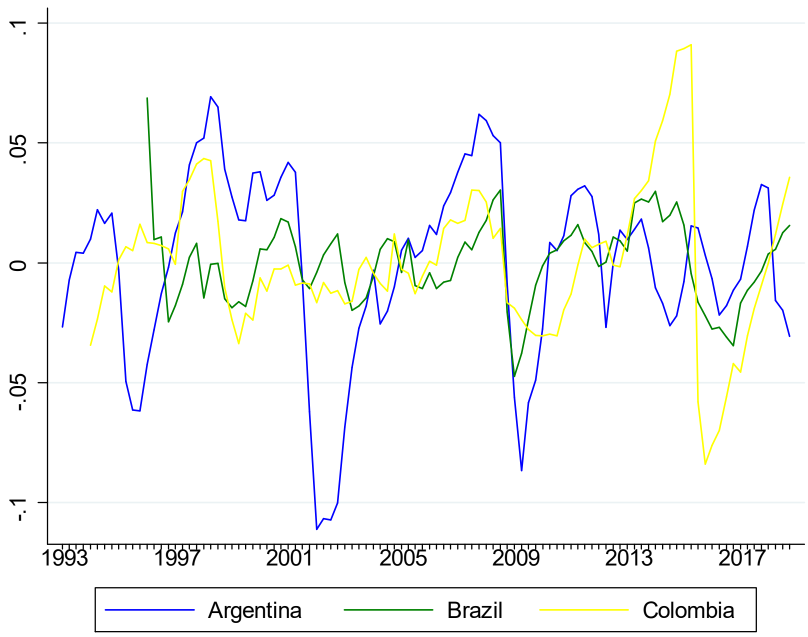


FIGURE 2: ECONOMIC CYCLES

The percentage deviation of output with respect to the trend, that is, the cycle component, is illustrated in Figure 2 for each country. It is noteworthy that, throughout the period of interest, the number of times the module of the Argentine cycle was 0.5 or more is 5. In contrast, for Brazil that number

[1] Brazil and Colombia are the South American countries closest to Argentina in terms of income level, according to the International Monetary Fund (IMF) data available at https://www.imf.org/external/datamapper/NGDPD@WEO/OEMDC/ADVEC/WEOWORLD

Table 1: standard deviation of output

Country	1993-2001	2002-2005	2006-2015	2016-2018
Argentina	3.53	4.81	3.33	2.14
	(0.59)	(1.20)	(0, 53)	(0.62)
Brazil	1.23	1.06	1.80	1.74
	(0.25)	(0.26)	(0.28)	(0.50)
Colombia	2.09	0.77	1.55	0.53
	(0.37)	(0.19)	(0.24)	(0.15)

Notes: HP cyclical component of logged deseasonalized real quaterly output, with smoothing parameter of 1600. The samples are: Argentina, 1993-2018; Brazil, 1996-2018; and Colombia, 1994-2018. Standard errors (S.E.) are shown in parenthesis. The standard deviation (S.D.) and S.E. are reported in percentage terms.

was 1 and for Colombia, 2. Moreover, Argentina cycle moves past the unit circle in one occasion, more precisely, during the quarters corresponding to the economic crash of 2001-2002, while no such event is present in the series of its regional counterparts considered in this study. Thus, business cycles in Argentina not only appear to suffer from added dispersion, but from excess amplitude.

The results summarized above survive the partition of the total time range into sub periods. Table 1 presents, for each country, the standard deviation (in percentage) of the output short term component across arbitrarily defined periods of time. Such periods are meant to be aligned with the various political and economical regimes Argentina experienced during the time range 1993-2018. Standard errors are in parenthesis. Again, the variance of economic cycles in the southern country is higher than in Brazil and Colombia, no matter the sub period of time that is considered. Particularly impressive is the period 2002-2005, in which Argentina´s business cycles were 4.5 times more volatile than the second most volatile in the same period, Brazil. At best, Argentina added volatility relative to Brazil was *only* 1.2 during the period 2016-2018.

The set of measures typically used to characterize economic cyles in emerging economies are presented in Table 2. Here, three features are underlined. First, the excess volatily of consumption relative to that of output is present in the three countries under study. Note, however, that consumption looks to be somewhat more volatile in Argentina than Brazil and Colombia. Added volatility for Argentina, 1.17, is 6 percentage points

Table 2: business cycles regularities

Moment	Argentina 1993-2018	Brazil 1996-2018	Colombia 1994-2018
$\sigma(y)$	3.91	1.60	1.62
	(0.31)	(0.12)	(0.12)
$\sigma(tb/y)$	5.96	2.55	4.01
	(0.42)	(0.12)	(0.18)
$\sigma(i)/\sigma(y)$	3.26	3.15	5.64
	(0.11)	(0.11)	(0.43)
$\sigma(c)/\sigma(y)$	1.17	1.11	1.02
	(0.03)	(0.09)	(0.04)
$\rho(tb/y, y)$	-0.46	-0.39	-0.27
	(0.09)	(0.07)	(0.07)
$\rho(c, y)$	0.96	0.75	0.91
	(0.01)	(0.05)	(0.02)
$\rho(i, y)$	0.96	0.90	0.81
	(0.01)	(0.03)	(0.04)

Notes: HP cyclical component of logged deseasonalized real quaterly series, with smoothing parameter of 1600. The samples are: Argentina, 1993-2018; Brazil, 1996-2018; and Colombia, 1994-2018. GMM estimated S.E. are reported in parentheses. The S.D. and S.E. are reported in percentage terms.

(p.p.) higher than in Brazil and 15 p.p. higher than in Colombia.

Second, a strong negative correlation between the trade balance-to-output ratio and output, the other typical empirical regularity in emerging markets, is also found in each case. Once again, the regularity looks to be more prevalent in Argentina than in its regional neighbours. While the statistical assosiation between output and the share of trade balace is -0.46 in Argentina, it is -0.39 in Brazil and -0.27 in Colombia. Thus, the movements of the Argentine trade balance appear to be assosiated to its economic cycles in greater magnitud that in other comparable countries.

Finally, Argentina also presents a much higher standard deviation (in percentage) of the trade balance-to-output ratio than Brazil and Colombia. The dispersion of 5.6 p.p. observed in the southern country is 1.49 times the measured volatility for Colombia and 2.34 times the observed variability in Brazil. This fact is probably linked to two previously analyzed features: a country with higher dispersion of the trade balance-to-output ratio and a higher correlation of the later with output is very likely to display higher

volatility in its economic cycle.
All in all, relative to countries in the region of similar income levels, Argentina economic cycles seem to be characterized by higher volatility of output, higher excess volatily of consumption relative to output and higher correlation between the tade balance-to-output ratio and output. Moreover, its trade balance-to-output ratio appears to be much more volatile, which might explain the observed higher volatility of short term output fluctuations.

II. THE MODEL ECONOMY

The economy is inhabited by households, firms in the tradable sector and firms in the non tradable sector. These agents interact with one another in the market of tradable goods, the market of non tradable goods, the labor market and, finally, the capital market. It is an open economy because, in the market of tradable goods, households participate in international assets markets by issuing debt in period t to be paid in $t+1$. At the same time, since agents decisions do not influence the international interest rate, the economy is said to be small. The purpose of this section is to introduce the primitives of each agent in this economy. The procedure to solve for the model policy functions is presented in the Appendix II to this study.

A. Households.

In each period t, households demand consumption goods, C_t, and investment goods, I_t and supply labor, L_t, and capital goods, K_t, to the firms of the economy. Additionally, they pay for the debt issued in period $t-1$ and taken as given in period t, D_t. If resources are not enough, households issue new debt to be paid in $t+1$, D_{t+1}, in the international assets market. Their preferences, defined over consumption goods and labor supply, are represented by the following utility function $u : \mathbb{R}^2_+ \longrightarrow \mathbb{R}$:

$$u(C_t, L_t) = \frac{1}{1-\sigma}\left(C_t - \upsilon\frac{L_t^\omega}{\omega}\right)^{1-\sigma} \tag{1}$$

where ω governs the elasticity of labor supply with respect to wages, $\sigma \neq 1$ captures the degree of relative risk aversion and υ is an auxiliary parameter.

As mentioned before, this functional form is due to Greenwood, Hercowitz, and Huffman (1988), and, by inducing a high degree of substitutability between leisure and consumption, eliminates the income effect on labor supply. In this way, the model is expected to produce large responses of consumption and labor to productivity shocks.

Hoseholds decisions are restricted to a budget constraint, all of which prices are relative to the price of tradable goods, normalized to 1:

$$p_t^C C_t + p_t^I I_t + (1 + r_t) D_t = w_t L_t + r_t^K K_t + D_{t+1} \tag{2}$$

where, for each period t, p_t^C is the relative price of consumption goods, p_t^I is the relative price of investment goods, r_t is the local interest rate, w_t is the relative price of labor, or wages, and r_t^K is the relative price of capital.

In each period t, the given stock of capital, net of depreciation, coupled with investment decisions made by households determine the stock of capital available in $t + 1$. In addition, the adjustment of the sotck of capital between periods is thought to be costly, a standard assumption in business cycles models to avoid excessive volatility of investment. Thus, the law of capital accumulation is given by:

$$K_{t+1} = (1 - \delta) K_t + I_t - \frac{\chi}{2} \left(\frac{K_{t+1}}{K_t} - 1 \right)^2 K_t \tag{3}$$

where δ is the rate of capital depreciation and χ regulates the sensitivity of the future stock of capital to capital adjustment costs.

The interest rate paid by households in this economy is influenced by the international interest rate, r_t^*, and a country specific risk premium, S_t, or the country spread. Additionally, in order to guarantee a unique stable solution to the model, the local interest rate is assumed to be elastic to the net rate of change of the stock of debt for period $t + 1$ with respect to its steady state value, $\bar{D}$. Hence, for each unit of debt issued one period earlier, in t households must pay:

$$r_t = S_t r_t^* + \gamma \left[\exp \left(D_{t+1} - \bar{D} \right) - 1 \right] \tag{4}$$

where γ governs the debt-elastic nature of the local interest rate. In their

decision making process, households are assumed to take r_t as given.
Following a very similar formulation to Neumeyer and Perri´s (2005) model, in this economy the country spread is associated to the expected total factor productivity of the tradable sector in period $t+1$:

$$S_t = \overline{\eta}\left(E_t A_{t+1}^T\right) \tag{5}$$

where $\overline{\eta}$ is a decreasing function that defines the relationship. As explained by the authors, this idea is based on models of default and incomplete markets in which default probabilities are high when expectations of productivity shocks are low.
It is well documented by the literature that fluctuations in the international interest rate play a key role in explaining business cycles in emerging countries. In this study, r_t^* is assumed to be exogenous and to follow an autoregressive process of order 1:

$$\ln r_{t+1}^* = (1-\psi_{r^*})\ln r^* + \psi_{r^*}\ln r_t^* + \varepsilon_{t+1}^{r^*} \tag{6}$$

where $\ln(\cdot)$ stands for natural logarithm, ψ_{r^*} is the first-order autoregressive parameter, r^* is the steady state value of the international interest rate and $\varepsilon_{t+1}^{r^*}$ is a zero-mean normally distributed random shock, with variance $\sigma_{r^*}^2$.
In a finite horizon economy, households are not allowed to hold positive amounts of debt in the last period. The analogous constraint in an infinite horizon economy, known as the no-Ponzi condition, is described by:

$$\lim_{j\to\infty} E_t \frac{D_{t+j}}{\prod_{s=0}^{j}(1+r_s)} \leq 0 \tag{7}$$

In each period t, given the chosen level for C_t, households decide the optimal combination of consumption goods coming from the tradable sector, C_t^T, and consumption goods coming from the non tradable sector, C_t^N. The combination technology of consumption goods is described by the following CES function:

$$C_t = \left[\theta_C^{\frac{1}{\phi}}\left(C_t^T\right)^{\frac{\phi-1}{\phi}} + (1-\theta_C)^{\frac{1}{\phi}}\left(C_t^N\right)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}} \tag{8}$$

where $\phi > 0$ is the elasticity of substitution between tradable and non tradable goods and $\theta_C \in (0,1)$ is a parameter that determines the share of tradable consumption goods in total consumption.
Analogously, given the chosen level for I_t, households decide the optimal combination of investment goods coming from the tradable sector, I_t^T, and investment goods coming from the non tradable sector, I_t^N. The combination technology of investment goods is described by the following CES function:

$$I_t = \left[\theta_I^{\frac{1}{\phi}} \left(I_t^T\right)^{\frac{\phi-1}{\phi}} + (1-\theta_I)^{\frac{1}{\phi}} \left(I_t^N\right)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}} \tag{9}$$

where $\theta_I \in (0,1)$ is a parameter that determines the share of tradable investment goods in total investment.

B. Firms.

In each period t, given the relative prices of production factors, w_t and r_t^K, firms in the tradable sector demand labor, L_t^T, and capital, K_t^T, to the households of the economy. After that, they combine those factors and use their technology to produce the tradable goods output of this economy, Y_t^T. The Cobb-Douglas production function representing the technology of firms in the tradable sector is:

$$Y_t^T = \left(A_t^T\right)^{\alpha_T} \left(L_t^T\right)^{\alpha_T} \left(K_t^T\right)^{1-\alpha_T} \tag{10}$$

where A_t^T is the total factor productivity in the tradable sector and $\alpha_T \in (0,1)$ is the share of labor in total output of the tradable sector.
The total factor productivity for the tradable sector, which is taken as given by the sectoral firms, is assumed to follow an autoregressive process of order 1:

$$\ln A_{t+1}^T = (1-\psi_T) \ln A^T + \psi_T \ln A_t^T + \varepsilon_{t+1}^T \tag{11}$$

where ψ_T is the first-order autoregressive parameter, A^T is the steady state value of the total factor productivity in the tradable sector and ε_{t+1}^T is a zero-mean normally distributed random shock, with variance σ_T^2.

Analogously, in each period t, given the relative prices of production factors, w_t and r_t^K, firms in the non tradable sector demand labor, L_t^N, and capital, K_t^N, to the households of the economy. After that, they combine those factors and use their technology to produce the non tradable goods output of this economy, Y_t^N. The Cobb-Douglas production function representing the technology of firms in the non tradable sector is:

$$Y_t^N = \left(A_t^N\right)^{\alpha_N} \left(L_t^T\right)^{\alpha_N} \left(K_t^N\right)^{1-\alpha_N} \quad (12)$$

where A_t^N is the total factor productivity in the non tradable sector and $\alpha_N \in (0,1)$ is the share of labor in total output of the non tradable sector. The total factor productivity for the non tradable sector, which is taken as given by the sectoral firms, is assumed to follow an autoregressive process of order 1:

$$\ln A_{t+1}^N = (1-\psi_N)\ln A^N + \psi_N \ln A_t^N + \varepsilon_{t+1}^N \quad (13)$$

where ψ_N is the first-order autoregressive parameter, A^N is the steady state value of the total factor productivity in the non tradable sector and ε_{t+1}^N is a zero-mean normally distributed random shock, with variance σ_N^2.

III. PARAMETER CALIBRATION

This section discusses the calibration of parameters in the model and in the exogenous processes. One standard procedure in the literature is to solve for the non stochastic steady state of the model and calibration at the same time[2]. Thus, some parameters will be set according to the steady state solution to the model, others will be calibrated using data for Argentina´s economy, and the rest will be taken from previous studies in the literature. The set of parameters values are summarized in Table 3.

To start with, consider the parameters in the utility function. As described in the previous section, ω governs the wage-elasticity of the labor supply, which in this model takes the form $1/(\omega-1)$. In general, as microeconometric estimations are somewhat contradictory to macroeconomic observations,

[2] Such procedure is developed in the Appendix II.B to this study.

TABLE 3: CALIBRATION OF PARAMETER VALUES

Parameter	Description	Value
ϕ	Non tradable/ tradable goods subs. elasticity	0.44
ω	Labor supply elasticity $1/(\omega - 1)$	1.60
σ	Intertemporal elasticity of substitution $1/\sigma$	0.99
α_T	Labor share of income in tradable sector	0.42
α_N	Labor share of income in non tradable sector	0.62
δ	Depreciation rate of capital (quaterly)	1.37
χ	Capital adjustment costs parameter	6
γ	Debt elastic interest rate parameter	0.001
η	Spread elasticity	1.04
σ_S^2	S.D. of country spread shock (%)	1.73
ψ_{r^*}	AR(1) coeff. in international interest rate process	0.81
σ_{r^*}	S.D. of international interest rate shock (%)	0.63
ψ_T	AR(1) coeff. in tradable sector TFP process	0.60
σ_T	S.D. of tradable sector TFP shock (%)	5.73
ψ_N	AR(1) coeff. in non tradable TFP process	0.84
σ_N	S.D. of non tradable sector TFP shock (%)	4.19
θ_C	Tradable goods share of consumption	0.23
θ_I	Tradable goods share of investment	0.53
υ	Auxiliary parameter in utility function	1.91
β	Discount factor	0.87

this parameter is assumed to be between 1 and 2 in the literature. We will follow Neumeyer and Perri (2005) in setting it to 1.60, which is in line with elasticities estimates found in some informal gray literature for Argentina, available online. The relative risk aversion parameter, σ, regulates the curvature of the utility function and, in general, is normalized to some value, which is then changed to perform sensitivity analysis. As it can not assume the value of 1, in this study σ is set to 0.99. The auxiliary parameter υ is defined by the steady state solution to the model, in particular the steady state values of wages, the relative price of consumption goods and labor. When computed, it is equal to 1.91. The calibration of the discount factor also comes from the solution to the balanced growth path, which gives β a value of 0.87.

In a work studying the sources of economic growth in Argentina, Coremberg (2009) measures the annual rate of capital depreciation to be 5.6 percent. As in the present study the unit of time is a quarter, the equivalent quaterly depreciation rate δ is set to be 1.37 percent. The capital stock adjustment cost parameter, χ, is given very dissimilar values across papers in the literature, from García-Cicco, Pancrazi and Uribe (2010), who find a median of 4.60 in their posterior distribution to their financial-friction model, to Neumeyer and Perri (2005), who set a value of 40 for the induced country spread version of their model. The benchmark value taken here will be 6. The parameter γ, which measures the sensitivity of the country interest-rate to deviations of external debt from the steady state value, is set to 0.001 as in García-Cicco, Pancrazi & Uribe (2010).

As explained in Section 3, the model economy, similar to Neumeyer and Perri (2005), considers a country spread that is associated to the expected total factor productivity of the tradable sector in period $t+1$. Despite not stating explicitly the functional form of such association in levels, the authors define a linear relationship in the log linearized version of the analogous in their paper of equation (5):

$$\widetilde{S}_t = -\eta E_t\left(\widetilde{A^T_{t+1}}\right) + \varepsilon^S_t$$

where ε^S_t is a zero-mean normally distributed random shock, with variance σ^2_S. The parameter η is calibrated equal to 1.04 and σ^2_S is set to 1.73 percent. The parameters for the international interest rate exogenous process are also referred to their study, in which ψ_{r^*} is set to 0.81, while σ_{r^*} is set to 0.63.

González-Rozada et al (2004) estimate the elasticity of substitution in the demand for non tradable goods relative to tradable goods in Argentina to be between 0.40 and 0.48. Thus, in the present study the value of ϕ is assumed to be 0.44. On the other hand, the parameters determining the share of tradable consumption and investment goods in total consumption and total investment, respectively, θ_C and θ_I, are defined by the steady state solution to the model. More specifically, θ_C depends on the steady state values of the relative price of non tradable goods and the ratio of non tradable-to-tradable consumption goods. The balanced growth path solution sets a value for θ_C equal to 0.23. Analogously, θ_I depends on the steady state values of the relative price of non tradable goods and the ratio of non tradable-to-tradable investment goods. Its computation gives

a value of 0.53.
The technology parameters for each sector, α_T and α_N, are calibrated using sectoral data[3] of the labor wage remuneration, gross mixed income and gross added value for Argentina. First, the participation of the labor wage remuneration in the sum of labor wage remuneration and gross exploitation surplus was computed for each sector. Then, that share was applied to the gross mixed income to approximate the part of it that corresponds to labor services, the "gross mixed labor income". Lastly, α_T and α_N were computed as the ratio of the sum of the labor wage remuneration and the gross mixed labor income to the gross added value net of production taxes net of subsidies, for each sector.
Finally, Coremberg (2009) calculates time series of total factor productivities for different sectors of the Argentine economy from 1990 to 2006. This study exploits those series to estimate the parameters of the autoregressive processes for A_t^T and A_t^N. Hence, thinking the dataset as a panel in which the individual unit of observation is the sector at time t, the robust fixed effects estimates for ψ_T and ψ_N are 0.60 and 0.84, respectively, while σ_T is estimated to be 5.73 percent and σ_N, 4.19 percent.

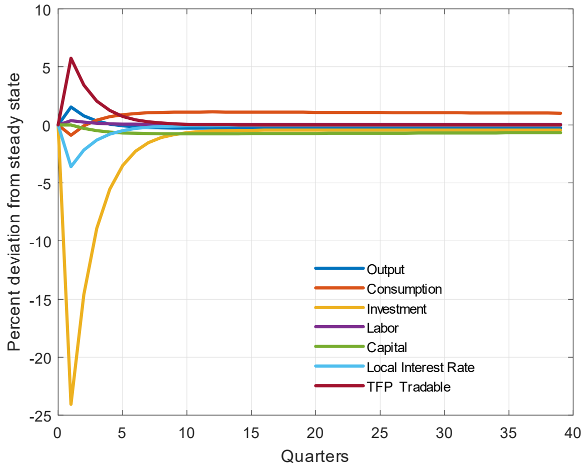
IV. RESULTS

This section analyzes the model in two ways. First, the mechanisms of the model are inspected through the study of the main macroeconomic aggregates impulse response to shocks to the total factor productivity in the tradable sector, the total factor productivity in the non tradable sector and the international interest rate. Second, the model is evaluated in terms of the extent to which it reproduces, under different scenarios, the main empirical regularities of Argentina´s economy, that is, the excess volatility of consumption relative to that of output and the negative correlation between the trade balance-to-output ratio and output.

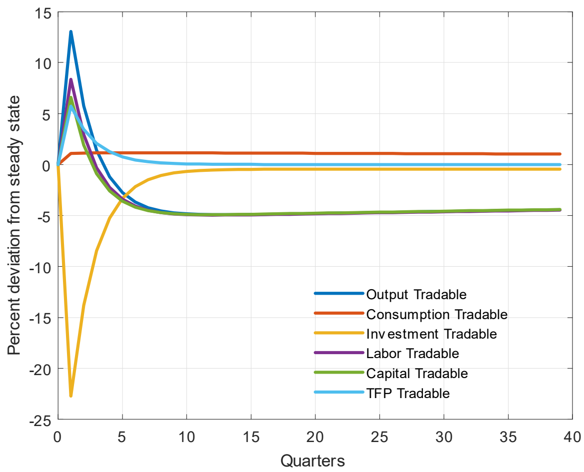
A. Impulse Response Functions.

Figure 3 illustrates the responses of the main macroeconomic aggregates to a shock to the total factor productivity in the tradable sector, in the

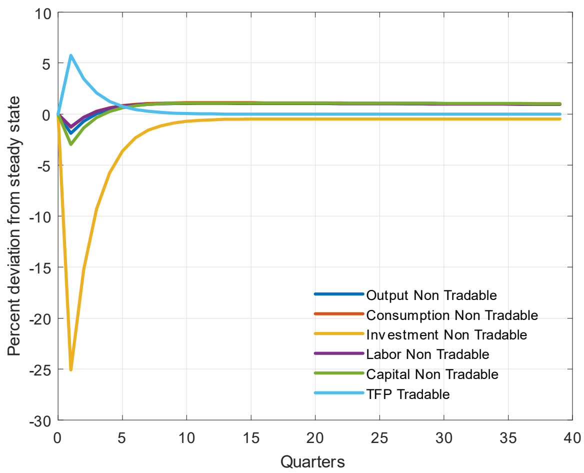
[3] See Appendix I for data sources used for all measures presented in this Section.



(A) ECONOMY

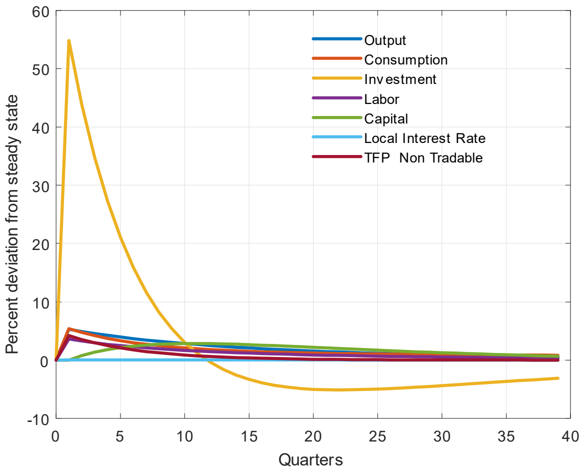


(B) TRADABLE SECTOR

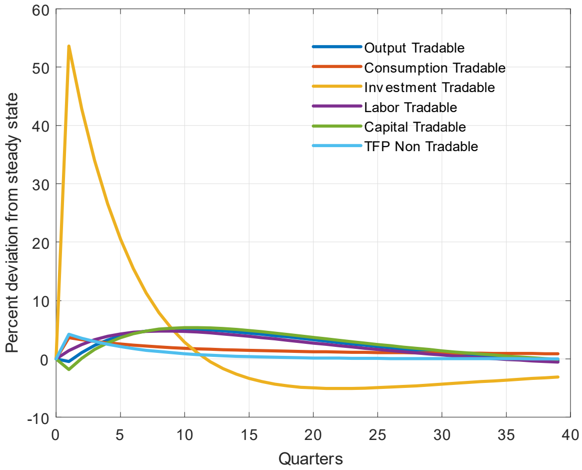


(C) NON TRADABLE SECTOR

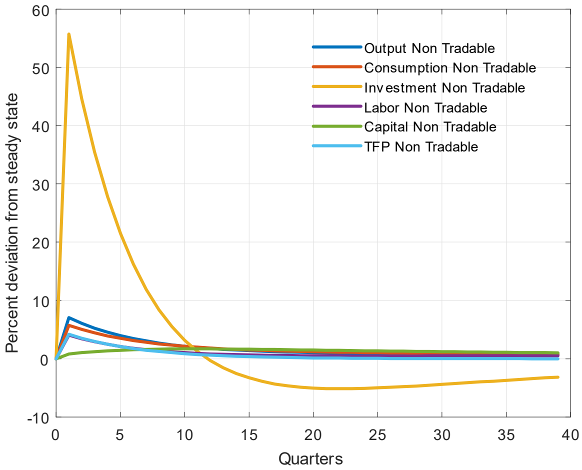
FIGURE 3: IMPULSE RESPONSES TO A TRADABLE SECTOR TFP SHOCK



(A) ECONOMY



(B) TRADABLE SECTOR



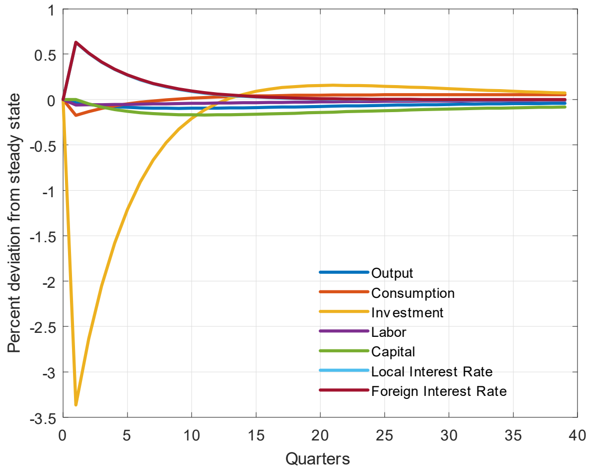
(C) NON TRADABLE SECTOR

FIGURE 4: IMPULSE RESPONSES TO A NON TRADABLE SECTOR TFP SHOCK

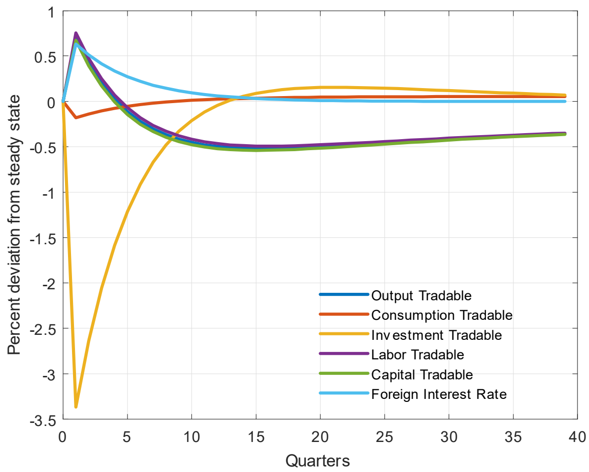
benchmark economy with the parameter values described in the previous section. The responses of variables in the economy as a whole are depicted in Panel A, while Panel B and Panel C show the impulse response functions of macroeconomic outcomes in the tradable and non tradable sector, respectively. A positive productivity shock in the tradable sector has a positive effect on the economy output, explained by a significant increase in the output of the tradable sector, which compensates for the decrease of output in the non tradable sector. A similar dynamic appears to be happening in the case of the labor market. On the other hand, a negative impact on the local interest rate is explained by the fact that the risk premium of this economy is modeled to be a negative function of A^T. This is likely to be linked to the observed increase in the consumption of tradable goods, which households have as substitutes of non tradable consumption goods. The fall in the later looks to be the driving force of a weak short term negative impact on total consumption. The capital stock market shows similar paths: a large increase in the tradable sector is compensated by a lower stock of capital in the non tradable sector in the aggregate economy.

The impulse responses to a shock to the total factor productivity in the non tradable sector are presented in Figure 4, again, for the benchmark economy with the parameter values defined in the previous section. Analogously, the responses of variables of the economy as a whole are depicted in Panel A, while Panel B and Panel C show the impulse response function of macroeconomic outcomes in the tradable and non tradable sector, respectively. A positive shock to productivity in the non tradable sector has a positive impact on all variables in Panel A, with the exception of the local interest rate, which does not experiment movements from its steady state value. Output, consumption, labor, investment and capital in the economy as a whole are all driven by their analogs in the non tradable sector, with minor short term decreases in output and capital in the tradable sector not having any influence whatsoever. As expected from a calibration that sets labor shares $\alpha_N > \alpha_T$, labor responses are much more in line with output responses in the non tradable sector than in the tradable sector.

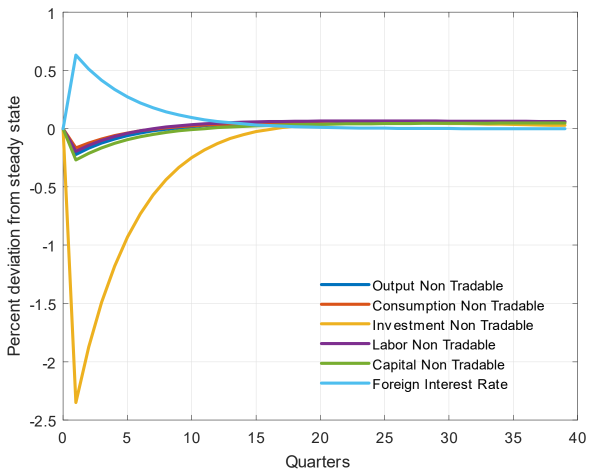
Finally, for the same benchmark economy of the two previous cases, the effects of a shock to the international interest rate on the main macroeconomic aggregates in this economy are depicted in Figure 5. Panel A shows



(A) ECONOMY



(B) TRADABLE SECTOR



(C) NON TRADABLE SECTOR

FIGURE 5: IMPULSE RESPONSES TO AN INTERNATIONAL INTEREST RATE SHOCK

the impulse responses for the economy as a whole, and Panel B and Panel C illustrate the same for the macroeconomic outcomes in the tradable and non tradable sector, respectively. In the aggregate economy depicted in Panel A, a positive shock to the international interest rate has a weak negative effect on total output. While output in the non tradable sector also decreases, in the tradable sector output is impacted positively, as well as capital and labor in the same sector. The local interest rate follows the same path as the international, which means that for each unit of debt the economy will need more tradable goods to pay for $D(1+r)$. While consumption and investment goods fall in both sectors, it appears that the tradable sector is more affected by the shock than the non tradable sector, especially in terms of investment.

B. Shock and Parameter Sensitivity Analysis.

Another form of model analysis in this context implies evaluating how well it reproduces the empirical regularities of economic cycles in emerging countries. In particular, the focus of this sub-section is on two measures: the volatility of consumption relative to that of output and the correlation between the trade balance-to-output ratio and output. As described in Section 2, for Argentina data during the period 1993-2018, those measures are 1.17 and −0.46, respectively. Table 4 presents the results for a number of simulation[4] settings, which consist of different combinations of shocks to the exogenous processes of the model: A_t^N, A_t^T and r_t^*.

Table 4: simulated business cycles

Setting	$\sigma(c)/\sigma(y)$	$\rho(tb/y, y)$
Data	1.17	−0.46
Shocks to A_t^N	1.00	−0.09
Shocks to A_t^N and A_t^T	1.00	−0.03
Shocks to A_t^N, A_t^T and r_t^* (b)	1.00	−0.03

(b) stands for benchmark. Parameters are set to values presented in Table 3.

For the benchmark values of parameters, no experiment results in a satisfactory reproduction of Argentina´s regularities. For all settings, consump-

[4] The simulated trade balance was computed as $TB_t = Y_t^T + D_{t+1} - C_t^T + I_t^T - (1+r_t)D_t$.

tion is as volatile as output and the tade balance-to-output ratio correlation with output, while negative, is not as strong as it is in observational data.

TABLE 5: SENSITIVITY ANALYSIS

Setting	$\sigma(c)/\sigma(y)$	$\rho(tb/y, y)$
Data	1.17	-0.46
$\omega = 1.60$ (b)	1.00	-0.03
$\omega = 1.20$	0.97	-0.04
$\omega = 4.00$	1.12	-0.01
$\chi = 6$ (b)	1.00	-0.03
$\chi = 3$	0.96	-0.09
$\chi = 12$	1.01	-0.01
$\eta = 1.04$ (b)	1.00	-0.03
$\eta = 0.01$	1.01	-0.03
$\eta = 14$	1.16	-0.06

(b) stands for benchmark. All three shocks are considered.

In order to see how the model works, an alternative exercise consists of manipulating the parameter values set in the benchmark specification. Table 5 shows the results of changing the values of three parameters of interest: the wage elasticity of labor supply, the capital adjustment cost parameter and the spread elasticity with respect to the expected value of total factor productivity in the tradable sector. Despite not finding any significant change in the strength of the negative correlation between the trade balance-to-output ratio and output, some advances are found with regards to the consumption-output relative volatility. Changing ω from 1.60 to 4 has an impact of 12 p.p. in $\sigma(c)/\sigma(y)$, which value becomes 5 p.p. shy of the data realization. Such change implies a significant reduction in the wage elasticity of labor supply, from 1.67 to 0.33. Thus, under $\omega = 4$, an equal change in wages (hence, in income) results in a weaker movement of labor supply, thus provoking larger volatility in consumption decisions. On the other hand, it seems that the manipulation of the adjustment capital cost parameter does not have a significant effect on the empirical regularities of interest, although its reduction induces a stronger (negative) correlation of output with the trade balance-to-output ratio. Finally, the model performance is inspected through the setting of parameter η to extreme values 0.01 and 14. In relation to its benchmark value of 1.04, increasing the sensitivity of the country spread to changes in the expected TFP of the tradable sector to 14 approximates better the empirical regularities that emerge from the data. While matching the excess volatility of consumption with respect to output, the manipulation also results in a higher module of $\rho\,(tb/y, y)$. This may be pointing out that making country spreads endogenous to the economy dynamics is a feature in need of a deeper exploration in the literature. Such as models have departed from the "only TFP shocks" setting in the original RBC frameworks, a more comprehensive structural depiction of the interest rate risk premium might prove useful in enhancing the ability of the economic discipline to understand economic cycles in emerging economies.

V. CONCLUSIONS

Understanding economic cycles has been one of the most challenging items in the macroeconomist research agenda for decades. While the twentieth century was largely dominated by idea that mainly nominal factors drive economic fluctuations, from Kydland and Prescott (1982) on, real factors have played an increasingly important role in the litereature. Emerging market economies posed new challenges to researchers, as uncommon empirical regularities were identified in observational data. In concrete, emerging countries, vis-a-vis their developed counterparts, are characterized by a higher variability of consumption relative to output and a strong countercyclicality of the trade balance. As a consequence, the focus of the literature on emerging economies has been put in the development of models which could potentially approximate such empirical regularities.

The aim os this study is to build a dynamic stochastic general equilibrium RBC model capable of reproducing the empirical regularities of economic cycles in Argentina. Two main features are highlighted. First, the recognition of heterogeneous productivity levels across country sectors, summarized in a tradable sector and a non tradable sector. Second, the importance of shocks to international interest rates in explaining numerous episodes of recessions in the country economic history. In addition, local interest rates are impacted by the country risk premium, which in turn is elastic to the expected total factor productivity of the tradable sector. As standard in the literature, the solution to the non stochastic steady state and calibration are done simultaneously. Thus, some parameters are set according to the steady state solution to the model, others are calibrated using data for Argentina´s economy, and the rest is taken from previous studies in the literature.

Shocks to total factor productivity in the non tradable sector induce better outcomes than that of the tradable sector. Meanwhile, shocks to international interest rates affect positively the production of tradable goods but has a negative impact on the non tradable sector, which drives the economy as a whole in the reduction of total outcome. On the other hand, the model does a poor job at reproducing the two main empirical regularities for Argentina. However, parameter sensitivity analysis shows that the manipulation of the country spread elasticity to expected TFP in the tradable sector, η, makes the model match the excess volatility of consumption and moves the trade balance-to-output correlation with output in the right direction. This may suggest the need of a more comprehensive structural depiction of the interest rate risk premium, so that the economic discipline could improve its ability to explain economic cycles in emerging economies.

APPENDIX I: DATA SOURCES

Argentina:

- National Accounts: Instituto Nacional de Estadísticas y Censos-INDEC. Quaterly deseasonalized data for gross domestic product, private consumption, public consumption, investment, exports and imports can be obtained in [1] for the period 2004-2018 at 2004 prices and in [2] for the period 1993-2003 at 1993 prices. Series at 1993 prices were computed at 2004 prices by replicating growth rates of the series during 1993-2004 in the series at 2004 prices, from 2004 backwards.
- Country Risk: Local newspaper Ambito Financiero publishes historical data in its website link [3].
- Income distribution: INDEC. Data for the calibration of α_T and α_N comes from the Cuenta de generación del ingreso (CGI) for I16-IV18, which is publisehd in [4].
- Total factor productivity for each sector: Coremberg (2009).

Brazil:

- National Accounts: Instituto Brasileiro de Geografia e Estatística-IBGE. Quaterly deseasonalized data for gross domestic product, private consumption, public consumption, investment, exports and imports can be obtained in [5] for the period 1996-2018 at 1995 prices.

Colombia:

- National Accounts: Departamento Administrativo Nacional de Estadística -DANE. Quaterly deseasonalized data for gross domestic product, private consumption, public consumption, investment, exports and imports was obtained in [6] for the period 2005-2018 at 2015

[1] *https://www.indec.gob.ar/nivel4_default.asp?id_tema_1=3&id_tema_2=9&id_tema_3=47*
[2] *https://www.indec.gob.ar/informacion-de-archivo.asp?solapa=5*
[3] *https://www.ambito.com/contenidos/riesgo-pais-historico.html*
[4] *https://www.indec.gob.ar/nivel4_default.asp?id_tema_1=3&id_tema_2=9&id_tema_3=49*
[5] *https://sidra.ibge.gov.br/tabela/6613*
[6] *https://www.dane.gov.co/index.php/estadisticas-por-tema/cuentas-nacionales/cuentas-nacionales-trimestrales*

prices and for the period 1994-2005 at 1994 prices. Series at 1994 prices were computed at 2015 prices by replicating growth rates of the series during 1994-2005 in the series at 2015 prices, from 2005 backwards.

USA:

- Interest Rates: Federal Reserve Bank of St. Louis Economic Data-FRED

Series for the 3-Month US Treasury Bill are published in [7].

APPENDIX II: SOLUTION OF THE MODEL

The objective of this section is to show how the model is solved. First, we will find the equilibrium conditions which form a system of non linear stochastic difference equations. Then, we will calibrate and show the non stochastic steady state of the model. A first order Taylor approximation will be applied to the equilibrium conditions to log linearize the system around the economy balanced growth path. Finally, the approximate policy functions will be obtained by use of the Matlab program "solab.m", due to Paul Klein, which explodes the QZ Theorem or Generalized Schur Decomposition.
We will start by listing the primitives of the economy, which were presented previously in the body of this study. The preferences of households are represented by the utility function:

$$u(C_t, L_t) = \frac{1}{1-\sigma}\left(C_t - \upsilon\frac{L_t^{\omega}}{\omega}\right)^{1-\sigma} \tag{A.1}$$

The budget constraint is:

$$p_t^C C_t + p_t^I I_t + (1+r_t)D_t = w_t L_t + r_t^K K_t + D_{t+1} \tag{A.2}$$

The law of capital accumulation is described by:

$$K_{t+1} = (1-\delta)K_t + I_t - \frac{\chi}{2}\left(\frac{K_{t+1}}{K_t} - 1\right)^2 K_t \tag{A.3}$$

[7] *https://fred.stlouisfed.org/series/TB3MS*

Households pay (charge) an interest rate for issuing (buying) debt that follows:

$$r_t = S_t r_t^* + \gamma \left[\exp\left(D_{t+1} - \bar{D}\right) - 1\right] \tag{A.4}$$

The spread between local and international interest rates is assosiated to the expected total productivity shock in the tradable sector:

$$S_t = \overline{\eta}\left(E_t A_{t+1}^T\right) \tag{A.5}$$

The international interest rate is assumed to follow an AR(1) process:

$$\ln r_{t+1}^* = (1 - \psi_{r^*}) \ln r^* + \psi_{r^*} \ln r_t^* + \varepsilon_{t+1}^{r^*} \tag{A.6}$$

with $\varepsilon_{t+1}^{r^*} \sim \mathcal{N}\left(0, \sigma_{r^*}^2\right)$.

Demands for tradable and non tradable consumption goods make up the aggregate demand for consumption goods according to:

$$C_t = \left[\theta_C^{\frac{1}{\phi}} \left(C_t^T\right)^{\frac{\phi-1}{\phi}} + (1 - \theta_C)^{\frac{1}{\phi}} \left(C_t^N\right)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}} \tag{A.7}$$

Demands for tradable and non tradable investment goods make up the aggregate demand for investment goods according to:

$$I_t = \left[\theta_I^{\frac{1}{\phi}} \left(I_t^T\right)^{\frac{\phi-1}{\phi}} + (1 - \theta_I)^{\frac{1}{\phi}} \left(I_t^N\right)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}} \tag{A.8}$$

The no-Ponzi condition is:

$$\lim_{j \to \infty} E_t \frac{D_{t+j}}{\prod_{s=0}^{j} (1 + r_s)} \leq 0 \tag{A.9}$$

The technology of firms in the tradable sector is described by:

$$Y_t^T = \left(A_t^T\right)^{\alpha_T} \left(L_t^T\right)^{\alpha_T} \left(K_t^T\right)^{1-\alpha_T} \tag{A.10}$$

The total factor productivity in the tradable sector is assumed to follow an

AR(1) process:

$$\ln A_{t+1}^T = (1-\psi_T)\ln A^T + \psi_T \ln A_t^T + \varepsilon_{t+1}^T \tag{A.11}$$

with $\varepsilon_{t+1}^T \sim \mathcal{N}(0, \sigma_T^2)$.

The technology of firms in the non tradable sector is described by:

$$Y_t^N = \left(A_t^N\right)^{\alpha_N}\left(L_t^N\right)^{\alpha_N}\left(K_t^N\right)^{1-\alpha_N} \tag{A.12}$$

The total factor productivity in the non tradable sector is assumed to follow an AR(1) process:

$$\ln A_{t+1}^N = (1-\psi_N)\ln A^N + \psi_N \ln A_t^N + \varepsilon_{t+1}^N \tag{A.13}$$

with $\varepsilon_{t+1}^N \sim \mathcal{N}(0, \sigma_N^2)$.

A. Equilibrium Conditions

The study of households decision process will be divided into two parts. First, we will separately characterize households decisions over $\left(C_t^T, C_t^N\right)$ and $\left(I_t^T, I_t^N\right)$ for all t, given the non tradable goods relative price and aggregate values for C_t and I_t, respectively. Then, we will study how households optimally choose the set $(C_t, L_t, K_{t+1}, D_{t+1})$ for all t, by maximizing the expected utility function given the economy relative prices and state variables K_t and D_t.

The household problem of selecting demand for consumption goods in each sector is described by:

$$\min_{C_t^T, C_t^N} C_t^T + p_t^N C_t^N$$

subject to

$$\left[\theta_C^{\frac{1}{\phi}}\left(C_t^T\right)^{\frac{\phi-1}{\phi}} + (1-\theta_C)^{\frac{1}{\phi}}\left(C_t^N\right)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}} = C_t$$

where p_t^N is the price of non-tradable goods relative to that of tradable goods. The Lagrange function for this problem is:

$$\mathcal{L}_C^H = C_t^T + p_t^N C_t^N + \lambda_t^C \left\{ C_t - \left[\theta_C^{\frac{1}{\phi}} \left(C_t^T\right)^{\frac{\phi-1}{\phi}} + (1-\theta_C)^{\frac{1}{\phi}} \left(C_t^N\right)^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}} \right\}$$

where λ_t^C is the Lagrange multiplier. The multiplier represents the increment in total cost due to an unit increase in C_t, that is, the relative price of consumption goods, p_t^C.

The first order conditions for this problem are:

$$\left[C_t^T\right] : 1 - \lambda_t^C \left[\theta_C^{\frac{1}{\phi}} \left(C_t^T\right)^{\frac{\phi-1}{\phi}} + (1-\theta_C)^{\frac{1}{\phi}} \left(C_t^N\right)^{\frac{\phi-1}{\phi}} \right]^{\frac{1}{\phi-1}} \theta_C^{\frac{1}{\phi}} \left(C_t^T\right)^{\frac{-1}{\phi}} = 0$$

$$1 = \lambda_t^C \theta_C^{\frac{1}{\phi}} \left(\frac{C_t}{C_t^T} \right)^{\frac{1}{\phi}} \tag{A.14}$$

$$\left[C_t^N\right] : p_t^N - \lambda_t^C \left[\theta_C^{\frac{1}{\phi}} \left(C_t^T\right)^{\frac{\phi-1}{\phi}} + (1-\theta_C)^{\frac{1}{\phi}} \left(C_t^N\right)^{\frac{\phi-1}{\phi}} \right]^{\frac{1}{\phi-1}} \ldots$$

$$\ldots (1-\theta_C)^{\frac{1}{\phi}} \left(C_t^N\right)^{\frac{-1}{\phi}} = 0$$

$$p_t^N = \lambda_t^C (1-\theta_C)^{\frac{1}{\phi}} \left(\frac{C_t}{C_t^N} \right)^{\frac{1}{\phi}} \tag{A.15}$$

$$\left[\lambda_t^C\right] : C_t - \left[\theta_C^{\frac{1}{\phi}} \left(C_t^T\right)^{\frac{\phi-1}{\phi}} + (1-\theta_C)^{\frac{1}{\phi}} \left(C_t^N\right)^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}} = 0 \tag{A.16}$$

Where $[x]$ denotes the partial derivative of the Lagrange function with respect to the generic variable x. Replace x accordingly in each optimization problem presented in this study.

Dividing (A.14) by (A.15):

$$\frac{1}{p_t^N} = \left(\frac{\theta_C}{1-\theta_C} \right)^{\frac{1}{\phi}} \left(\frac{C_t^T}{C_t^N} \right)^{\frac{-1}{\phi}}$$

$$\left(\frac{1-\theta_C}{\theta_C} \right)^{\frac{1}{\phi}} \frac{1}{p_t^N} = \left(\frac{C_t^N}{C_t^T} \right)^{\frac{1}{\phi}}$$

$$\frac{C_t^N}{C_t^T} = \frac{1-\theta_C}{\theta_C}\left(p_t^N\right)^{-\phi} \tag{A.17}$$

Equations (A.16) and (A.17) fully characterize the household optimal decisions over C_t^T and C_t^N, given p_t^N and C_t. The relative price of consumption goods, which is equal to λ_t^C, can be obtained as follows:

$$C_t^T + p_t^N C_t^N \equiv p_t^C C_t$$

$$1 + p_t^N \frac{C_t^N}{C_t^T} = p_t^C \frac{C_t}{C_t^T}$$

Plug in equations (A.17) and (A.14):

$$1 + p_t^N \left(\frac{1-\theta_C}{\theta_C}\left(p_t^N\right)^{-\phi}\right) = p_t^C \left(\left(\lambda_t^C\right)^{-\phi}\theta_C^{-1}\right)$$

Finally, use that $\lambda_t^C \equiv p_t^C$:

$$p_t^C = \left[\theta_C + (1-\theta_C)\left(p_t^N\right)^{1-\phi}\right]^{\frac{1}{1-\phi}} \tag{A.18}$$

The household problem of selecting demand for investment goods in each sector is described by:

$$\min_{I_t^T, I_t^N} I_t^T + p_t^N I_t^N$$

subject to

$$\left[\theta_I^{\frac{1}{\phi}}\left(I_t^T\right)^{\frac{\phi-1}{\phi}} + (1-\theta_I)^{\frac{1}{\phi}}\left(I_t^N\right)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}} = I_t \tag{A.19}$$

If the procedure followed in the previous problem is applied, we obtain:

$$\frac{I_t^N}{I_t^T} = \frac{1-\theta_I}{\theta_I}\left(p_t^N\right)^{-\phi} \tag{A.20}$$

Hence, equations (A.19) and (A.20) fully characterize the household optimal decisions over I_t^T and I_t^N, given p_t^N and I_t. The relative price of investment goods is:

$$p_t^I = \left[\theta_I + (1-\theta_I)\left(p_t^N\right)^{1-\phi}\right]^{\frac{1}{1-\phi}} \tag{A.21}$$

The second part of the analysis of households decision problems involves the choice of $(C_t, L_t, K_{t+1}, D_{t+1})$ for all t, given the economy relative prices and state variables K_t and D_t.
The household problem of selecting supply of labor and demand for consumption, investment and debt is described by:

$$\max_{\{C_t,L_t,I_t,D_{t+1}\}_{t=0}^{\infty}} E_0\left[\sum_{t=0}^{\infty}\beta^t\frac{1}{1-\sigma}\left(C_t-\upsilon\frac{L_t^{\omega}}{\omega}\right)^{1-\sigma}\right]$$

subject to

$$p_t^C C_t + p_t^I I_t + (1+r_t)D_t = w_t L_t + r_t^K K_t + D_{t+1}$$

where E_0 denotes the mathematical expectation conditional on information at $t=0$ and $\beta \in (0,1)$ is a discount factor.
A standard procedure is to assume that households internalize the law of capital accumulation, so the problem becomes:

$$\max_{\{C_t,L_t,K_{t+1},D_{t+1}\}_{t=0}^{\infty}} E_0\left[\sum_{t=0}^{\infty}\beta^t\frac{1}{1-\sigma}\left(C_t-\upsilon\frac{L_t^{\omega}}{\omega}\right)^{1-\sigma}\right]$$

subject to

$$p_t^C C_t + p_t^I K_{t+1} - (1-\delta)p_t^I K_t + \frac{\chi}{2}\left(\frac{K_{t+1}}{K_t}-1\right)^2 p_t^I K_t ...$$

$$... + (1+r_t)D_t = w_t L_t + r_t^K K_t + D_{t+1}$$

The Lagrange function for this problem is:

$$\begin{aligned}\mathcal{L}^H \;=\; & E_0\sum_{t=0}^{\infty}\beta^t\left\{\frac{1}{1-\sigma}\left(C_t-\upsilon\frac{L_t^{\omega}}{\omega}\right)^{1-\sigma}+\lambda_t\left[w_tL_t+r_t^KK_t+D_{t+1}-p_t^CC_t...\right.\right.\\ & \left.\left.... - p_t^I K_{t+1} + (1-\delta)p_t^I K_t - \frac{\chi}{2}\left(\frac{K_{t+1}}{K_t}-1\right)^2 p_t^I K_t - (1+r_t)D_t\right]\right\}\end{aligned}$$

where λ_t is the Lagrange multiplier. The first order conditions for this problem are:

$$[C_t] : \left(C_t-\upsilon\frac{L_t^{\omega}}{\omega}\right)^{-\sigma} - \lambda_t p_t^C = 0$$

(A.22)
$$\left(C_t - \upsilon\frac{L_t^{\omega}}{\omega}\right)^{-\sigma} = \lambda_t p_t^C$$

$$[L_t] : \left(C_t - \upsilon\frac{L_t^{\omega}}{\omega}\right)^{-\sigma}\left(-\upsilon L_t^{\omega-1}\right) + \lambda_t w_t = 0$$

(A.23)
$$\left(C_t - \upsilon\frac{L_t^{\omega}}{\omega}\right)^{-\sigma}\upsilon L_t^{\omega-1} = \lambda_t w_t$$

$$[K_{t+1}]: \lambda_t\left[-p_t^I - \frac{\chi}{2}p_t^I K_t 2\left(\frac{K_{t+1}}{K_t} - 1\right)\frac{1}{K_t}\right] + E_t\beta\lambda_{t+1}\left[r_{t+1}^K + (1-\delta)\,p_{t+1}^I\right.$$

$$\left. ... - \frac{\chi}{2}p_{t+1}^I\left(\frac{K_{t+2}}{K_{t+1}} - 1\right)^2 - \frac{\chi}{2}p_{t+1}^I K_{t+1}2\left(\frac{K_{t+2}}{K_{t+1}} - 1\right)\left(\frac{-K_{t+2}}{K_{t+1}^2}\right)\right] = 0$$

Doing some algebra results in:
(A.24)
$$1 = E_t\beta\frac{\lambda_{t+1}}{\lambda_t}\frac{r_{t+1}^K + p_{t+1}^I\left\{1 - \delta + \chi\left(\frac{K_{t+2}}{K_{t+1}} - 1\right)\left[\frac{K_{t+2}}{K_{t+1}} - \frac{1}{2}\left(\frac{K_{t+2}}{K_{t+1}} - 1\right)\right]\right\}}{p_t^I\left[1 + \chi\left(\frac{K_{t+1}}{K_t} - 1\right)\right]}$$

$$[D_{t+1}] : \lambda_t - E_t\beta\lambda_{t+1}\left(1 + r_{t+1}\right) = 0$$

(A.25)
$$1 = E_t\beta\frac{\lambda_{t+1}}{\lambda_t}\left(1 + r_{t+1}\right)$$

(A.26)
$$[\lambda_t] : p_t^C C_t + p_t^I K_{t+1} - (1-\delta)p_t^I K_t + \frac{\chi}{2}\left(\frac{K_{t+1}}{K_t} - 1\right)^2 p_t^I K_t ...$$

$$... + (1 + r_t)D_t = w_t L_t + r_t^K K_t + D_{t+1}$$

Divide (A.23) by (A.22) to obtain:

(A.27)
$$\upsilon L_t^{\omega-1} = \frac{w_t}{p_t^C}$$

Equations (A.22), (A.24), (A.25), (A.26) and (A.27) fully characterize the household optimal decisions over $(C_t, L_t, K_{t+1}, D_{t+1})$ for all t, given the economy relative prices and state variables K_t and D_t.

On the other hand, firms participate in perfectly competitive factors markets, demanding labor and capital to households. In each sector, given factors prices and the state variable A_t^i, $i = T, N$, they choose the demand for labor and capital so that their benefits are maximized. Thus, the tradable sector firm problem is described by:

$$\max_{L_t^T, K_t^T} \left(A_t^T\right)^{\alpha_T} \left(L_t^T\right)^{\alpha_T} \left(K_t^T\right)^{1-\alpha_T} - w_t L_t^T - r_t^K K_t^T$$

The first order conditions for this problem are:

$$\left[L_t^T\right] : \left(A_t^T\right)^{\alpha_T} \alpha_T \left(L_t^T\right)^{\alpha_T - 1} \left(K_t^T\right)^{1-\alpha_T} - w_t = 0$$

$$w_t = \alpha_T \frac{\left(A_t^T\right)^{\alpha_T} \left(L_t^T\right)^{\alpha_T} \left(K_t^T\right)^{1-\alpha_T}}{L_t^T}$$

$$w_t = \alpha_T \frac{Y_t^T}{L_t^T} \tag{A.28}$$

$$\left[K_t^T\right] : \left(A_t^T\right)^{\alpha_T} \left(L_t^T\right)^{\alpha_T} \left(1-\alpha_T\right) \left(K_t^T\right)^{-\alpha_T} - r_t^K = 0$$

$$r_t^K = \left(1-\alpha_T\right) \frac{\left(A_t^T\right)^{\alpha_T} \left(L_t^T\right)^{\alpha_T} \left(K_t^T\right)^{1-\alpha_T}}{K_t^T}$$

$$r_t^K = \left(1-\alpha_T\right) \frac{Y_t^T}{K_t^T} \tag{A.29}$$

Equations (A.28) and (A.29) fully characterize the tradable sector firm optimal decisions over L_t^T and K_t^T, given w_t, r_t^K and A_t^T.

The non tradable sector firm problem is described by:

$$\max_{L_t^N, K_t^N} p_t^N \left(A_t^N\right)^{\alpha_N} \left(L_t^N\right)^{\alpha_N} \left(K_t^N\right)^{1-\alpha_N} - w_t L_t^N - r_t^K K_t^N$$

The first order conditions for this problem are:

$$\left[L_t^N\right] : p_t^N \left(A_t^N\right)^{\alpha_N} \alpha_N \left(L_t^N\right)^{\alpha_N - 1} \left(K_t^N\right)^{1-\alpha_N} - w_t = 0$$

$$w_t = \alpha_N p_t^N \frac{\left(A_t^N\right)^{\alpha_N} \left(L_t^N\right)^{\alpha_N} \left(K_t^N\right)^{1-\alpha_N}}{L_t^N}$$

(A.30)
$$w_t = \alpha_N \frac{p_t^N Y_t^N}{L_t^N}$$

$$\left[K_t^N\right] : p_t^N \left(A_t^N\right)^{\alpha_N} \left(L_t^N\right)^{\alpha_N} (1-\alpha_N) \left(K_t^N\right)^{-\alpha_N} - r_t^K = 0$$

$$r_t^K = (1-\alpha_N) p_t^N \frac{\left(A_t^N\right)^{\alpha_N} \left(L_t^N\right)^{\alpha_N} \left(K_t^N\right)^{1-\alpha_N}}{K_t^N}$$

(A.31)
$$r_t^K = (1-\alpha_N) \frac{p_t^N Y_t^N}{K_t^N}$$

Hence, equations (A.30) and (A.31) fully characterize the non tradable sector firm optimal decisions over L_t^N and K_t^N, given w_t, r_t^K and A_t^N.

After defining their optimal decision rules, households and firms in each sector interact with one another in the factors and goods markets. The economy aggregate equilibrium outcomes will be set once an equilibrium is met in each of those markets. Equilibrium in the labor market is met when:

(A.32)
$$L_t^T + L_t^N = L_t$$

Equilibrium in the capital market is defined by:

(A.33)
$$K_t^T + K_t^N = K_t$$

In the market for tradable goods, equilibrium is met when:

(A.34)
$$C_t^T + I_t^T = Y_t^T + D_{t+1} - (1+r_t)D_t$$

In the market for non tradable goods, equilibrium is defined by:

(A.35)
$$C_t^N + I_t^N = Y_t^N$$

The autoregressive processes for A_t^T, A_t^N and r_t^*, the last four equations and each of the equations that characterize households and firms optimal decisions define the equilibrium conditions of the economy. It can be shown that equation (A.26), the household budget constraint, is implied by a subset of the rest of the equations. To see that, use equations (A.28) and (A.29) to obtain:

$$Y_t^T = w_t L_t^T + r_t^K K_t^T$$

Analogously, the use of equations (A.30) and (A.31) results in:

$$p_t^N Y_t^N = w_t L_t^N + r_t^K K_t^N$$

Plug in the last two expressions in (A.34) and (A.35), respectively:

$$C_t^T + I_t^T - D_{t+1} + (1+r_t)D_t = w_t L_t^T + r_t^K K_t^T$$

$$p_t^N C_t^N + p_t^N I_t^N = w_t L_t^N + r_t^K K_t^N$$

Sum each term of the last two expressions:

$$C_t^T + p_t^N C_t^N + I_t^T + p_t^N I_t^N + (1+r_t)D_t = w_t \left(L_t^T + L_t^N\right) + r_t^K \left(K_t^T + K_t^N\right) + D_{t+1}$$

Use equations (A.32) and (A.33) and the definition of $p_t^C C_t$ and $p_t^I I_t$ to obtain:

$$p_t^C C_t + p_t^I I_t + (1+r_t)D_t = w_t L_t + r_t^K K_t + D_{t+1}$$

Finally, by replacing I_t using equation (A.3), the budget constraint is gotten:

$$p_t^C C_t + p_t^I K_{t+1} - (1-\delta)p_t^I K_t + \frac{\chi}{2}\left(\frac{K_{t+1}}{K_t} - 1\right)^2 p_t^I K_t ...$$

$$... + (1+r_t)D_t = w_t L_t + r_t^K K_t + D_{t+1}$$

Thus, the system of non linear stochastic difference equations that is formed by the equilibrium conditions (EC) of this economy is:

(EC.1) $$\frac{C_t^N}{C_t^T} = \frac{1-\theta_C}{\theta_C}\left(p_t^N\right)^{-\phi}$$

(EC.2) $$\left[\theta_C^{\frac{1}{\phi}}\left(C_t^T\right)^{\frac{\phi-1}{\phi}} + (1-\theta_C)^{\frac{1}{\phi}}\left(C_t^N\right)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}} = C_t$$

(EC.3)
$$p_t^C = \left[\theta_C + (1-\theta_C)\left(p_t^N\right)^{1-\phi}\right]^{\frac{1}{1-\phi}}$$

(EC.4)
$$\frac{I_t^N}{I_t^T} = \frac{1-\theta_I}{\theta_I}\left(p_t^N\right)^{-\phi}$$

(EC.5)
$$\left[\theta_I^{\frac{1}{\phi}}\left(I_t^T\right)^{\frac{\phi-1}{\phi}} + (1-\theta_I)^{\frac{1}{\phi}}\left(I_t^N\right)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}} = I_t$$

(EC.6)
$$p_t^I = \left[\theta_I + (1-\theta_I)\left(p_t^N\right)^{1-\phi}\right]^{\frac{1}{1-\phi}}$$

(EC.7)
$$\left(C_t - \upsilon\frac{L_t^\omega}{\omega}\right)^{-\sigma} = \lambda_t p_t^C$$

(EC.8)
$$\upsilon L_t^{\omega-1} = \frac{w_t}{p_t^C}$$

(EC.9)
$$w_t = \alpha_T\frac{Y_t^T}{L_t^T}$$

(EC.10)
$$r_t^K = (1-\alpha_T)\frac{Y_t^T}{K_t^T}$$

(EC.11)
$$w_t = \alpha_N\frac{p_t^N Y_t^N}{L_t^N}$$

(EC.12)
$$r_t^K = (1-\alpha_N)\frac{p_t^N Y_t^N}{K_t^N}$$

(EC.13)
$$L_t^T + L_t^N = L_t$$

(EC.14)
$$K_t^T + K_t^N = K_t$$

(EC.15)
$$Y_t^T = \left(A_t^T\right)^{\alpha_T}\left(L_t^T\right)^{\alpha_T}\left(K_t^T\right)^{1-\alpha_T}$$

(EC.16)
$$Y_t^N = \left(A_t^N\right)^{\alpha_N}\left(L_t^N\right)^{\alpha_N}\left(K_t^N\right)^{1-\alpha_N}$$

(EC.17)
$$C_t^N + I_t^N = Y_t^N$$

(EC.18)
$$Y_t = Y_t^T + p_t^N Y_t^N$$

(EC.19)
$$1 = E_t\beta\frac{\lambda_{t+1}}{\lambda_t}\frac{r_{t+1}^K + p_{t+1}^I\left\{1-\delta+\chi\left(\frac{K_{t+2}}{K_{t+1}}-1\right)\left[\frac{K_{t+2}}{K_{t+1}}-\frac{1}{2}\left(\frac{K_{t+2}}{K_{t+1}}-1\right)\right]\right\}}{p_t^I\left[1+\chi\left(\frac{K_{t+1}}{K_t}-1\right)\right]}$$

(EC.20)
$$1 = E_t\beta\frac{\lambda_{t+1}}{\lambda_t}\left(1+r_{t+1}\right)$$

(EC.21)
$$C_t^T + I_t^T = Y_t^T + D_{t+1} - (1+r_t)D_t$$

(EC.22)
$$I_t = K_{t+1} - (1-\delta)K_t + \frac{\chi}{2}\left(\frac{K_{t+1}}{K_t}-1\right)^2 K_t$$

(EC.23)
$$r_t = S_t r_t^* + \gamma\left[\exp\left(D_{t+1}-\bar{D}\right)-1\right]$$

(EC.24)
$$S_t = \overline{\eta}\left(E_t A_{t+1}^T\right)$$

(EC.25)
$$\ln A_{t+1}^T = (1-\psi_T)\ln A^T + \psi_T \ln A_t^T + \varepsilon_{t+1}^T$$

$$\ln A^N_{t+1} = (1-\psi_N)\ln A^N + \psi_N \ln A^N_t + \varepsilon^N_{t+1} \tag{EC.26}$$

$$\ln r^*_{t+1} = (1-\psi_{r^*})\ln r^* + \psi_{r^*} \ln r^*_t + \varepsilon^{r^*}_{t+1} \tag{EC.27}$$

The set of all variables in the model is:

$$\left(\begin{array}{c} C^N_t, C^T_t, p^N_t, C_t, p^C_t, I^N_t, I^T_t, I_t, p^I_t, L_t, \lambda_t, w_t, Y^T_t, L^T_t, \\ r^K_t, K^T_t, Y^N_t, L^N_t, K^N_t, K_t, A^T_t, A^N_t, Y_t, r_t, D_t, S_t, r^*_t \end{array}\right)$$

Then, the system includes 27 equations and 27 variables, which guarantees the existence of a unique solution.

B. Non Stochastic Steady State and Calibration

First, recall Table 3 in the body of the study, in which parameter values resulting from calibration are presented.

Parameter	Description	Value
ϕ	Non tradable/ tradable goods subs. elasticity	0.44
ω	Labor supply elasticity $1/(\omega-1)$	1.60
σ	Intertemporal elasticity of substitution $1/\sigma$	0.99
α_T	Labor share of income in tradable sector	0.42
α_N	Labor share of income in non tradable sector	0.62
δ	Depreciation rate of capital (quaterly)	1.37
χ	Capital adjustment costs parameter	6
γ	Debt elastic interest rate parameter	0.001
η	Spread elasticity	1.04
σ^2_S	S.D. of country spread shock (%)	1.73
ψ_{r^*}	AR(1) coeff. in international interest rate process	0.81
σ_{r^*}	S.D. of international interest rate shock (%)	0.63
ψ_T	AR(1) coeff. in tradable sector TFP process	0.60
σ_T	S.D. of tradable sector TFP shock (%)	5.73
ψ_N	AR(1) coeff. in non tradable TFP process	0.84
σ_N	S.D. of non tradable sector TFP shock (%)	4.19
θ_C	Tradable goods share of consumption	0.23
θ_I	Tradable goods share of investment	0.53
υ	Auxiliary parameter in utility function	1.91
β	Discount factor	0.87

In the process of calibrating and solving for the balanced growth path presented in this section, a group of variables and ratios values will be taken as given. The steady state value of the real international interest rate is computed as the average over the series which construction is described in Section III. That gives a value for $\overline{r^*}$ equal to 0.00117. The local interest rate value in the balanced growth path is also calculated using averages over long times series. In this case, for each quarter from IV-98 to IV-18, the nominal local interest rate is computed as the 90-day U.S. T-bill rate plus the J.P. Morgan EMBI Global Spread. Real rates are obtained by subtracting expected U.S. GDP deflator inflation from the nominal dollar rate. Expected inflation in period t is computed as the average of inflation in the current period and in the three preceding periods. Thus, the value of $\overline{r}$ is 0.1541. On the other hand, the data published by the INDEC regarding the net added value in each sector allows for the calibration of the ratio of output in the tradable sector over total output. The steady state value of $\overline{Y^T}/\overline{Y}$ is calculated to be 0.23. In this study, the steady state value of the debt-to-output ratio, $\overline{D}/\overline{Y}$, is taken from Neumeyer and Perri (2005), who set it to be 0.42. On the other hand, a subset of steady state values will be defined as targets or normalizations. That will be the cases for $\overline{Y}$, $\overline{L}$, $\overline{I^N}/\overline{I^T}$ and $\overline{p^N}$, whose target values are presented in the following Table.

Measure	Value	Source
$\overline{r^*}$	0.00117	Data
$\overline{r}$	0.1541	Data
$\overline{Y^T}/\overline{Y}$	0.23	Data
$\overline{D}/\overline{Y}$	0.42	Neumeyer and Perri (2005)
$\overline{Y}$	1	Target
$\overline{L}$	1/3	Target
$\overline{I^N}/\overline{I^T}$	2/3	Target
$\overline{p^N}$	2	Target

After having set and calibrated a group of parameters, target ratios and variables, we move on with the procedure to get the steady state solution and calibration of the rest of the model parameters. The steady state is defined as a vector of values for model variables, which:

- For each generic variable x_t, satisfy[8] $x_t = x_{t+1} = \overline{x}$, and

[8] Letters with an upper bar denote variables in the steady state.

- Solve the system of equations (EC.1)-(EC.27).

Thus, take the system of equations (EC.1)-(EC.27) and write it in its steady state form:

(S.S.1)
$$\frac{\overline{C^N}}{\overline{C^T}} = \frac{1-\theta_C}{\theta_C}\left(\overline{p^N}\right)^{-\phi}$$

(S.S.2)
$$\left[\theta_C^{\frac{1}{\phi}}\left(\overline{C^T}\right)^{\frac{\phi-1}{\phi}} + (1-\theta_C)^{\frac{1}{\phi}}\left(\overline{C^N}\right)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}} = \overline{C}$$

(S.S.3)
$$\overline{p^C} = \left[\theta_C + (1-\theta_C)\left(\overline{p^N}\right)^{1-\phi}\right]^{\frac{1}{1-\phi}}$$

(S.S.4)
$$\frac{\overline{I^N}}{\overline{I^T}} = \frac{1-\theta_I}{\theta_I}\left(\overline{p^N}\right)^{-\phi}$$

(S.S.5)
$$\left[\theta_I^{\frac{1}{\phi}}\left(\overline{I^T}\right)^{\frac{\phi-1}{\phi}} + (1-\theta_I)^{\frac{1}{\phi}}\left(\overline{I^N}\right)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}} = \overline{I}$$

(S.S.6)
$$\overline{p^I} = \left[\theta_I + (1-\theta_I)\left(\overline{p^N}\right)^{1-\phi}\right]^{\frac{1}{1-\phi}}$$

(S.S.7)
$$\left(\overline{C} - \upsilon\frac{\overline{L}^{\omega}}{\omega}\right)^{-\sigma} = \overline{\lambda}\overline{p^C}$$

(S.S.8)
$$\upsilon\overline{L}^{\omega-1} = \frac{\overline{w}}{\overline{p^C}}$$

(S.S.9)
$$\overline{w} = \alpha_T\frac{\overline{Y^T}}{\overline{L^T}}$$

$$\overline{r^K} = (1-\alpha_T)\,\frac{\overline{Y^T}}{\overline{K^T}} \tag{S.S.10}$$

$$\overline{w} = \alpha_N \frac{\overline{p^N Y^N}}{\overline{L^N}} \tag{S.S.11}$$

$$\overline{r^K} = (1-\alpha_N)\,\frac{\overline{p^N Y^N}}{\overline{K^N}} \tag{S.S.12}$$

$$\overline{L^T} + \overline{L^N} = \overline{L} \tag{S.S.13}$$

$$\overline{K^T} + \overline{K^N} = \overline{K} \tag{S.S.14}$$

$$\overline{Y^T} = \left(\overline{A^T}\right)^{\alpha_T} \left(\overline{L^T}\right)^{\alpha_T} \left(\overline{K^T}\right)^{1-\alpha_T} \tag{S.S.15}$$

$$\overline{Y^N} = \left(\overline{A^N}\right)^{\alpha_N} \left(\overline{L^N}\right)^{\alpha_N} \left(\overline{K^N}\right)^{1-\alpha_N} \tag{S.S.16}$$

$$\overline{C^N} + \overline{I^N} = \overline{Y^N} \tag{S.S.17}$$

$$\overline{Y} = \overline{Y^T} + \overline{p^N Y^N} \tag{S.S.18}$$

$$1 = \beta \frac{\overline{r^K} + \overline{p^I}(1-\delta)}{\overline{p^I}} \tag{S.S.19}$$

$$1 = \beta\,(1+\overline{r}) \tag{S.S.20}$$

$$\overline{C^T} + \overline{I^T} = \overline{Y^T} - \overline{r}\overline{D} \tag{S.S.21}$$

(S.S.22) $$\overline{I} = \delta\overline{K}$$

(S.S.23) $$\overline{r} = \overline{S}\overline{r^*}$$

(S.S.24) $$\overline{S} = \overline{\eta}\left(\overline{A^T}\right)$$

(S.S.25) $$\overline{A^T} = A^T$$

(S.S.26) $$\overline{A^N} = A^N$$

(S.S.27) $$\overline{r^*} = r^*$$

Having calibrated the values of $\overline{r}$ and $\overline{r^*}$ enables the use of equation (S.S.23) to get $\overline{S}$. The calibrated value for $\overline{r}$ also permits calibrating for β by use of equation (S.S.20):

$$1 = \beta(1+\overline{r})$$

$$\beta = \frac{1}{1+\overline{r}}$$

Given ϕ, equation (S.S.4) coupled with the target for $\overline{I^N}/\overline{I^T}$ and the normalization of $\overline{p^N}$ allows for calibrating θ_I:

$$\frac{\overline{I^N}}{\overline{I^T}} = \frac{1-\theta_I}{\theta_I}\left(\overline{p^N}\right)^{-\phi}$$

$$\left(\overline{p^N}\right)^{\phi}\frac{\overline{I^N}}{\overline{I^T}} = \frac{1}{\theta_I} - 1$$

$$1 + \left(\overline{p^N}\right)^{\phi}\frac{\overline{I^N}}{\overline{I^T}} = \frac{1}{\theta_I}$$

$$\theta_I = \left[1 + \left(\overline{p^N}\right)^{\phi}\frac{\overline{I^N}}{\overline{I^T}}\right]^{-1}$$

Once θ_I is known, equation (S.S.6) facilitates $\overline{p^I}$. Then, given δ, equation

(S.S.19) can be used to solve for $\bar{r}^K$:

$$1 = \beta \frac{\bar{r}^K + \bar{p}^I(1-\delta)}{\bar{p}^I}$$

$$1 = \left(\frac{1}{1+\bar{r}}\right) \frac{\bar{r}^K + \bar{p}^I(1-\delta)}{\bar{p}^I}$$

$$1 + \bar{r} = \frac{\bar{r}^K}{\bar{p}^I} + 1 - \delta$$

$$\bar{r}^K = \bar{p}^I \left(\bar{r} + \delta\right)$$

Hence, given α_T, equation (S.S.10) provides the value for the capital-to-ouput ratio in the tradable sector:

$$\overline{r^K} = (1-\alpha_T) \frac{\overline{Y^T}}{\overline{K^T}}$$

$$\frac{\overline{K^T}}{\overline{Y^T}} = \frac{(1-\alpha_T)}{\overline{r^K}}$$

The same can be done for the capital-to-ouput ratio in the non tradable sector using equation (S.S.12), given α_N:

$$\overline{r^K} = \overline{p^N} \left(1-\alpha_N\right) \frac{\overline{Y^N}}{\overline{K^N}}$$

$$\frac{\overline{K^N}}{\overline{Y^N}} = (1-\alpha_N) \frac{\overline{p^N}}{\overline{r^K}}$$

Now, we will solve for $\overline{A^N}/\overline{A^T}$. First, use equation (S.S.15) to solve for:

$$\frac{\overline{Y^T}}{\overline{K^T}} = \frac{\left(\overline{A^T}\right)^{\alpha_T} \left(\overline{L^T}\right)^{\alpha_T} \left(\overline{K^T}\right)^{1-\alpha_T}}{\overline{K^T}}$$

$$\frac{\overline{Y^T}}{\overline{K^T}} = \left(\frac{\overline{A^T L^T}}{\overline{K^T}}\right)^{\alpha_T}$$

$$\frac{\overline{K^T}}{\overline{A^T L^T}} = \left(\frac{\overline{K^T}}{\overline{Y^T}}\right)^{\frac{1}{\alpha_T}}$$

Analogously for the non tradable sector, use equation (S.S.16):

$$\frac{\overline{K^N}}{\overline{A^N L^N}} = \left(\frac{\overline{K^N}}{\overline{Y^N}}\right)^{\frac{1}{\alpha_N}}$$

Then, equal equations (S.S.9) and (S.S.11):

$$\alpha_T \frac{\overline{Y^T}}{\overline{L^T}} = \alpha_N \frac{\overline{p^N Y^N}}{\overline{L^N}}$$

$$\alpha_T \frac{\left(\overline{A^T}\right)^{\alpha_T}\left(\overline{L^T}\right)^{\alpha_T}\left(\overline{K^T}\right)^{1-\alpha_T}}{\overline{L^T}} = \alpha_N \frac{\overline{p^N}\left(\overline{A^N}\right)^{\alpha_N}\left(\overline{L^N}\right)^{\alpha_N}\left(\overline{K^N}\right)^{1-\alpha_N}}{\overline{L^N}}$$

$$\alpha_T \overline{A^T}\left(\frac{\overline{K^T}}{\overline{A^T L^T}}\right)^{1-\alpha_T} = \alpha_N \overline{p^N A^N}\left(\frac{\overline{K^N}}{\overline{A^N L^N}}\right)^{1-\alpha_N}$$

Replace the previously obtained expressions for $\overline{K^T}/\overline{A^T L^T}$ and $\overline{K^N}/\overline{A^N L^N}$:

$$\alpha_T \overline{A^T}\left(\frac{\overline{K^T}}{\overline{Y^T}}\right)^{\frac{1-\alpha_T}{\alpha_T}} = \alpha_N \overline{p^N A^N}\left(\frac{\overline{K^N}}{\overline{Y^N}}\right)^{\frac{1-\alpha_N}{\alpha_N}}$$

$$\frac{\overline{A^N}}{\overline{A^T}} = \frac{\alpha_T\left(\frac{\overline{K^T}}{\overline{Y^T}}\right)^{\frac{1-\alpha_T}{\alpha_T}}}{\alpha_N \overline{p^N}\left(\frac{\overline{K^N}}{\overline{Y^N}}\right)^{\frac{1-\alpha_N}{\alpha_N}}}$$

Equation (S.S.18) and the target $\overline{Y^T}/\overline{Y} \equiv sh_T$ determine the between-sector output ratio:

$$\overline{Y} = \overline{Y^T} + \overline{p^N Y^N}$$

$$\frac{\overline{Y}}{\overline{Y}} = sh_T + \overline{p^N}\frac{\overline{Y^N}}{\overline{Y}}$$

$$1 - sh_T = \overline{p^N}\frac{\overline{Y^N}}{\overline{Y}}$$

$$\frac{\overline{Y^N}}{\overline{Y}} = \frac{1 - sh_T}{\overline{p^N}}$$

Dividing the last expression by $\overline{Y^T}/\overline{Y} \equiv sh_T$:

$$\frac{\overline{Y^N}}{\overline{Y^T}} = \frac{1 - sh_T}{sh_T \overline{p^N}}$$

The labor for each sector can be computed by taking the steps that follow. First, use (S.S.15) to solve for the output-to-labor ratio in the tradable sector:

$$\frac{\overline{Y^T}}{\overline{L^T}} = \frac{\left(\overline{A^T}\right)^{\alpha_T}\left(\overline{L^T}\right)^{\alpha_T}\left(\overline{K^T}\right)^{1-\alpha_T}}{\overline{L^T}}$$

$$\frac{\overline{Y^T}}{\overline{L^T}} = \overline{A^T}\left(\frac{\overline{K^T}}{\overline{A^T}\,\overline{L^T}}\right)^{1-\alpha_T}$$

(S.S.28)
$$\frac{\overline{Y^T}}{\overline{L^T}} = \overline{A^T}\left(\frac{\overline{K^T}}{\overline{Y^T}}\right)^{\frac{1-\alpha_T}{\alpha_T}}$$

Analogously for the non tradable sector, use (S.S.16) to solve for:

(S.S.29)
$$\frac{\overline{Y^N}}{\overline{L^N}} = \overline{A^N}\left(\frac{\overline{K^N}}{\overline{Y^N}}\right)^{\frac{1-\alpha_N}{\alpha_N}}$$

Then, divide the last two expressions:

$$\frac{\overline{L^N}}{\overline{L^T}} \equiv l_N = \left(\frac{\overline{A^N}}{\overline{A^T}}\right)^{-1}\left(\frac{\overline{K^T}}{\overline{Y^T}}\right)^{\frac{1-\alpha_T}{\alpha_T}}\left(\frac{\overline{K^N}}{\overline{Y^N}}\right)^{-\frac{1-\alpha_N}{\alpha_N}}\frac{\overline{Y^N}}{\overline{Y^T}}$$

Now, use the normalized value $\overline{L}$ and equation (S.S.13) to get:

$$\overline{L^T} + \overline{L^N} = \overline{L}$$

$$1 + l_N = \frac{\overline{L}}{\overline{L^T}}$$

$$\overline{L^T} = \frac{1}{1 + l_N}\overline{L}$$

Analogously for the non tradable sector:

$$\overline{L^N} = \frac{l_N}{1+l_N}\overline{L}$$

Obtain the expression for $\overline{A^T}$ as follows. Take equations (S.S.28) and (S.S.29):

$$\overline{Y^T} = \overline{A^T L^T}\left(\frac{\overline{K^T}}{\overline{Y^T}}\right)^{\frac{1-\alpha_T}{\alpha_T}}$$

$$\overline{Y^N} = \overline{A^N L^N}\left(\frac{\overline{K^N}}{\overline{Y^N}}\right)^{\frac{1-\alpha_N}{\alpha_N}}$$

Then, plug-in the last two expressions in equation (S.S.18):

$$\overline{Y} = \overline{Y^T} + \overline{p^N Y^N}$$

$$\overline{Y} = \overline{A^T L^T}\left(\frac{\overline{K^T}}{\overline{Y^T}}\right)^{\frac{1-\alpha_T}{\alpha_T}} + \overline{p^N A^N L^N}\left(\frac{\overline{K^N}}{\overline{Y^N}}\right)^{\frac{1-\alpha_N}{\alpha_N}}$$

$$\overline{Y} = \overline{A^T L^T}\left(\frac{\overline{K^T}}{\overline{Y^T}}\right)^{\frac{1-\alpha_T}{\alpha_T}} + \overline{p^N A^N L^N}\left(\frac{\overline{K^N}}{\overline{Y^N}}\right)^{\frac{1-\alpha_N}{\alpha_N}}$$

$$\overline{Y} = \overline{A^T}\left[\overline{L^T}\left(\frac{\overline{K^T}}{\overline{Y^T}}\right)^{\frac{1-\alpha_T}{\alpha_T}} + \overline{p^N}\left(\frac{\overline{A^N}}{\overline{A^T}}\right)\overline{L^N}\left(\frac{\overline{K^N}}{\overline{Y^N}}\right)^{\frac{1-\alpha_N}{\alpha_N}}\right]$$

$$\overline{A^T} = \frac{\overline{Y}}{\left[\overline{L^T}\left(\frac{\overline{K^T}}{\overline{Y^T}}\right)^{\frac{1-\alpha_T}{\alpha_T}} + \overline{p^N}\left(\frac{\overline{A^N}}{\overline{A^T}}\right)\overline{L^N}\left(\frac{\overline{K^N}}{\overline{Y^N}}\right)^{\frac{1-\alpha_N}{\alpha_N}}\right]}$$

where the target value for $\overline{Y}$ is used.

Now, obtain the level of $\overline{A^N}$:

$$\overline{A^N} = \left(\frac{\overline{A^N}}{\overline{A^T}}\right)\overline{A^T}$$

Thus, the level values of $\overline{Y^T}$ and $\overline{Y^N}$ can be computed.

$\overline{D}$ can be found by exploiting the target value $\overline{D}/\overline{Y} = x_Y^D$:

$$\overline{D} = x_Y^D \overline{Y}$$

Futhermore, either equation (S.S.9) or equation (S.S.11) enables getting $\overline{w}$. We can also use equation (S.S.10) to find $\overline{K^T}$:

$$\overline{r^K} = (1-\alpha_T)\frac{\overline{Y^T}}{\overline{K^T}}$$

$$\overline{K^T} = (1-\alpha_T)\frac{\overline{Y^T}}{\overline{r^K}}$$

Analogously for the non tradable sector:

$$\overline{K^N} = (1-\alpha_N)\frac{\overline{p^N Y^N}}{\overline{r^K}}$$

Now, get $\overline{K}$ by the use of equation (S.S.14) and $\overline{I}$ using equation (S.S.22). Equation (S.S.5) and target value $\overline{I^N}/\overline{I^T} = x_{I_T^N}$ facilitates $\overline{I^T}$:

$$\left[\theta_I^{\frac{1}{\phi}}\left(\overline{I^T}\right)^{\frac{\phi-1}{\phi}} + (1-\theta_I)^{\frac{1}{\phi}}\left(\overline{I^N}\right)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}} = \overline{I}$$

$$\left[\theta_I^{\frac{1}{\phi}}\left(\overline{I^T}\right)^{\frac{\phi-1}{\phi}} + (1-\theta_I)^{\frac{1}{\phi}}\left(x_{I_T^N}\right)^{\frac{\phi-1}{\phi}}\left(\overline{I^T}\right)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}} = \overline{I}$$

$$\left[\left(\overline{I^T}\right)^{\frac{\phi-1}{\phi}}\left(\theta_I^{\frac{1}{\phi}} + (1-\theta_I)^{\frac{1}{\phi}}\left(x_{I_T^N}\right)^{\frac{\phi-1}{\phi}}\right)\right]^{\frac{\phi}{\phi-1}} = \overline{I}$$

$$\overline{I^T} = \frac{\overline{I}}{\left(\theta_I^{\frac{1}{\phi}} + (1-\theta_I)^{\frac{1}{\phi}}\left(x_{I_T^N}\right)^{\frac{\phi-1}{\phi}}\right)^{\frac{\phi}{\phi-1}}}$$

$\overline{I^N}$ is found:

$$\overline{I^N} = x_{I_T^N}\overline{I^T}$$

Use equation (S.S.11) to find $\overline{C^N}$:

$$\overline{C^N} + \overline{I^N} = \overline{Y^N}$$

$$\overline{C^N} = \overline{Y^N} - \overline{I^N}$$

What follows is the procedure to obtain the between-sector consumption ratio, used later on to solve for the rest of the steady state values of variables in levels. To start with, take equation (S.S.21):

$$\overline{C^T} + \overline{I^T} = \overline{Y^T} - \overline{r}\overline{D}$$

$$\frac{\overline{C^T}}{\overline{Y^T}} + \frac{\overline{I^T}}{\overline{Y^T}} = 1 - \overline{r}\frac{\overline{D}}{\overline{Y^T}}\frac{\overline{Y}}{\overline{Y}}$$

$$\frac{\overline{C^T}}{\overline{Y^T}} + \frac{\overline{I^T}}{\overline{Y^T}} = 1 - \overline{r}\frac{\overline{D}}{\overline{Y}}\frac{\overline{Y}}{\overline{Y^T}}$$

Replace the target $\overline{D}/\overline{Y} \equiv \overline{d}$ and use $\overline{Y}/\overline{Y^T} = 1 + \overline{p^N Y^N}/\overline{Y^T}$ from equation (S.S.18):

$$\frac{\overline{C^T}}{\overline{Y^T}} + \frac{\overline{I^T}}{\overline{Y^T}} = 1 - \overline{r}\overline{d}\left(1 + \frac{\overline{p^N Y^N}}{\overline{Y^T}}\right)$$

$$\frac{\overline{C^T}}{\overline{Y^T}} = 1 - \overline{r}\overline{d} - \overline{r}\overline{d}\frac{\overline{p^N Y^N}}{\overline{Y^T}} - \frac{\overline{I^T}}{\overline{Y^T}}$$

Now, take equation (S.S.17):

$$\overline{C^N} + \overline{I^N} = \overline{Y^N}$$

$$\frac{\overline{C^N}}{\overline{Y^N}} = 1 - \frac{\overline{I^N}}{\overline{Y^N}}$$

Thus, dividing the two previously obtained expressions:

$$\frac{\overline{C^N}}{\overline{C^T}} = \frac{\left(1 - \frac{\overline{I^N}}{\overline{Y^N}}\right)\frac{\overline{Y^N}}{\overline{Y^T}}}{1 - \overline{r}\overline{d} - \overline{r}\overline{d}\left(\frac{\overline{p^N Y^N}}{\overline{Y^T}}\right) - \frac{\overline{I^T}}{\overline{Y^T}}}$$

This ratio can be plugged in equation (S.S.1) to calibrate θ_C:

$$\frac{\overline{C^N}}{\overline{C^T}} = \frac{1-\theta_C}{\theta_C}\left(\overline{p^N}\right)^{-\phi}$$

$$\theta_C = \left[1 + \left(\overline{p^N}\right)^{\phi}\frac{\overline{C^N}}{\overline{C^T}}\right]^{-1}$$

Given that $\overline{C^N}$ had already been found, the ratio can also provide the steady state value of consumption of tradable goods:

$$\overline{C^T} = \left(\frac{\overline{C^N}}{\overline{C^T}}\right)^{-1} \overline{C^N}$$

All the objects in the left hand side of equation (S.S.2) were obtained, so use it to compute $\overline{C}$. $\overline{p^C}$ can be obtained straight forward from equation (S.S.3). Hence, given ω, equation (S.S.8) enables the calibration for υ:

$$\upsilon \overline{L}^{\omega-1} = \frac{\overline{w}}{\overline{p^C}}$$

$$\upsilon = \frac{\overline{w}\,\overline{L}^{1-\omega}}{\overline{p^C}}$$

Lastly, given σ, make use of equation (S.S.7) to solve for the steady state value of the Lagrange multiplier:

$$\left(\overline{C} - \upsilon \frac{\overline{L}^{\omega}}{\omega}\right)^{-\sigma} = \overline{\lambda}\,\overline{p^C}$$

$$\overline{\lambda} = \left[\overline{p^C}\left(\overline{C} - \upsilon \frac{\overline{L}^{\omega}}{\omega}\right)^{\sigma}\right]^{-1}$$

C. Log Linearization of Equilibrium Conditions

The solution strategy used to obtain the policy functions will be based on a system of equations which are the result of the log linearization of the original non linear system around the non stochastic steady state. After having solved for the later, we proceed with the log linearization of the equilibrium conditions.

To start with, define the period t log deviation of the generic variable V_t from its non stochastic steady state value[9]:

$$\widetilde{V}_t \equiv \ln V_t - \ln \overline{V} \tag{D.1}$$

[9] The only variable for which this definition will not be used is D_t. In this case, linearization is donde in levels $\widetilde{D}_t \equiv D_t - \overline{D}$.

This definition allows writing V_t as follows:

$$V_t = \overline{V} e^{\widetilde{V}_t}$$

To see this, use properties of the logarithmic and exponencial functions and conduct some algebra as shown below:

$$\overline{V} e^{\widetilde{V}_t} = \overline{V} e^{\left(\ln V_t - \ln \overline{V}\right)}$$

$$\overline{V} e^{\widetilde{V}_t} = \overline{V} e^{\left(\ln \frac{V_t}{\overline{V}}\right)}$$

$$\overline{V} e^{\widetilde{V}_t} = \overline{V} \frac{V_t}{\overline{V}}$$

$$\overline{V} e^{\widetilde{V}_t} = V_t$$

In addition, recall that a first order Taylor linear approximation of a function $f(x, y)$ around $(\overline{x}, \overline{y})$ is written:

(D.2) $$f(x, y) \approx f(\overline{x}, \overline{y}) + f_x(\overline{x}, \overline{y})\,(x - \overline{x}) + f_y(\overline{x}, \overline{y})\,(y - \overline{y})$$

Even though the log linearization exercise can be done using the previous formula, as will be the case for some equations, a trio of results will be helpful to make the process more efficient. Consider the log linearization of functions of generic variables $\widetilde{X}_t$ and $\widetilde{Y}_t$ around $(0, 0)$ in the following three escenarios.

Result 1:

$$e^{\widetilde{X}_t + a\widetilde{Y}_t} \approx 1 + \widetilde{X}_t + a\widetilde{Y}_t$$

Proof:

$$e^{\widetilde{X}_t + a\widetilde{Y}_t} \approx e^{0+a0} + e^{0+a0}\left(\widetilde{X}_t - 0\right) + ae^{0+a0}\left(\widetilde{Y}_t - 0\right)$$

$$e^{\widetilde{X}_t + a\widetilde{Y}_t} \approx 1 + \widetilde{X}_t + a\widetilde{Y}_t$$

Corollary:

$$e^{\widetilde{X}_t} \approx 1 + \widetilde{X}_t$$

Result 2:

$$\widetilde{X}_t \widetilde{Y}_t \approx 0$$

<u>*Proof*</u>:

$$\widetilde{X}_t\widetilde{Y}_t \approx 0 + 0\left(\widetilde{X}_t - 0\right) + 0\left(\widetilde{Y}_t - 0\right)$$

$$\widetilde{X}_t\widetilde{Y}_t \approx 0$$

<u>*Result 3*</u>:

$$E\left(ae^{\widetilde{X}_{t+1}}\right) \approx a + aE\left(\widetilde{X}_{t+1}\right)$$

<u>*Proof*</u>:

$$E\left(ae^{\widetilde{X}_{t+1}}\right) \approx E\left[a\left(1 + \widetilde{X}_{t+1}\right)\right]$$

$$E\left(ae^{\widetilde{X}_{t+1}}\right) \approx E\left(a + a\widetilde{X}_{t+1}\right)$$

$$E\left(ae^{\widetilde{X}_{t+1}}\right) \approx a + aE\left(\widetilde{X}_{t+1}\right)$$

where in the first line we used result 1 and in the third one we used the linear operator property of the mathematical expectation.

Now, we proceed to log linearize the 27 equations that characterize the equilibrium conditions (EC.1)-(EC.27) of this model. In all cases[10], three general steps will be followed. First, apply definition (D.1) to all variables with a t subscript in the equation. Second, employ one or more of the three results just presented. If convenient, use (D.2)[11]. Finally, simplify the expression as much as possible with the help of equations in steady state form in system (SS.1)-(SS.27).

<u>Equation (EC.1)</u>:

$$\frac{C_t^N}{C_t^T} = \frac{1-\theta_C}{\theta_C}\left(p_t^N\right)^{-\phi}$$

$$\frac{\overline{C^N}e^{\widetilde{C_t^N}}}{\overline{C^T}e^{\widetilde{C_t^T}}} = \frac{1-\theta_C}{\theta_C}\left(\overline{p^N}e^{\widetilde{p_t^N}}\right)^{-\phi}$$

$$\frac{\overline{C^N}}{\overline{C^T}}e^{\widetilde{C_t^N}-\widetilde{C_t^T}} = \frac{1-\theta_C}{\theta_C}\left(\overline{p^N}\right)^{-\phi}e^{-\phi\widetilde{p_t^N}}$$

$$1 + \widetilde{C_t^N} - \widetilde{C_t^T} = 1 - \phi\widetilde{p_t^N}$$

$$\widetilde{C_t^T} - \widetilde{C_t^N} = \phi\widetilde{p_t^N}$$

[10] The autoregressive processes for A_t^T, A_t^N and r_t^* are already in log linear form, so in these cases the steps are not necessary.

[11] Notational simplification: $=$ instead of $\approx$.

Equation (EC.4):

$$\widetilde{I_t^T} - \widetilde{I_t^N} = \phi \widetilde{p_t^N}$$

Equation (EC.2):

In this case, we will use (D.2) to linearize around $\left(\widetilde{C_t^T}, \widetilde{C_t^N}, \widetilde{C}_t\right) = (0,0,0)$.

$$\left[\theta_C^{\frac{1}{\phi}} \left(C_t^T\right)^{\frac{\phi-1}{\phi}} + (1-\theta_C)^{\frac{1}{\phi}} \left(C_t^N\right)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}} = C_t$$

$$\left[\theta_C^{\frac{1}{\phi}} \left(\overline{C^T}\right)^{\frac{\phi-1}{\phi}} e^{\left(\frac{\phi-1}{\phi}\right)\widetilde{C_t^T}} + (1-\theta_C)^{\frac{1}{\phi}} \left(\overline{C^N}\right)^{\frac{\phi-1}{\phi}} e^{\left(\frac{\phi-1}{\phi}\right)\widetilde{C_t^N}}\right]^{\frac{\phi}{\phi-1}} - \overline{C} e^{\widetilde{C}_t} = 0$$

$$\left[\theta_C^{\frac{1}{\phi}} \left(\overline{C^T}\right)^{\frac{\phi-1}{\phi}} + (1-\theta_C)^{\frac{1}{\phi}} \left(\overline{C^N}\right)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}} - \overline{C}...$$

$$... + \left(\frac{\phi}{\phi-1}\right) \left[\theta_C^{\frac{1}{\phi}} \left(\overline{C^T}\right)^{\frac{\phi-1}{\phi}} + (1-\theta_C)^{\frac{1}{\phi}} \left(\overline{C^N}\right)^{\frac{\phi-1}{\phi}}\right]^{\frac{1}{\phi-1}} \theta_C^{\frac{1}{\phi}} \left(\overline{C^T}\right)^{\frac{\phi-1}{\phi}} ...$$

$$\left(\frac{\phi-1}{\phi}\right) \left(\widetilde{C_t^T} - 0\right) + \left(\frac{\phi}{\phi-1}\right) \left[\theta_C^{\frac{1}{\phi}} \left(\overline{C^T}\right)^{\frac{\phi-1}{\phi}} + (1-\theta_C)^{\frac{1}{\phi}} \left(\overline{C^N}\right)^{\frac{\phi-1}{\phi}}\right]^{\frac{1}{\phi-1}}$$

$$...(1-\theta_C)^{\frac{1}{\phi}} \left(\overline{C^N}\right)^{\frac{\phi-1}{\phi}} \left(\frac{\phi-1}{\phi}\right) \left(\widetilde{C_t^N} - 0\right) + \left(-\overline{C}\right) \left(\widetilde{C}_t - 0\right) = 0$$

$$\overline{C}^{\frac{1}{\phi}} \left[\theta_C^{\frac{1}{\phi}} \left(\overline{C^T}\right)^{\frac{\phi-1}{\phi}} \widetilde{C_t^T} + (1-\theta_C)^{\frac{1}{\phi}} \left(\overline{C^N}\right)^{\frac{\phi-1}{\phi}} \widetilde{C_t^N}\right] = \overline{C}\widetilde{C}_t$$

$$\theta_C^{\frac{1}{\phi}} \left(\frac{\overline{C^T}}{\overline{C}}\right)^{\frac{\phi-1}{\phi}} \widetilde{C_t^T} + (1-\theta_C)^{\frac{1}{\phi}} \left(\frac{\overline{C^N}}{\overline{C}}\right)^{\frac{\phi-1}{\phi}} \widetilde{C_t^N} = \widetilde{C}_t$$

Equation (EC.5):

Take the same steps as for (EC.2).

$$\theta_I^{\frac{1}{\phi}} \left(\frac{\overline{I^T}}{\overline{I}}\right)^{\frac{\phi-1}{\phi}} \widetilde{I_t^T} + (1-\theta_I)^{\frac{1}{\phi}} \left(\frac{\overline{I^N}}{\overline{I}}\right)^{\frac{\phi-1}{\phi}} \widetilde{I_t^N} = \widetilde{I}_t$$

Equation (EC.3):

In this case, we will use (D.2) to linearize around $\left(\widetilde{p_t^C}, \widetilde{p_t^N}\right) = (0,0)$.

$$p_t^C = \left[\theta_C + (1-\theta_C)\left(p_t^N\right)^{1-\phi}\right]^{\frac{1}{1-\phi}}$$

$$\left[\theta_C + (1-\theta_C)\left(\overline{p^N}\right)^{1-\phi} e^{(1-\phi)\widetilde{p_t^N}}\right]^{\frac{1}{1-\phi}} - \overline{p^C} e^{\widetilde{p_t^C}} = 0$$

$$\left[\theta_C + (1-\theta_C)\overline{p^N}^{1-\phi}\right]^{\frac{1}{1-\phi}} - \overline{p^C} + \frac{1}{1-\phi}\left[\theta_C + (1-\theta_C)\overline{p^N}^{1-\phi}\right]^{\frac{\phi}{1-\phi}} (1-\theta_C)...$$

$$...\left(\overline{p^N}\right)^{1-\phi}(1-\phi)\left(\widetilde{p_t^N} - 0\right) + \left(-\overline{p^C}\right)\left(\widetilde{p_t^C} - 0\right) = 0$$

$$\overline{p^C}^{\phi}(1-\theta_C)\left(\overline{p^N}\right)^{1-\phi}\widetilde{p_t^N} = \overline{p^C}\widetilde{p_t^C}$$

$$(1-\theta_C)\left(\frac{\overline{p^N}}{\overline{p^C}}\right)^{1-\phi}\widetilde{p_t^N} = \widetilde{p_t^C}$$

<u>Equation (EC.6):</u>

Take the same steps as for (EC.3).

$$(1-\theta_I)\left(\frac{\overline{p^N}}{\overline{p^I}}\right)^{1-\phi}\widetilde{p_t^N} = \widetilde{p_t^I}$$

<u>Equation (EC.7):</u>

In this case, use (D.2) to linearize around $\left(\widetilde{C_t}, \widetilde{L_t}, \widetilde{\lambda_t}, \widetilde{p_t^C}\right) = (0,0,0,0)$

$$\left(C_t - \upsilon\frac{L_t^{\omega}}{\omega}\right)^{-\sigma} = \lambda_t p_t^C$$

$$\left(\overline{C} e^{\widetilde{C_t}} - \upsilon\frac{\overline{L}^{\omega}}{\omega} e^{\omega\widetilde{L_t}}\right)^{-\sigma} - \overline{\lambda} e^{\widetilde{\lambda_t}}\overline{p^C} e^{\widetilde{p_t^C}} = 0$$

$$\left(\overline{C} e^{\widetilde{C_t}} - \upsilon\frac{\overline{L}^{\omega}}{\omega} e^{\omega\widetilde{L_t}}\right)^{-\sigma} - \overline{\lambda}\overline{p^C} e^{\widetilde{\lambda_t} + \widetilde{p_t^C}} = 0$$

$$\left(\overline{C} - \upsilon\frac{\overline{L}^{\omega}}{\omega}\right)^{-\sigma} - \overline{\lambda}\overline{p^C} + (-\sigma)\left(\overline{C} - \upsilon\frac{\overline{L}^{\omega}}{\omega}\right)^{-\sigma-1}\overline{C}\left(\widetilde{C_t} - 0\right)...$$

$$... + (-\sigma)\left(\overline{C} - \upsilon\frac{\overline{L}^{\omega}}{\omega}\right)^{-\sigma-1}(-\upsilon)\frac{\overline{L}^{\omega}}{\omega}\omega\left(\widetilde{L}_t - 0\right)...$$

$$... + \left(-\overline{\lambda}\overline{p^C}\right)\left(\widetilde{\lambda}_t - 0\right) + \left(-\overline{\lambda}\overline{p^C}\right)\left(\widetilde{p_t^C} - 0\right) = 0$$

$$\sigma\left(\overline{\lambda}\overline{p^C}\right)^{1+\frac{1}{\sigma}}\left(\upsilon\overline{L}^{\omega}\widetilde{L}_t - \overline{C}\widetilde{C}_t\right) = \overline{\lambda}\overline{p^C}\left(\widetilde{\lambda}_t + \widetilde{p_t^C}\right)$$

$$\sigma\overline{\lambda}^{\frac{1}{\sigma}}\overline{p^C}^{\frac{1}{\sigma}}\left(\upsilon\overline{L}^{\omega}\widetilde{L}_t - \overline{C}\widetilde{C}_t\right) = \widetilde{\lambda}_t + \widetilde{p_t^C}$$

$$\left(\sigma\overline{\lambda}^{\frac{1}{\sigma}}\overline{p^C}^{\frac{1}{\sigma}}\upsilon\overline{L}^{\omega}\right)\widetilde{L}_t - \left(\sigma\overline{\lambda}^{\frac{1}{\sigma}}\overline{p^C}^{\frac{1}{\sigma}}\overline{C}\right)\widetilde{C}_t = \widetilde{\lambda}_t + \widetilde{p_t^C}$$

Equation (EC.8):

$$\upsilon L_t^{\omega-1} = \frac{w_t}{p_t^C}$$

$$\upsilon\overline{L}^{\omega-1}e^{(\omega-1)\widetilde{L_t}} = \frac{\overline{w}}{\overline{p^C}}\frac{e^{\widetilde{w_t}}}{e^{\widetilde{p_t^C}}}$$

$$e^{(\omega-1)\widetilde{L_t}} = e^{\widetilde{w_t}-\widetilde{p_t^C}}$$

$$(\omega-1)\widetilde{L}_t = \widetilde{w}_t - \widetilde{p_t^C}$$

Equation (EC.9):

$$w_t = \alpha_T\frac{Y_t^T}{L_t^T}$$

$$\overline{w}e^{\widetilde{w_t}} = \alpha_T\frac{\overline{Y^T}}{\overline{L^T}}\frac{e^{\widetilde{Y_t^T}}}{e^{\widetilde{L_t^T}}}$$

$$e^{\widetilde{w_t}} = e^{\widetilde{Y_t^T}-\widetilde{L_t^T}}$$

$$\widetilde{w}_t = \widetilde{Y_t^T} - \widetilde{L_t^T}$$

Equation (EC.10):

Take the same steps as for (EC.9).

$$\widetilde{r_t^K} = \widetilde{Y_t^T} - \widetilde{K_t^T}$$

Equation (EC.11):

$$w_t = p_t^N\alpha_N\frac{Y_t^N}{L_t^N}$$

$$\overline{w}e^{\widetilde{w_t}} = \overline{p^N}\alpha_N \frac{\overline{Y^N}}{\overline{L^N}} \frac{e^{\widetilde{p_t^N}} e^{\widetilde{Y_t^N}}}{e^{\widetilde{L_t^N}}}$$

$$e^{\widetilde{w_t}} = e^{\widetilde{p_t^N}+\widetilde{Y_t^N}-\widetilde{L_t^N}}$$

$$\widetilde{w}_t = \widetilde{p_t^N} + \widetilde{Y_t^N} - \widetilde{L_t^N}$$

Equation (EC.12):

Take the same steps as for (EC.11).

$$\widetilde{r_t^K} = \widetilde{p_t^N} + \widetilde{Y_t^N} - \widetilde{K_t^N}$$

Equation (EC.13):

$$L_t^T + L_t^N = L_t$$

$$\overline{L^T}e^{\widetilde{L_t^T}} + \overline{L^N}e^{\widetilde{L_t^N}} = \overline{L}e^{\widetilde{L_t}}$$

$$\overline{L^T}\left(1 + \widetilde{L_t^T}\right) + \overline{L^N}\left(1 + \widetilde{L_t^N}\right) = \overline{L}\left(1 + \widetilde{L}_t\right)$$

$$\overline{L^T} + \overline{L^T}\widetilde{L_t^T} + \overline{L^N} + \overline{L^N}\widetilde{L_t^N} = \overline{L} + \overline{L}\widetilde{L}_t$$

$$\overline{L^T}\widetilde{L_t^T} + \overline{L^N}\widetilde{L_t^N} = \overline{L}\widetilde{L}_t$$

$$\frac{\overline{L^T}}{\overline{L}}\widetilde{L_t^T} + \frac{\overline{L^N}}{\overline{L}}\widetilde{L_t^N} = \widetilde{L}_t$$

Equation (EC.14):

Take the same steps as for (EC.13).

$$\frac{\overline{K^T}}{\overline{K}}\widetilde{K_t^T} + \frac{\overline{K^N}}{\overline{K}}\widetilde{K_t^N} = \widetilde{K_t}$$

Equation (EC.17):

Take the same steps as for (EC.13).

$$\frac{\overline{C^N}}{\overline{Y^N}}\widetilde{C_t^N} + \frac{\overline{I^N}}{\overline{Y^N}}\widetilde{I_t^N} = \widetilde{Y_t^N}$$

Equation (EC.15):

$$Y_t^T = A_t^T \left(L_t^T\right)^{\alpha_T} \left(K_t^T\right)^{1-\alpha_T}$$

$$\overline{Y^T}e^{\widetilde{Y_t^T}} = \overline{A^T}e^{\widetilde{A_t^T}} \left(\overline{L^T}\right)^{\alpha_T} e^{\alpha_T \widetilde{L_t^T}} \left(\overline{K^T}\right)^{1-\alpha_T} e^{(1-\alpha_T)\widetilde{K_t^T}}$$

$$e^{\widetilde{Y_t^T}} = e^{\widetilde{A_t^T} + \alpha_T \widetilde{L_t^T} + (1-\alpha_T)\widetilde{K_t^T}}$$

$$\widetilde{Y_t^T} = \widetilde{A_t^T} + \alpha_T \widetilde{L_t^T} + (1-\alpha_T)\widetilde{K_t^T}$$

Equation (EC.16):

Take the same steps as for (EC.15).

$$\widetilde{Y_t^N} = \widetilde{A_t^N} + \alpha_N \widetilde{L_t^N} + (1-\alpha_N)\widetilde{K_t^N}$$

Equation (EC.18):

$$Y_t = Y_t^T + p_t^N Y_t^N$$

$$\overline{Y} e^{\widetilde{Y}_t} = \overline{Y^T} e^{\widetilde{Y_t^T}} + \overline{p^N Y^N} e^{\widetilde{p_t^N} + \widetilde{Y_t^N}}$$

$$\overline{Y}\left(1 + \widetilde{Y}_t\right) = \overline{Y^T}\left(1 + \widetilde{Y_t^T}\right) + \overline{p^N Y^N}\left(1 + \widetilde{p_t^N} + \widetilde{Y_t^N}\right)$$

$$\overline{Y} + \overline{Y}\widetilde{Y}_t = \overline{Y^T} + \overline{Y^T}\widetilde{Y_t^T} + \overline{p^N Y^N} + \overline{p^N Y^N}\widetilde{p_t^N} + \overline{p^N Y^N}\widetilde{Y_t^N}$$

$$\widetilde{Y}_t = \frac{\overline{Y^T}}{\overline{Y}}\widetilde{Y_t^T} + \frac{\overline{p^N Y^N}}{\overline{Y}}\widetilde{p_t^N} + \frac{\overline{p^N Y^N}}{\overline{Y}}\widetilde{Y_t^N}$$

The INDEC provides quaterly data of real GDP measured at constant prices of 2004. Hence, the structure of relative prices remains constant across the quarters of the series, which implies that $\widetilde{p_t^N} = 0$ in the log-linearized equation of aggregrate output:

$$\widetilde{Y}_t = \frac{\overline{Y^T}}{\overline{Y}}\widetilde{Y_t^T} + \frac{\overline{p^N Y^N}}{\overline{Y}}\widetilde{Y_t^N}$$

Equation (EC.19):

$$1 = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{r_{t+1}^K + p_{t+1}^I \left\{1 - \delta + \chi\left(\frac{K_{t+2}}{K_{t+1}} - 1\right)\left[\frac{K_{t+2}}{K_{t+1}} - \frac{1}{2}\left(\frac{K_{t+2}}{K_{t+1}} - 1\right)\right]\right\}}{p_t^I\left[1 + \chi\left(\frac{K_{t+1}}{K_t} - 1\right)\right]}$$

$$1 = \beta E_t \frac{\overline{\lambda} e^{\widetilde{\lambda_{t+1}}}}{\overline{\lambda} e^{\widetilde{\lambda}_t}} \dots$$

$$\dots \frac{\overline{r^K} e^{\widetilde{r_{t+1}^K}} + \overline{p^I} e^{\widetilde{p_{t+1}^I}} \left\{1 - \delta + \chi\left(\frac{\overline{K} e^{\widetilde{K_{t+2}}}}{\overline{K} e^{\widetilde{K_{t+1}}}} - 1\right)\left[\frac{\overline{K} e^{\widetilde{K_{t+2}}}}{\overline{K} e^{\widetilde{K_{t+1}}}} - \frac{1}{2}\left(\frac{\overline{K} e^{\widetilde{K_{t+2}}}}{\overline{K} e^{\widetilde{K_{t+1}}}} - 1\right)\right]\right\}}{\overline{p^I} e^{\widetilde{p_t^I}}\left[1 + \chi\left(\frac{\overline{K} e^{\widetilde{K_{t+1}}}}{\overline{K} e^{\widetilde{K_t}}} - 1\right)\right]}$$

$$\overline{p^I}\left(1+\widetilde{p^I_t}\right)\left[1+\chi\left(\widetilde{K_{t+1}}-\widetilde{K_t}\right)\right]=\beta E_t\left(1+\widetilde{\lambda_{t+1}}-\widetilde{\lambda_t}\right)\left\{\overline{r^K}\left(1+\widetilde{r^K_{t+1}}\right)...\right.$$

$$\left....+\overline{p^I}\left(1+\widetilde{p^I_{t+1}}\right)\left\{1-\delta+\chi\left(\widetilde{K_{t+2}}-\widetilde{K_{t+1}}\right)\left[1+\frac{1}{2}\left(\widetilde{K_{t+2}}-\widetilde{K_{t+1}}\right)\right]\right\}\right\}$$

$$\overline{p^I}+\overline{p^I}\widetilde{p^I_t}+\overline{p^I}\chi\widetilde{K_{t+1}}-\overline{p^I}\chi\widetilde{K_t}=E_t\left(1+\widetilde{\lambda_{t+1}}-\widetilde{\lambda_t}\right)\left[\beta\overline{r^K}+\beta\overline{r^K}\widetilde{r^K_{t+1}}...\right.$$

$$\left....+\beta\overline{p^I}\left(1-\delta\right)+\beta\overline{p^I}\left(1-\delta\right)\widetilde{p^I_{t+1}}+\beta\overline{p^I}\chi\widetilde{K_{t+2}}-\beta\overline{p^I}\chi\widetilde{K_{t+1}}\right]$$

$$\widetilde{p^I_t}+\left(1+\beta\right)\chi\widetilde{K_{t+1}}-\chi\widetilde{K_t}=...$$

$$...E_t\left[\widetilde{\lambda_{t+1}}-\widetilde{\lambda_t}+\left(1-\beta(1-\delta)\right)\widetilde{r^K_{t+1}}+\beta(1-\delta)\widetilde{p^I_{t+1}}+\beta\chi\widetilde{K_{t+2}}\right]$$

<u>Equation (EC.20)</u>:

$$1=E_t\beta\frac{\lambda_{t+1}}{\lambda_t}\left(1+r_{t+1}\right)$$

$$1=E_t\beta\frac{\overline{\lambda}}{\overline{\lambda}}e^{\widetilde{\lambda_{t+1}}-\widetilde{\lambda_t}}\left(1+\overline{r}e^{\widetilde{r_{t+1}}}\right)$$

$$1=E_t\beta\left(1+\widetilde{\lambda_{t+1}}-\widetilde{\lambda_t}\right)\left[1+\overline{r}\left(1+\widetilde{r_{t+1}}\right)\right]$$

$$1=E_t\left[\beta\left(1+\overline{r}\right)+\beta\left(1+\overline{r}\right)\widetilde{\lambda_{t+1}}-\beta\left(1+\overline{r}\right)\widetilde{\lambda_t}+\beta\overline{r}\widetilde{r_{t+1}}\right]$$

$$0=E_t\left[\widetilde{\lambda_{t+1}}-\widetilde{\lambda_t}+\left(1-\beta\right)\widetilde{r_{t+1}}\right]$$

<u>Equation (EC.21)</u>:

$$C^T_t+I^T_t=Y^T_t+D_{t+1}-(1+r_t)D_t$$

$$C^T_t+I^T_t=Y^T_t+D_{t+1}-D_t-r_tD_t$$

$$C^T_t+I^T_t=Y^T_t+D_{t+1}-\overline{D}-D_t+\overline{D}-r_t\left(D_t-\overline{D}+\overline{D}\right)$$

$$\overline{C^T}e^{\widetilde{C^T_t}}+\overline{I^T}e^{\widetilde{I^T_t}}=\overline{Y^T}e^{\widetilde{Y^T_t}}+\widetilde{D_{t+1}}-\widetilde{D_t}-\overline{r}e^{\widetilde{r_t}}\left(\widetilde{D_t}+\overline{D}\right)$$

$$\overline{C^T}\left(1+\widetilde{C^T_t}\right)+\overline{I^T}\left(1+\widetilde{I^T_t}\right)=\overline{Y^T}\left(1+\widetilde{Y^T_t}\right)+\widetilde{D_{t+1}}...$$

$$...-\widetilde{D_t}-\overline{r}\left(1+\widetilde{r_t}\right)\widetilde{D_t}-\overline{r}\left(1+\widetilde{r_t}\right)\overline{D}$$

$$\overline{C^T}+\overline{C^T}\widetilde{C^T_t}+\overline{I^T}+\overline{I^T}\widetilde{I^T_t}=\overline{Y^T}+\overline{Y^T}\widetilde{Y^T_t}+\widetilde{D_{t+1}}-\widetilde{D_t}-\overline{r}\widetilde{D_t}-\overline{r}\overline{D}-\overline{r}\overline{D}\widetilde{r_t}$$

$$\overline{C^T}\widetilde{C_t^T} + \overline{I^T}\widetilde{I_t^T} = \overline{Y^T}\widetilde{Y_t^T} + \widetilde{D_{t+1}} - (1+\overline{r})\widetilde{D_t} - \overline{r}\overline{D}\widetilde{r_t}$$

Equation (EC.22):

$$I_t = K_{t+1} - (1-\delta)K_t + \frac{\chi}{2}\left(\frac{K_{t+1}}{K_t} - 1\right)^2 K_t$$

$$\overline{I}e^{\widetilde{I_t}} = \overline{K}e^{\widetilde{K_{t+1}}} - (1-\delta)\overline{K}e^{\widetilde{K_t}} + \frac{\chi}{2}\left(\frac{\overline{K}}{\overline{K}}e^{\widetilde{K_{t+1}}-\widetilde{K_t}} - 1\right)^2 \overline{K}e^{\widetilde{K_t}}$$

$$\overline{I}\left(1+\widetilde{I_t}\right) = \overline{K}\left(1+\widetilde{K_{t+1}}\right) - (1-\delta)\overline{K}\left(1+\widetilde{K_t}\right)...$$

$$... + \frac{\chi}{2}\left(\widetilde{K_{t+1}} - \widetilde{K_t}\right)^2 \overline{K}\left(1+\widetilde{K_t}\right)$$

$$\overline{I} + \overline{I}\widetilde{I_t} = \overline{K} + \overline{K}\widetilde{K_{t+1}} - (1-\delta)\overline{K} - (1-\delta)\overline{K}\widetilde{K_t}$$

$$\frac{\overline{I}}{\overline{K}}\widetilde{I_t} = \widetilde{K_{t+1}} - (1-\delta)\widetilde{K_t}$$

Equation (EC.23):

$$r_t = S_t r_t^* + \gamma\left[e^{\left(D_{t+1}-\bar{D}\right)} - 1\right]$$

$$\overline{r}e^{\widetilde{r_t}} = \overline{S}\overline{r^*}e^{\widetilde{S_t}+\widetilde{r_t^*}} + \gamma e^{\widetilde{D_{t+1}}} - \gamma$$

$$\overline{r}\left(1+\widetilde{r_t}\right) = \overline{S}\overline{r^*}\left(1+\widetilde{S_t}+\widetilde{r_t^*}\right) + \gamma\left(1+\widetilde{D_{t+1}}\right) - \gamma$$

$$\overline{r} + \overline{r}\widetilde{r_t} = \overline{S}\overline{r^*} + \overline{S}\overline{r^*}\widetilde{S_t} + \overline{S}\overline{r^*}\widetilde{r_t^*} + \gamma + \gamma\widetilde{D_{t+1}} - \gamma$$

$$\overline{r}\widetilde{r_t} = \overline{r}\widetilde{S_t} + \overline{r}\widetilde{r_t^*} + \gamma\widetilde{D_{t+1}}$$

Equation (EC.24):

Following Neumeyer and Perri (2005), the especification for the country spread in its log linearized version is:

$$\widetilde{S_t} = -\eta E_t\left(\widetilde{A_{t+1}^T}\right)$$

Equation (EC.25):

$$\ln A_{t+1}^T = (1-\psi_T)\ln A^T + \psi_T \ln A_t^T + \varepsilon_{t+1}^T$$

$$\ln A^T_{t+1} - \ln \overline{A^T} = \psi_T \left(\ln A^T_t - \ln \overline{A^T} \right) + \varepsilon^T_{t+1}$$

$$\widetilde{A^T_{t+1}} = \psi_T \widetilde{A^T_t} + \varepsilon^T_{t+1}$$

Equation (EC.26):

Take the same steps as for (EC.25).

$$\widetilde{A^N_{t+1}} = \psi_N \widetilde{A^N_t} + \varepsilon^N_{t+1}$$

Equation (EC.27):

Take the same steps as for (EC.25).

$$\widetilde{r^*_{t+1}} = \psi_{r^*} \widetilde{r^*_t} + \varepsilon^{r^*}_{t+1}$$

The set of derived equations constitutes a new system of linear stochastic difference equations whose solution approximates the one of the original non linear system, as long as we are close enough to the non stochastic steady state.

D. Policy Functions

What follows is the construction of the matrix form of the previously log linearized system, which is required to get the approximate policy functions through the use of the Matlab program "solab.m". The log linearized conditions will be written in a way such that all variables indexed with a t appear on the right side, while the rest of the variables are kept on the left side. In adittion, a mathematical expectation operator conditional on information available at time t will be included on the left side. For computational reasons, an auxiliary equation (L.L.28) needs to be added because the system includes terms indexed by $t + 2$.

The system of log linearized stochastic difference equations, written in appropiate form, is:

(L.L.1) $$0 = \widetilde{C^N_t} - \widetilde{C^T_t} + \phi \widetilde{p^N_t}$$

(L.L.2) $$0 = \theta_C^{\frac{1}{\phi}} \left(\frac{\overline{C^T}}{\overline{C}} \right)^{\frac{\phi-1}{\phi}} \widetilde{C_t^T} + (1-\theta_C)^{\frac{1}{\phi}} \left(\frac{\overline{C^N}}{\overline{C}} \right)^{\frac{\phi-1}{\phi}} \widetilde{C_t^N} - \widetilde{C}_t$$

(L.L.3) $$0 = (1-\theta_C) \left(\frac{\overline{p^N}}{\overline{p^C}} \right)^{1-\phi} \widetilde{p_t^N} - \widetilde{p_t^C}$$

(L.L.4) $$0 = \widetilde{I_t^N} - \widetilde{I_t^T} + \phi \widetilde{p_t^N}$$

(L.L.5) $$0 = \theta_I^{\frac{1}{\phi}} \left(\frac{\overline{I^T}}{\overline{I}} \right)^{\frac{\phi-1}{\phi}} \widetilde{I_t^T} + (1-\theta_I)^{\frac{1}{\phi}} \left(\frac{\overline{I^N}}{\overline{I}} \right)^{\frac{\phi-1}{\phi}} \widetilde{I_t^N} - \widetilde{I}_t$$

(L.L.6) $$0 = (1-\theta_I) \left(\frac{\overline{p^N}}{\overline{p^I}} \right)^{1-\phi} \widetilde{p_t^N} - \widetilde{p_t^I}$$

(L.L.7) $$0 = \left(\sigma \overline{\lambda}^{\frac{1}{\sigma}} \overline{p^C}^{\frac{1}{\sigma}} \upsilon \overline{L}^{\omega} \right) \widetilde{L}_t - \left(\sigma \overline{\lambda}^{\frac{1}{\sigma}} \overline{p^C}^{\frac{1}{\sigma}} \overline{C} \right) \widetilde{C}_t - \widetilde{\lambda}_t - \widetilde{p_t^C}$$

(L.L.8) $$0 = \widetilde{w}_t - \widetilde{p_t^C} - (\omega - 1) \widetilde{L}_t$$

(L.L.9) $$0 = \widetilde{Y_t^T} - \widetilde{L_t^T} - \widetilde{w}_t$$

(L.L.10) $$0 = \widetilde{Y_t^T} - \widetilde{K_t^T} - \widetilde{r_t^K}$$

(L.L.11) $$0 = \widetilde{p_t^N} + \widetilde{Y_t^N} - \widetilde{L_t^N} - \widetilde{w}_t$$

(L.L.12) $$0 = \widetilde{p_t^N} + \widetilde{Y_t^N} - \widetilde{K_t^N} - \widetilde{r_t^K}$$

$$0=\frac{\overline{L^T}}{\overline{L}}\widetilde{L_t^T}+\frac{\overline{L^N}}{\overline{L}}\widetilde{L_t^N}-\widetilde{L}_t \tag{L.L.13}$$

$$0=\frac{\overline{K^T}}{\overline{K}}\widetilde{K_t^T}+\frac{\overline{K^N}}{\overline{K}}\widetilde{K_t^N}-\widetilde{K_t} \tag{L.L.14}$$

$$0=\widetilde{A_t^T}+\alpha_T\widetilde{L_t^T}+(1-\alpha_T)\,\widetilde{K_t^T}-\widetilde{Y_t^T} \tag{L.L.15}$$

$$0=\widetilde{A_t^N}+\alpha_N\widetilde{L_t^N}+(1-\alpha_N)\,\widetilde{K_t^N}-\widetilde{Y_t^N} \tag{L.L.16}$$

$$0=\frac{\overline{C^N}}{\overline{Y^N}}\widetilde{C_t^N}+\frac{\overline{I^N}}{\overline{Y^N}}\widetilde{I_t^N}-\widetilde{Y_t^N} \tag{L.L.17}$$

$$0=\frac{\overline{Y^T}}{\overline{Y}}\widetilde{Y_t^T}+\frac{\overline{p^NY^N}}{\overline{Y}}\widetilde{Y_t^N}-\widetilde{Y}_t \tag{L.L.18}$$

$$E_t\Big[\widetilde{\lambda_{t+1}}+(1-\beta(1-\delta))\,\widetilde{r_{t+1}^K}+\beta(1-\delta)\widetilde{p_{t+1}^I}... \tag{L.L.19}$$

$$...+\beta\chi\widetilde{K_{t+2}}-(1+\beta)\,\chi\widetilde{K_{t+1}}\Big]=\widetilde{\lambda}_t+\widetilde{p_t^I}-\chi\widetilde{K_t}$$

$$E_t\Big[\widetilde{\lambda_{t+1}}+(1-\beta)\,\widetilde{r_{t+1}}\Big]=\widetilde{\lambda}_t \tag{L.L.20}$$

$$E_t\left(\widetilde{D_{t+1}}\right)=\overline{C^T}\widetilde{C_t^T}+\overline{I^T}\widetilde{I_t^T}-\overline{Y^T}\widetilde{Y_t^T}+(1+\overline{r})\,\widetilde{D_t}+\overline{r}\overline{D}\widetilde{r_t} \tag{L.L.21}$$

$$E_t\left(\widetilde{K_{t+1}}\right)=\frac{\overline{I}}{\overline{K}}\widetilde{I}_t+(1-\delta)\widetilde{K_t} \tag{L.L.22}$$

$$E_t\left(\gamma\widetilde{D_{t+1}}\right)=\overline{r}\widetilde{r_t}-\overline{r}\widetilde{S_t}-\overline{r}\widetilde{r_t^*} \tag{L.L.23}$$

(L.L.24) $$E_t\left(-\eta\widetilde{A^T_{t+1}}\right)=\widetilde{S}_t$$

(L.L.25) $$E_t\left(\widetilde{A^T_{t+1}}\right)=\psi_T\widetilde{A^T_t}$$

(L.L.26) $$E_t\left(\widetilde{A^N_{t+1}}\right)=\psi_N\widetilde{A^N_t}$$

(L.L.27) $$E_t\left(\widetilde{r^*_{t+1}}\right)=\psi_{r^*}\widetilde{r^*_t}$$

(L.L.28) $$E_t\left(\widetilde{K_{t+1}}\right)=\widetilde{X}_t$$

Now, we proceed to cast this system of equations in matrix form. For that purposes, define the 28x1 column vector of all variables in the system:

$$\mathbf{x}_t=\begin{bmatrix}\widetilde{K}_t & \widetilde{D}_t & \widetilde{A^T_t} & \widetilde{A^N_t} & \widetilde{r^*_t} & \widetilde{Y}_t & \widetilde{Y^T_t}... \\ ...\widetilde{Y^N_t} & \widetilde{C}_t & \widetilde{C^T_t} & \widetilde{C^N_t} & \widetilde{I}_t & \widetilde{I^T_t} & \widetilde{I^N_t}... \\ ...\widetilde{L}_t & \widetilde{L^T_t} & \widetilde{L^N_t} & \widetilde{K^T_t} & \widetilde{K^N_t} & \widetilde{S}_t & \widetilde{p^N_t}... \\ ...\widetilde{p^C_t} & \widetilde{p^I_t} & \widetilde{w}_t & \widetilde{r^K_t} & \widetilde{r}_t & \widetilde{\lambda}_t & \widetilde{X}_t\end{bmatrix}^T$$

The 5x1 column vector of state variables in the system is:

$$\mathbf{k}_t=\begin{bmatrix}\widetilde{K}_t & \widetilde{D}_t & \widetilde{A^T_t} & \widetilde{A^N_t} & \widetilde{r^*_t}\end{bmatrix}^T$$

The 23x1 column vector of control variables in the system is:

$$\mathbf{u}_t=\begin{bmatrix}\widetilde{Y}_t & \widetilde{Y^T_t} & \widetilde{Y^N_t} & \widetilde{C}_t & \widetilde{C^T_t} & \widetilde{C^N_t} & \widetilde{I}_t & \widetilde{I^T_t} & \widetilde{I^N_t} & \widetilde{L}_t & \widetilde{L^T_t} & \widetilde{L^N_t}... \\ ...\widetilde{K^T_t} & \widetilde{K^N_t} & \widetilde{S}_t & \widetilde{p^N_t} & \widetilde{p^C_t} & \widetilde{p^I_t} & \widetilde{w}_t & \widetilde{r^K_t} & \widetilde{r}_t & \widetilde{\lambda}_t & \widetilde{X}_t & \end{bmatrix}^T$$

The matrix form of the log linear system (L.L.1)-(L.L.28) is written as follows:

$$\mathbf{a}E_t\left(\mathbf{x}_{t+1}\right)=\mathbf{b}\mathbf{x}_t$$

where **a** and **b** are 28x28 matrices of parameters. The entries of these matrices are:

$$\mathbf{a} = \begin{pmatrix} \cdots & \cdots & \cdots & \mathbf{0}_{18x28} & \cdots & \cdots & \cdots & \cdots \\ -(1+\beta)\chi & \mathbf{0}_{1x21} & \beta(1-\delta) & 0 & 1-\beta(1-\delta) & 0 & 1 & \beta\chi \\ \cdots & \cdots & \mathbf{0}_{1x25} & \cdots & \cdots & (1-\beta) & 1 & 0 \\ 0 & 1 & \cdots & \cdots & \mathbf{0}_{1x26} & \cdots & \cdots & \cdots \\ 1 & \cdots & \cdots & \cdots & \mathbf{0}_{1x27} & \cdots & \cdots & \cdots \\ 0 & \gamma & \cdots & \cdots & \mathbf{0}_{1x26} & \cdots & \cdots & \cdots \\ \mathbf{0}_{1x2} & -\eta & \cdots & \cdots & \mathbf{0}_{1x25} & \cdots & \cdots & \cdots \\ \mathbf{0}_{1x2} & 1 & \cdots & \cdots & \mathbf{0}_{1x25} & \cdots & \cdots & \cdots \\ \mathbf{0}_{1x3} & 1 & \cdots & \cdots & \mathbf{0}_{1x24} & \cdots & \cdots & \cdots \\ \mathbf{0}_{1x4} & 1 & \cdots & \cdots & \mathbf{0}_{1x23} & \cdots & \cdots & \cdots \\ 1 & \cdots & \cdots & \cdots & \mathbf{0}_{1x27} & \cdots & \cdots & \cdots \end{pmatrix}$$

$$
\mathbf{b}=\left(\begin{array}{ccccccccccc}
\mathbf{0}_{1x9} & -1 & 1 & \mathbf{0}_{1x9} & \phi & \cdots & \cdots & \mathbf{0}_{1x7} & \cdots & \cdots & \cdots \\
\mathbf{0}_{1x8} & -1 & \left[\theta_C^{(1/\phi)}\left(\frac{\overline{C^T}}{\overline{C}}\right)^{(\phi-1)/\phi}\right] & \left[(1-\theta_C)^{(1/\phi)}\left(\frac{\overline{C^N}}{\overline{C}}\right)^{(\phi-1)/\phi}\right] & \cdots & \cdots & \cdots & \mathbf{0}_{1x17} & \cdots & \cdots & \cdots \\
\mathbf{0}_{1x20} & \left[(1-\theta_C)\left(\frac{\overline{p^N}}{\overline{p^C}}\right)^{(1-\phi)}\right] & -1 & \cdots & \cdots & \cdots & \cdots & \mathbf{0}_{1x6} & \cdots & \cdots & \cdots \\
\mathbf{0}_{1x12} & -1 & 1 & \mathbf{0}_{1x6} & \phi & \cdots & \cdots & \mathbf{0}_{1x7} & \cdots & \cdots & \cdots \\
\mathbf{0}_{1x11} & -1 & \left[\theta_I^{(1/\phi)}\left(\frac{\overline{I^T}}{\overline{I}}\right)^{(\phi-1)/\phi}\right] & \left[(1-\theta_I)^{(1/\phi)}\left(\frac{\overline{I^N}}{\overline{I}}\right)^{(\phi-1)/\phi}\right] & \cdots & \cdots & \cdots & \mathbf{0}_{1x14} & \cdots & \cdots & \cdots \\
\mathbf{0}_{1x20} & \left[(1-\theta_I)\left(\frac{\overline{p^N}}{\overline{p^I}}\right)^{(1-\phi)}\right] & 0 & -1 & \cdots & \cdots & \cdots & \mathbf{0}_{1x5} & \cdots & \cdots & \cdots \\
\mathbf{0}_{1x8} & \left[-\sigma\overline{\lambda}^{(1/\sigma)}\overline{p^C}^{(1/\sigma)}\overline{C}\right] & \mathbf{0}_{1x5} & \left[\sigma\overline{\lambda}^{(1/\sigma)}\overline{p^C}^{(1/\sigma)}\upsilon\overline{L}^{\omega}\right] & \mathbf{0}_{1x6} & -1 & \cdots & \mathbf{0}_{1x4} & \cdots & -1 & 0 \\
\mathbf{0}_{1x14} & -(\omega-1) & \mathbf{0}_{1x6} & -1 & 0 & 1 & \cdots & \mathbf{0}_{1x4} & \cdots & \cdots & \cdots \\
\mathbf{0}_{1x6} & 1 & \mathbf{0}_{1x8} & -1 & \mathbf{0}_{1x7} & -1 & \cdots & \mathbf{0}_{1x4} & \cdots & \cdots & \cdots \\
\mathbf{0}_{1x6} & 1 & \mathbf{0}_{1x10} & -1 & \mathbf{0}_{1x6} & -1 & \cdots & \mathbf{0}_{1x3} & \cdots & \cdots & \cdots \\
\mathbf{0}_{1x7} & 1 & \mathbf{0}_{1x8} & -1 & \mathbf{0}_{1x3} & 1 & \mathbf{0}_{1x2} & -1 & \cdots & \mathbf{0}_{1x4} & \cdots \\
\mathbf{0}_{1x7} & 1 & \mathbf{0}_{1x10} & -1 & 0 & 1 & \mathbf{0}_{1x3} & -1 & \cdots & \mathbf{0}_{1x3} & \cdots \\
\mathbf{0}_{1x14} & -1 & \left(\frac{\overline{L^T}}{\overline{L}}\right) & \left(\frac{\overline{L^N}}{\overline{L}}\right) & \cdots & \cdots & \cdots & \mathbf{0}_{1x11} & \cdots & \cdots & \cdots \\
-1 & \mathbf{0}_{1x16} & \left(\frac{\overline{K^T}}{\overline{K}}\right) & \left(\frac{\overline{K^N}}{\overline{K}}\right) & 0 & \cdots & \cdots & \mathbf{0}_{1x7} & \cdots & \cdots & \cdots \\
\mathbf{0}_{1x2} & 1 & \mathbf{0}_{1x3} & -1 & \mathbf{0}_{1x8} & \alpha_T & 0 & (1-\alpha_T) & \cdots & \mathbf{0}_{1x10} & \cdots \\
\mathbf{0}_{1x3} & 1 & \mathbf{0}_{1x3} & -1 & \mathbf{0}_{1x8} & \alpha_N & 0 & (1-\alpha_N) & \cdots & \mathbf{0}_{1x9} & \cdots \\
\mathbf{0}_{1x7} & -1 & \mathbf{0}_{1x2} & \left(\frac{\overline{C^N}}{\overline{Y^N}}\right) & \mathbf{0}_{1x2} & \left(\frac{\overline{I^N}}{\overline{Y^N}}\right) & \cdots & \mathbf{0}_{1x14} & \cdots & \cdots & \cdots \\
\mathbf{0}_{1x5} & -1 & \left(\frac{\overline{Y^T}}{\overline{Y}}\right) & \left(\frac{\overline{p^N Y^N}}{\overline{Y}}\right) & \cdots & \cdots & \cdots & \mathbf{0}_{1x20} & \cdots & \cdots & \cdots \\
-\chi & \cdots & \cdots & \mathbf{0}_{1x21} & \cdots & 1 & \cdots & \mathbf{0}_{1x3} & \cdots & 1 & 0 \\
\cdots & \cdots & \cdots & \mathbf{0}_{1x26} & \cdots & \cdots & \cdots & \cdots & \cdots & 1 & 0 \\
0 & (1+\overline{r}) & \mathbf{0}_{1x4} & -\overline{Y^T} & \mathbf{0}_{1x2} & \overline{C^T} & \mathbf{0}_{1x2} & \overline{I^T} & \mathbf{0}_{1x12} & \overline{r}\overline{D} & \mathbf{0}_{1x2} \\
(1-\delta) & \cdots & \mathbf{0}_{1x10} & \cdots & \left(\frac{\overline{I}}{\overline{K}}\right) & \cdots & \cdots & \mathbf{0}_{1x16} & \cdots & \cdots & \cdots \\
\mathbf{0}_{1x4} & -\overline{r} & \mathbf{0}_{1x14} & -\overline{r} & \cdots & \cdots & \mathbf{0}_{1x5} & \cdots & \overline{r} & \cdots & \mathbf{0}_{1x2} \\
\cdots & \cdots & \mathbf{0}_{1x19} & \cdots & 1 & \cdots & \cdots & \mathbf{0}_{1x8} & \cdots & \cdots & \cdots \\
\mathbf{0}_{1x2} & \psi_T & \cdots & \cdots & \cdots & \cdots & \cdots & \mathbf{0}_{1x25} & \cdots & \cdots & \cdots \\
\mathbf{0}_{1x3} & \psi_N & \cdots & \cdots & \cdots & \cdots & \cdots & \mathbf{0}_{1x24} & \cdots & \cdots & \cdots \\
\mathbf{0}_{1x4} & \psi_{r^*} & \cdots & \cdots & \cdots & \cdots & \cdots & \mathbf{0}_{1x23} & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \mathbf{0}_{1x27} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 1
\end{array}\right)
$$

The Matlab program "solab.m" uses matrices **a** and **b** as inputs in applying the Generalized Schur decomposition to solve for the policy functions. For insights regarding the analytical procedure, refer to Schmitt-Grohé & Uribe (2016, p.143).

Thus, the set of approximate policy functions, which characterize the solution of the model, is:

(P.F.1) $\widetilde{Y_t} = 0.6551\widetilde{K_t} - 0.0245\widetilde{D_t} + 0.2728\widetilde{A_t^T} + 1.2478\widetilde{A_t^N} - 0.0464\widetilde{r_t^*}$

(P.F.2) $\widetilde{Y_t^T} = 0.8590\widetilde{K_t} + 0.5220\widetilde{D_t} + 2.1719\widetilde{A_t^T} + 0.2171\widetilde{A_t^N} + 0.9887\widetilde{r_t^*}$

(P.F.3) $\widetilde{Y_t^N} = 0.5942\widetilde{K_t} - 0.1877\widetilde{D_t} - 0.2944\widetilde{A_t^T} + 1.5557\widetilde{A_t^N} - 0.3556\widetilde{r_t^*}$

(P.F.4) $\widetilde{C_t} = 0.6818\widetilde{K_t} - 0.2014\widetilde{D_t} - 0.1514\widetilde{A_t^T} + 1.2633\widetilde{A_t^N} - 0.2680\widetilde{r_t^*}$

(P.F.5) $\widetilde{C_t^T} = 0.7243\widetilde{K_t} - 0.2059\widetilde{D_t} + 0.1934\widetilde{A_t^T} + 0.8377\widetilde{A_t^N} - 0.2766\widetilde{r_t^*}$

(P.F.6) $\widetilde{C_t^N} = 0.6733\widetilde{K_t} - 0.2005\widetilde{D_t} - 0.2206\widetilde{A_t^T} + 1.3487\widetilde{A_t^N} - 0.2663\widetilde{r_t^*}$

(P.F.7) $\widetilde{I_t} = -2.3693\widetilde{K_t} + 0.2917\widetilde{D_t} - 2.9045\widetilde{A_t^T} + 9.1510\widetilde{A_t^N} - 3.7315\widetilde{r_t^*}$

(P.F.8) $\widetilde{I_t^T} = -2.3401\widetilde{K_t} + 0.2886\widetilde{D_t} - 2.6680\widetilde{A_t^T} + 8.8590\widetilde{A_t^N} - 3.7374\widetilde{r_t^*}$

(P.F.9) $\widetilde{I_t^N} = -2.3911\widetilde{K_t} + 0.2940\widetilde{D_t} - 3.0819\widetilde{A_t^T} + 9.3700\widetilde{A_t^N} - 3.7270\widetilde{r_t^*}$

(P.F.10) $\widetilde{L_t} = 0.3991\widetilde{K_t} - 0.0427\widetilde{D_t} + 0.0746\widetilde{A_t^T} + 0.8325\widetilde{A_t^N} - 0.0808\widetilde{r_t^*}$

(P.F.11) $\widetilde{L_t^T} = 0.5230\widetilde{K_t} + 0.5579\widetilde{D_t} + 1.3434\widetilde{A_t^T} + 0.6851\widetilde{A_t^N} + 1.0568\widetilde{r_t^*}$

$$\widetilde{L_t^N} = 0.3741\widetilde{K_t} - 0.1642\widetilde{D_t} - 0.1821\widetilde{A_t^T} + 0.8623\widetilde{A_t^N} - 0.3110\widetilde{r_t^*} \tag{P.F.12}$$

$$\widetilde{K_t^T} = 1.1023\widetilde{K_t} + 0.4960\widetilde{D_t} + 1.0477\widetilde{A_t^T} - 0.1217\widetilde{A_t^N} + 0.9395\widetilde{r_t^*} \tag{P.F.13}$$

$$\widetilde{K_t^N} = 0.9533\widetilde{K_t} - 0.2261\widetilde{D_t} - 0.4777\widetilde{A_t^T} + 0.0555\widetilde{A_t^N} - 0.4283\widetilde{r_t^*} \tag{P.F.14}$$

$$\widetilde{S_t} = -0.6240\widetilde{A_t^T} + \varepsilon_t^S \tag{P.F.15}$$

$$\widetilde{p_t^N} = 0.1159\widetilde{K_t} - 0.0124\widetilde{D_t} + 0.9409\widetilde{A_t^T} - 1.1614\widetilde{A_t^N} - 0.0235\widetilde{r_t^*} \tag{P.F.16}$$

$$\widetilde{p_t^C} = 0.0965\widetilde{K_t} - 0.0103\widetilde{D_t} + 0.7838\widetilde{A_t^T} - 0.9674\widetilde{A_t^N} - 0.0195\widetilde{r_t^*} \tag{P.F.17}$$

$$\widetilde{p_t^I} = 0.0662\widetilde{K_t} - 0.0071\widetilde{D_t} + 0.5376\widetilde{A_t^T} - 0.6636\widetilde{A_t^N} - 0.0134\widetilde{r_t^*} \tag{P.F.18}$$

$$\widetilde{w_t} = 0.3360\widetilde{K_t} - 0.0359\widetilde{D_t} + 0.8285\widetilde{A_t^T} - 0.4679\widetilde{A_t^N} - 0.0680\widetilde{r_t^*} \tag{P.F.19}$$

$$\widetilde{r_t^K} = -0.2433\widetilde{K_t} + 0.0260\widetilde{D_t} + 1.1242\widetilde{A_t^T} + 0.3389\widetilde{A_t^N} + 0.0493\widetilde{r_t^*} \tag{P.F.20}$$

$$\widetilde{r_t} = -0.0008\widetilde{K_t} + 0.0065\widetilde{D_t} - 0.6276\widetilde{A_t^T} - 0.0014\widetilde{A_t^N} + 0.9983\widetilde{r_t^*} \tag{P.F.21}$$

$$\widetilde{\lambda_t} = -0.7998\widetilde{K_t} + 0.2970\widetilde{D_t} - 0.4563\widetilde{A_t^T} - 0.2383\widetilde{A_t^N} + 0.3758\widetilde{r_t^*} \tag{P.F.22}$$

$$\widetilde{K_{t+1}} = 0.9538\widetilde{K_t} + 0.0040\widetilde{D_t} - 0.0398\widetilde{A_t^T} + 0.1254\widetilde{A_t^N} - 0.0511\widetilde{r_t^*} \tag{P.F.23}$$

$$\widetilde{D_{t+1}} = -0.1236\widetilde{K_t} + 1.0078\widetilde{D_t} - 0.5508\widetilde{A_t^T} + 0.2082\widetilde{A_t^N} - 0.2601\widetilde{r_t^*} \tag{P.F.24}$$

(P.F.25) $$\widetilde{A^T_{t+1}} = 0.6000\widetilde{A^T_t} + \varepsilon^T_{t+1}$$

(P.F.26) $$\widetilde{A^N_{t+1}} = 0.8400\widetilde{A^N_t} + \varepsilon^N_{t+1}$$

(P.F.27) $$\widetilde{r^*_{t+1}} = 0.8100\widetilde{r^*_t} + \varepsilon^{r^*_t}_{t+1}$$

The benchmarck model results in this study is based on this system of policy functions.

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