Inflation Targeting: An international comparison of policy determinacy

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Abstract

This paper explores inflation targeting policy determinacy across sixteen economies explicitly following such rule. I find that these countries do not necessarily manage their monetary policy in a way that leads to determinacy, which is desirable from a welfare perspective. Data coming from developing economies tend to be less consistent with determinacy immediately after inflation targeting implementation, only increasing its "activeness" later in the period under scrutiny. Contrarily, developed economies tend to show a strong tendency towards determinacy in the first stage of inflation targeting implementation while showing an indeterminate solution after the target is achieved. I suggest a link between economic development and monetary policy effectiveness, which in turn is related with credibility.

1.1 Introduction

Inflation targeting had become an increasingly common technique to lower and control inflation levels during the 1990's. First implemented by developed economies, it quickly spread among multiple developing countries later in that decade and in the 2000's. Extensive work covering inflation targeting has been done in several respects. Those include its transparency and discipline (Bernanke and Mishkin, 1997), its optimality (Giannoni and Woodford, 2003) and the experience of inflation targeting in transition economies (Jonas and Mishkin, 2003) among other key studies. In this chapter I will focus on the performance of the inflation targeting framework in two groups of countries, developed and developing economies. I believe this focus is relevant since there is a visible difference in inflation dynamics among these two groups that, in principle, may suggest intrinsic reasons worth to be explored. In order to measure the inflation targeting performance for each country, I consider a New Keynesian model where the equilibrium is indeterminate if monetary policy is passive, meaning that interest rate increases to lower inflation are not strong enough. On the other hand, if monetary policy is active, it should yield to determinacy, which in turn is wealth improving. I estimate the model across sixteen economies for periods in which they explicitly declared to follow an inflation targeting rule. So, the specific questions I would like to answer for these groups of counties are: 1) Are inflation targeting economies consistent with unique equilibria? 2) Are there behavioral differences among inflation targeting economies in terms of policy conduction? 3) Is there a visible cost of policymaking when allowing for indeterminacy?

In order to address these questions it is important to review recent literature which tackles these issues. Levin et al. (2004) analyze whether the implementation of inflation targeting has an influence on the formation of inflation expectations and inflation dynamics, both in emerging and industrialized economies. Using the methods of Stock (1991) and Hansen (1999) to obtain measures of persistence for consumer price inflation, they find that the monetary rule was meaningful in keeping inflation expectations at bay and reducing inflation persistence, especially for industrialized economies. As far as the emerging markets are concerned, the implementation of inflation targeting did not produce a fast convergence of inflation expectations to the target, as it was also the case of industrialized countries. In addition, even though succeeding in reducing its inflation level, emerging economies still produced high levels of inflation volatility.

Garín, Lester and Sims (2015) analyze welfare properties of nominal GDP targeting contrasting different policy rules (monetary and non-monetary) through the lens of a New Keynesian model. They find that nominal GDP targeting is significantly better than inflation targeting in welfare terms.

Florio and Gobbi (2015) focus on the level of the inflation target and its effect on determinacy of equilibrium and learnability of rational expectations under different policy combinations, both fiscal and monetary, in a new Keynesian model. They also explore the role of central bank transparency. They find that, in a non-Ricardian setup, the determinacy of equilibrium and learning process remains unaltered to changes in trend inflation and to transparency issues. However, a higher inflation target destabilizes expectations under active monetary policy.

On the other hand, under a Ricardian regime, increasing the inflation target calls for a more aggressive monetary authority in its response to inflation changes to achieve unique equilibrium. The authors show that determinacy implies a learning process only when agents are aware of both the inflation target and the central bank reaction function. The lack of awareness of any of these leads to a less stable regime in terms of determinacy. They conclude that full disclosure of the reaction function, including the target inflation rate, increases central bank's effectiveness and therefore determinacy of equilibrium.

I chose a typical Dynamic Stochastic General Equilibrium (DSGE) model. A contribution by Lubik and Schorfheide (2004) shows an innovative way to test for indeterminacy by extending the typical Linear Rational Expectations model (LRE) into a New Keynesian Dynamic Stochastic General Equilibrium (DSGE) model. The model allows for both indeterminacy of equilibrium and sunspot fluctuations and, as a result of the Bayesian approach, it gives probabilities of determinacy and indeterminacy of the model solution. Their finding, which is key for my study, directly relates active monetary policy with determinacy of equilibrium at the same time as passive monetary policy leads to indeterminacy of equilibrium. An active monetary policy performed by the monetary authority means to increase interest rates sufficiently enough in response to inflation so that self-fulfilling beliefs are suppressed. In this sense, King (2000) and Woodford (2003) show that indeterminacy can arise when monetary policy is passive. As Lubik and Schorfheide mention, indeterminacy, or multiple equilibria, has two basic undesirable properties. One is that the propagation of fundamental shocks is not uniquely determined. The other is that sunspot shocks can induce business-cycle fluctuations that would not have been present under determinacy. So, determinacy is a desirable model solution any central bank should seek.

Central banks following an inflation targeting rule should particularly fall in the determinacy category, considering that they explicitly announce their desired level of inflation the economy should converge. It is in this sense that I am interested in testing these groups of economies and evaluate their performance in terms of determinacy and propagation of shocks.

I estimate the DSGE model for sixteen economies, seven developed (New Zealand, Canada, UK, Sweden, Australia, Iceland and Norway) and nine developing (Israel, Czech Rep., Poland, Brazil, Colombia, South Africa, S. Korea, Mexico and Hungary), which follow an inflation targeting rule[2].

The chapter continues as follows. Section 1.2 deals with the methodology, which includes a description of the log-linearized model, the parameter estimation, the Bayesian approach, a simplified example and the econometric inference.

[2] On target? The international experience with achieving inflation targets, Scott Roger and Mark Stone, IMF Working Paper, August 2005

Section 1.3 shows my empirical results where I address the question on whether inflation targeting economies are consistent with unique equilibria. Section 1.4 tackles the question on behavioral differences among inflation targeting economies in terms of policy conduction. In section 1.5 I show the restricted vs. unrestricted models which deal with shock propagation and cost of policy. Section 1.6 shows a sensitivity analysis with respect to changes in prior distribution parameters. In section 1.7 I conclude. Sections 1.8 and 1.9 present two relevant appendices.

1.1.1 The Data

The data used to fit the model were extracted from IFS (IMF) database. I fit the model using output, interest rates and inflation for each country mentioned above. The period used varies between countries only considering inflation targeting rule adoption. Output is expressed as percentage deviations from the trend (log real per capita GDP HP de-trended). Inflation is expressed as annualized percentage changes. The nominal interest rates are quarterly averages of the benchmark rate in each country.

1.2 Methodology

1.2.1 The Log-linearized model

The model by Lubik and Schorfheide I selected to conduct the monetary policy analysis can be summarized below in the following three simple equations:

The first one is the Euler equation which reflects the demand side. Assuming that $\log(1+i_t) = R_t$ and defining $\varkappa$ as output we obtain the Euler Equation[3]:

$$\widetilde{\varkappa} = E_t(\widetilde{\varkappa_{t+1}}) - \tau(R_t - E_t(\widetilde{\pi_{t+1}})) + g_t, \qquad (1.1)$$

where ~indicates deviation from steady state and $\varkappa$ is both the level of aggregate consumption and output, considering a closed economy.

Equation (1.1) surges from the household's optimal decision between consumption and bond holding. Given the aggregate consistency conditions and the fact that there is no investment in this model, consumption can be expressed as the product minus G_t (an exogenous process reflecting government spending at time t). The variable g_t captures all real disturbances of the product's natural rate, the preference shock or government spending, and $\tau>0$ the inter-temporal substitution elasticity.

The second relevant equation is the so-called New Keynesian Phillips curve which reflects the supply side of this simplified economy[4]:

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[3] See equation 8 in appendix

[4] Combining equations 10 and 11 in appendix.

$$\widetilde{\pi} = \beta E_t(\widetilde{\pi_{t+1}}) + k(\widetilde{x_t} - z_t) \qquad (1.2)$$

The Phillips curve equation (1.2) shown above describes the inflation dynamics where k is the slope and z_t is a process that captures the exogenous changes of the marginal cost of production and the parameter $0 < \beta < 1$ is the household discount factor. The relationship between inflation, expected inflation and output is derived by introducing nominal rigidities, where only a portion of firms can adjust prices every period. In this way, monetary policy decisions can affect real variables in the short term.

Finally, the monetary policy rule is determined by the following equation (1.3):

$$\widetilde{R_t} = \rho_R \widetilde{R_{t-1}} + (1 - \rho_R)(\psi_1 \widetilde{\pi_t} + \psi_2(\widetilde{x_t} - z_t)) + \epsilon_{R,t} \qquad (1.3)$$

The monetary authority follows an interest rate rule, which is based on its previous behavior plus the component of inflation and product deviations with respect to their long-term values. Finally, the white noise shock $\epsilon_{R,t}$ can be interpreted as an unanticipated change in monetary policy or an error of such policy. Every time inflation falls below (above) its long-term target, all else equal, the nominal interest rate should come down (up). At the same time, when the output gap widens (narrows), the nominal interest rate should decrease (increase).

In this model, depending on the combination of parameters, indeterminacy of equilibrium may arise. This means that, there may be solutions of the model in which, after an exogenous shock, the path of adjustment of endogenous variables is not unique. Importantly, some adjustment paths have large deviations with respect to the adjustments coming from fundamental deviations. In addition, the possibility of exploring indeterminacy regions of the solution opens the possibility of solutions in which agent beliefs, which are not based on fundamentals (e.g. inflation forecast errors), can influence the return to the equilibrium path of endogenous variables.

1.2.2 Parameter Estimation

The Dynamic Stochastic General Equilibrium model DSGE (see King 2000 and Woodford 2003) that I use to evaluate the monetary policy in different economies has lately become a benchmark in macroeconomic analysis. The most relevant equations in order to estimate the parameters are the following.

Equations (1.1)-(1.3) are the base model for their econometric inference. They assume that shocks g_t and z_t follow univariate processes AR(1):

$$z_t = \rho_z z_{t-1} + \epsilon_{z,t} \ , \tag{1.4}$$

$$g_t = \rho_g g_{t-1} + \epsilon_{g,t} \ , \tag{1.5}$$

Where they assume that $\rho_{g,z} = 0$, meaning zero correlation between innovations. Given this specification, they obtain the DSGE model parameters:

$$\theta = [\psi_1, \psi_2, \rho_R, \beta, k, \tau, \rho_g, \rho_z, \rho_{gz}, \sigma_R, \sigma_g, \sigma_z] , \quad (1.6)$$

For the sake of clarity, the model can be expressed in the canonical form[5]:

$$\Gamma_0(\theta)s_t = \Gamma_1(\theta)s_{t-1} + \Psi(\theta)\epsilon_t + \Pi(\theta)\eta_t , \quad (1.7)$$

where,

$$s_t = \left[\widetilde{x}_t, \widetilde{\pi}_t, \widetilde{R}_t, E_t[\widetilde{x_{t+1}}], E_t[\widetilde{\pi_{t+1}}], g_t, z_t \right] , \quad (1.8)$$

$$\epsilon_t = [\epsilon_{R,t}, \epsilon_{g,t}, \epsilon_{z,t}] , \quad (1.9)$$

$$\eta_t = [(\widetilde{x}_t - E_{t-1}[\widetilde{x}_t]), (\widetilde{\pi}_t - E_{t-1}[\widetilde{\pi}_t])] , \quad (1.10)$$

The dimensions of the vectors in the above equations are: $s_t : n = 7$ for state variables , $Shocks : l = 3$ for shocks that influence endogenous variables and, $n_t : k = 2,$ for the forecast error.

The authors also assume an additional non-fundamental (sunspot) shock, ζ_t , which agents observe and cannot be associated to any of the fundamental variables.

[5] See equations 13-16

Given the linearity of the model, they express the forecast error as a linear function of the two sources of uncertainty of the model ϵ_t and ζ_t:

$$\eta_t = A_1\epsilon_t + A_2\zeta_t \tag{1.11}$$

where A_1 is kxl and A_2 is $kx1$.

This type of model allows for self-fulfilling expectations solutions consistent with the equilibrium. There are three different possible solutions for this model: 1) non-existent, implying s_t does not satisfy the transversality condition; 2) existence of a unique and stable solution (determinate equilibrium), where A_1 is uniquely determined and $A_2 = 0$; 3) existence of multiple equilibria, where A_1 is not uniquely determined and A_2 may be different from zero.

The parameter ψ_1, which represents the nominal interest rate sensitivity with respect to changes in the inflation rate, will be key in determining whether the solution will be determinate or indeterminate. As a rule of thumb, every time $\psi_1 > 1$, the model solution balances towards determinacy. In terms of the central bank policy, an interest rate hike proportionally higher than the increase in inflation hike ($\psi_1 > 1$) is considered as an active monetary policy. Otherwise is considered as passive. An active monetary policy favors determinacy of equilibrium. However, as the model established, ψ_1 is not the only relevant parameter to assure determinacy. ψ_2, which represents the nominal interest rate sensitivity with respect to the output gap, also has a relevant role. However, ψ_1 has a higher relevance than ψ_2 as far as determinacy is concerned (see Woodford pp. 256).

1.2.3 The Bayesian approach

This section shows an example by the authors on how to establish whether the data comes from a determinate or indeterminate equilibrium. When using DSGE models (see King (2000) and Woodford (2003)), indeterminacy can be a consequence of passive monetary policy (i.e. $\psi_1 < 1$). In addition, sunspot fluctuations can lead to significant welfare deterioration (Christiano and Harris, 1999). The natural conclusion at this point would be that the monetary authority should seek a policy that leads to determinacy in order to produce a social optimum.

One of the key novelties of this model is the Bayesian approach which enables to estimate the probability of both a determinate and indeterminate solution. Among other things, such feature allows to see the propagation of shocks in such region in order to measure the cost of policy in those cases were the equilibrium in not uniquely determined. Of course, this was not possible in pre-Bayesian models.

1.2.4 A simplified example

The following single equation model made by the authors has the objective to discuss these new features:

$$y_t = \frac{1}{\theta}E_t[y_{t+1}] + \epsilon_t \qquad \epsilon \sim iid(0,1) \qquad \theta \in \Theta = [0,2] \qquad (1.12)$$

This model can be cast in the canonical form equation (1.7) by introducing the conditional expectation $\xi_t = E_t[y_{t+1}]$ and the forecast error $\eta_t = y_t - \xi_{t-1}$. Thus,

$$\xi_t = \theta\xi_{t-1} - \theta\epsilon_t + \theta\eta_t \qquad (1.13)$$

The stability of equation (1.12) depends on the parameter θ. For $\theta > 1$ we have the unique and stable solution (determinate equilibrium) of the form $\xi_t = 0$, which obtains if $\eta_t = \epsilon_t$ and solving forward looking

$$y_t = \epsilon_t \qquad (1.14)$$

We see that for $\theta > 1$ the endogenous variable follows an *iid* process and its stochastic properties does not depend on the parameter θ. Based on this, they define the determinate region of the space parameter with $\Theta^D = (1,2]$. If, on the contrary, $\theta \leq 1$ (indeterminate equilibrium for $\Theta^I = [0,1]$), solving backwards they obtain the following new specification of the forecast error:

$$\eta_t = \widetilde{M}\epsilon_t + \zeta_t \qquad (1.15)$$

where $\widetilde{M}$ is a parameter non-related with θ that arises when the model is not determinate. In the above equation they add the sunspot shock ζ_t, considering that part of the forecast error could come from shocks not correlated with the fundamental error ϵ_t.

In the absence of a sunspot shock, and using (1.12) they obtain:

$$y_t = \theta y_{t-1} = \widetilde{M}\epsilon_t - \theta\epsilon_{t-1} \tag{1.16}$$

$\widetilde{M} = 1$ indicates determinacy of the equilibrium. They normalize $\widetilde{M} = 1 + M$ and focus on indeterminate solutions around $\widetilde{M} = 1$.

1.2.5 Econometric inference

Using the model presented by Lubik and Schorfheide, I estimate parameters θ and M, which would indicate whether the model solution seems to be in a determinate or indeterminate region. The following likelihood is the pillar of the analysis. It combines the space parameter of the indeterminacy and determinacy regions with the corresponding likelihood. They consider the joint probability of both parameters θ and M as $L(\theta, M \mid Y^T)$.

Assuming the error ϵ_t is normally distributed, both the likelihood function for the indeterminate and determinate region can be expressed in terms of the normal distribution.

Let $f(x) = \{x \in X\}$ be a function that takes the value 1 if $x \in X$ and 0 otherwise. Then,

$$L(\theta, M \mid Y^T) = \{\theta \in \Theta^I\} L_I(\theta, M \mid Y^T) + \{\theta \in \Theta^D\} L_D(Y^T) \tag{1.17}$$

where

$$L_D(Y^T) = (2\pi)^{\frac{-T}{2}} \exp\left[-\frac{1}{2} Y^{T'} Y^T\right] \tag{1.18}$$

$$L_I(\theta, M \mid Y^T) = (2\pi)^{\frac{-T}{2}} |\Gamma_Y(\theta, M)|^{-\frac{1}{2}} \exp\left[-\frac{1}{2} Y^{T'} \Gamma_Y^{-1}(\theta, M) Y^T \right] \quad (1.19)$$

and $\Gamma_Y(\theta, M)$ denotes the matrix of variances and covariances of vector Y^T under the ARMA (1,1) representation of equation (1.13). The determinate region of the likelihood is flat given that is invariant to θ and M. This implies that these parameters cannot be identified. Note that for $M = 0$:

$$L_I(\theta, M = 0 \mid Y^T) = L_D(Y^T) \quad (1.20)$$

If the observations correspond to the determinate region, we should expect close to zero correlation. So, for M significantly different from zero (indicating serial correlation) and $\theta \leq 1$ (indicating indeterminate equilibrium), the likelihood will be smaller. The absence of correlation is therefore interpreted as evidence of determinate equilibrium.

The novelty of the analysis presented by Lubik and Schorfheide comes from the possibility of weighting the determinate and indeterminate regions of the parameter space conditional to the observed data in order to estimate the parameters of the model. The probabilities used to weight the parameters estimation are coming from the Bayesian analysis.

By defining a prior distribution with density $p(\theta, M)$ over parameters θ and M it is possible to conduct inference based on the posterior distribution of the parameters given the data Y^T (see Table 1)[6].

Appling the Bayes theorem allows to get the posterior distribution of the parameters given the data.

$$p(\theta, M \mid Y^T) = \frac{[\{\theta \epsilon \Theta^I\} L_I(\theta, M \mid Y^T) + \{\theta \in \Theta^D\} L_D(Y^T)] p(\theta,}{\int L(\theta, M \mid Y^T) p(\theta, M) d\theta dM} \quad (1.21)$$

Then, the posterior probability of an indeterminate equilibrium is given by:

$$\pi_T = \int \{\theta \epsilon \Theta^I\} p(\theta, M \mid Y^T) d\theta dM \quad (1.22)$$

Importantly, there are combinations of parameters where the lack of serial correlation creates a bias towards the determinate region even when they belong to the indeterminate region. A clear example is when $\theta < 1$ and $M = 0.$ One way to solve this bias is assigning M a continuous prior distribution with probability zero to $M = 0.$ In the case of significant serial correlation, there will be values of $\theta < 1$ and $M \neq 0$ where the likelihood is higher than the determinate region. So, for $\theta \epsilon \Theta^I, \pi_T(I) \to 1.$ Meaning that the posterior probability provides a consistent test to detect an indeterminate equilibrium.

1.3 Empirical results: Are inflation targeting economies consistent with unique equilibria?

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[6] In section 4 we discuss how sensitive the parameter estimation could be depending on the choosing of the prior distribution.

Having explained the model by Lubik and Schorfheide, its log-linearization and econometric approach, I am now prepared to use the methodology to the set of countries pursuing an inflation targeting regime. The first question I would like to answer is: are these countries managing monetary policy in an efficient way to achieve their inflation target?

I estimate the log-linearized DSGE model, specified by equations (1.1)-(1.5), for sixteen economies categorized as inflation targeters by the IMF[7]. I choose four different sets of prior distributions for each country. Prior#1 refers to the unrestricted model, where both the indeterminacy region and sunspots are permitted. Prior#2 restricts the model to the determinacy region $M_{R\zeta} = M_{g\zeta} = M_{z\zeta} = 0$. Prior#3 considers no sunspot shocks ($\sigma_\zeta = 0$) and Prior#4 restricts both $M_{R\zeta} = M_{g\zeta} = M_{z\zeta} = 0$ *and* $\sigma_\zeta = 0.$ The selection of priors is detailed in Table 1.1. For each parameter, I report its range, distribution, mean, standard deviation and 90% confidence interval. For simplicity, I left the prior distributions chosen in the contribution by Lubik and Schorfheide (2004) untouched. However, I adjusted the mean and standard deviation of the inflation target π^*, as I consider it as a key parameter for this particular study.

Figure 1.6 presents all selected countries with their respective quarterly data on output, inflation and nominal interest rates. Data periods vary among countries as I chose the adoption of the inflation targeting rule as the starting data point. The

[7] Roger, Scott and Mark Stone, 2005, "On Target: The International Experience With Achieving Inflation Targets," IMF Working Paper. I decide to leave outside the analysis recent inflation targeters countries like Indonesia, Slovak Republic, Thailand, Peru and Romania due to too few observations.

quarterly sample data for each of the countries I've selected starts in the quarter where inflation targeting implementation was announced and finishes in the fourth quarter of the year 2005[8]. As a general observation, we see a disinflation trend along the sample period in most countries. In principle, such a stylized fact would suggest an active monetary policy and therefore a unique solution (determinacy), which in turn is welfare improving.

Table 1.2 reports estimated results for all sixteen countries under scrutiny considering full data samples (from beginning of inflation targeting implementation till 4Q05). My first finding is that, of the sixteen countries under scrutiny, nine of them show an indeterminate solution with probability 1, considering the unrestricted specification. My initial hypotheses was that, if the Volker-Greenspan period showed a clear increase in the probability of determinacy with respect to prior post-war periods, model estimates of pure inflation targeting periods, were monetary authorities explicitly announce a target, should be mostly determinate. So, Table 1.2 findings reject such hypothesis. I speculate that the diversity of results responds to different country characteristics not captured by the model. In particular, the macroeconomic context of each country at the moment of inflation targeting adoption, plus pending structural reforms, could be some of the reasons behind these findings. In terms of what can be tested in the present model, in Section 1.4 I consider visible characteristics that

[8] Lubik and Schorfheide (2003) compare the results of the model in different periods in the US (which is not an inflation targeter according to the IMF definition)

could contribute to the indeterminacy of equilibrium such as inflation volatility and periods of increasing inflation.

1.4 Are there behavioral differences among inflation targeting economies in terms of policy conduction?

In order to refine my analysis, I look for behavioral differences among inflation targeting economies in terms of policy conduction. I reconsider the data periods presented in Figure 1.6. As I mentioned previously, virtually all selected economies show a reduction in the rate of inflation by the end of the period (4Q05) with respect to the date of implementation of inflation targeting. However, there are visible differences in the inflation rate and its dynamics for different groups of countries which are worth to account for. I distinguish two main groups of countries, developed and developing economies. In principle, such categorization is relevant considering the presumable differences in central banks reputation and credibility at the moment of inflation targeting implementation.

As we can see in Figure 1.6, when comparing with developed economies, most developing economies take more time to show a decreasing trend in inflation towards the target over the period. In some cases like Poland, Brazil, South Korea, Czech Republic and Hungary, there are visible increases in the inflation rate before the adjustment takes place. On the other hand, while developed economies show a faster adjustment of inflation towards the target, some of them have a slight increase in inflation at the end of the selected periods. This is the case of New Zealand, Norway, Iceland, UK, Canada and Sweden.

Given the behavior of developing economies inflation dynamics, and in order to search for different type of results within the period under scrutiny, I split each of the sixteen inflation series in two periods and fit the model separately for each period. For developing economies, I fit both periods in order to look for a possible increase in the probability of determinacy in a second stage. For developed economies, I fit a single period after removing an increasing inflation trend at the end of the series. First periods start with the implementation of IT, which contains higher inflation volatility and even visible upward trends in the inflation rate. Second periods start after inflation peaked and a disinflationary trend dominates the dynamics. So, for each country, the length of the two periods is different.

Table 1.3 shows results for the above mentioned partial sample estimates. I find that, for developing economies, considering the unrestricted prior only, the second period shows a significant increase in the probability of determinacy. Developed economies, on the other hand, exhibit a visible increase in the probability of determinacy when short periods of increasing inflation were removed from the sample period. I speculate that this could be evidence that it is not sufficient for the inflation path to converge to the inflation target range in order to guarantee a higher probability of a unique equilibrium. The absence of inflation increase and erratic behavior of the series appears to be another condition needed to achieve determinacy. This was the case for New Zealand, Iceland and Sweden, Norway and Canada. UK and Australia, on the other hand, did not revert the high probability of indeterminacy when periods where split. I argue that both series of inflation have an erratic behavior along the whole sample and, even with an overall disinflation trend I could not find a splitting sample point with significant changes in the probability of determinacy.

I believe that a story consistent with these results has to do with higher credibility at the moment of the inflation targeting adoption. These developed economies honored such higher credibility by being hawkish in pursuing an active monetary policy. This means that Central Banks increased the nominal interest rate enough to suppress self-fulfilling beliefs and therefore reduced the inflation level towards the target for a reasonable period of time, building a good reputation on top of the already high credibility. As far as the late periods of increasing inflation are concerned, that story follows that central banks, having achieved its main goal, focus on a less relevant target (from the inflation targeting point of view), like the output gap, losing activeness to keep inflation low. Such story is fully consistent with passive monetary policy, where the monetary authority does not respond to increases in inflation with stronger increases in the nominal interest rate. The fact that the probability of a determinate equilibrium is significantly higher when periods of passive monetary policy are removed, gives us the idea that a credible central bank may manage a successful inflation targeting regime without conducting an active monetary policy after inflation is contained for some period.

In light of the estimated results for partial samples for both developed and developing economies, I derive three different scenarios in which countries can be characterized in terms of the inflation dynamics towards the inflation target.

1.- A quick and effective adjustment of the inflation. As expected, most of the selected developed economies fall into this category.

2.- IT policy is effective but the adjustment is slow. Most of developing economies fall into this category, considering the need of credibility built-up and possibly poor policy implementation. Importantly, all of these countries showed strong determinacy bias in the full sample results, consistent with active monetary policy (see Table 1.2).

3.- A poor performance of IT policy in the first period of implementation and a significantly more efficient one in the second period. Countries falling into this category were also developing (see Table 1.4).

South Africa, Australia and the UK could do not fall into any of these scenarios, presumably due to high inflation volatility and several episodes of increasing inflation along the sample which prevented from achieving a determinate equilibrium.

1.5 Restricted vs. Unrestricted models - shock propagation and cost of policy

In this section I address the question on the existence of costs in terms of output that may result from applying monetary policy in response to fundamental and sunspot shocks under different model specifications. I estimate the model and compute impulse response functions for monetary, demand, supply and sunspot shocks to output, inflation and the nominal interest rate under two different specifications. The first one is the unrestricted model which allows the solution to expand to the indeterminate region, also allowing for sunspot shocks. The second one, I run the model fully restricted using prior 4 where I impose $M_{R\zeta} = M_{g\zeta} = M_{z\zeta} = 0$ and $\sigma_{\zeta} = 0$.

I selected the cases of Iceland and the Czech Republic, in representation of developed and developing economies, respectively, to examine the propagation of shocks in developed and developing countries. The results of these impulse response exercises are different in the sense that, when running the model for the full sample using prior (unrestricted model), Iceland's solution was strongly determinate while Czech Republic data was consistent with indeterminacy of equilibrium. The results that follow for the two selected countries are, on average, representative for more developed economies in the case of Iceland, and developing economies for the case of the Czech Republic.

Table 1.5 contrasts model estimation results for Iceland under the unrestricted and restricted parameters. I find non-material differences in most of the estimated parameters. In terms of the central bank response to discrepancies of actual and desired inflation (ψ_1) , the posterior mean of the unrestricted model indicates a 1.2 percent versus a 1.28 percent for the restricted model. Output gap targeting (ψ_2) appears also similar in both specifications at around 0.25, in the same line as interest rate smoothing (ρ_R) at 0.55.The estimated steady state for the inflation rate is slightly lower for the restricted model, 3.73, compared to an estimation of 3.83 for the unrestricted one.

Results found for the Czech Republic, reported in Table 1.6, are quite different. First, the estimated monetary policy reaction to movements in the inflation rate are only slightly passive ($\psi_1 < 1$) in the unrestricted model while increasing above 1 when restricting the model to the determinacy region solution, implying that monetary policy becomes active only when determinacy restrictions are applied. This result shows how relevant is the expansion of the region solution towards indeterminacy given that, potentially indeterminate solutions may arise as determinate if the model is misspecified. Output gap targeting (ψ_2), on the other hand, did not showed a substantial change when restricting the model. ρ_R, the nominal interest rate auto-correlation, resulted higher when estimating the restricted model, suggesting a smoother adjustment towards the long-run equilibrium when restricting the solution to the determinacy region.

On average, I find that data coming from countries that were consistent with an indeterminate equilibrium for the full sample show bigger differences in parameter estimation between the unrestricted and restricted model specification than data consistent with determinacy.

Now, I derive impulse response functions to study potential differences in the propagation of shocks under the unrestricted and restricted specifications explained above. Figures 1.1 – 1.4 depict posterior means (with their 90-percent probability bands) for output, inflation and the nominal interest rate to fundamental and sunspot shocks. I compare Figure 1.1 and Figure 1.2, which depicts impulse responses for Iceland under the unrestricted specification and restricted specification respectively. Responses to an unexpected monetary tightening show no significant differences in output loss under the two different specifications. However, inflation falls slightly more under the unrestricted model for a similar, 100 basis points increase in the nominal interest rate, where the solution is extended to the indeterminacy region. A positive demand shock, which in this case takes the form of a positive shift in the Euler equation, have a slightly higher impact on all output, inflation and the nominal interest under the restricted specification in contrast to the restricted model. The supply shock also shows mild differences in the change of output when contrasting the two different specifications. Finally, the sunspot shock has a minimum positive effect on output for the unrestricted specification.

Turning to the comparison of Figure 1.3 and Figure 1.4, I find that the Czech Republic not only shows differences in the magnitude of output changes under the two different specifications but also the path towards the long-run equilibrium show different behavior. Starting with the monetary shock, a 50 basis points increase in the nominal interest rate reduces inflation by 0.3 percent under the restricted model while in the unrestricted specification, a 60-65 basis points increase in the nominal interest rate first reduces inflation by 0.1 percent, then increases 0.3 percent to only then converge to its long-run steady state. When a positive demand shock occurs, output increases by 0.6 percent under the restricted model while increasing below 0.5 percent under the unrestricted specification. Inflation and the nominal interest rate exhibit an interesting difference among the two different specifications. While the restricted model shows a 100 basis points increase in the nominal interest rate that adjusts inflation back to its long-run target in about ten periods, the behavior of these two variables under the unrestricted model is far from predictable. The nominal interest rate falls more than 100 basis points in the first eight periods followed by a slowly increase afterwards. At the same time, inflation falls by 1.0 percentage point, converging to the steady state in forty periods. A positive supply shock generates a 0.6 percent increase in output under prior 1 in contrast to an increase of only 0.3 percent under prior 4. Finally, the sunspot shock has only a marginal positive effect on output in the first two periods after the shock occurs.

Considering these two representative countries, I find differences between results coming from different model specifications much more compelling for data consistent with indeterminacy (the Czech Republic in this case) than the ones coming from Iceland (representing data consistent with determinacy of equilibrium). Differences in the loss/gain in output for the Czech Republic appear higher than the one I found for Iceland, possibly suggesting an over/under estimation of policy cost when the model is misspecified. In principle, I would argue that, forcing the model to estimate the parameters in the determinate region when in fact we already know the indeterminate region is relevant for the solution, creates some noise in the measure of output and inflation fluctuations.

1.6 How sensitive are model results to changes in prior distribution parameters?

In this section I explore how sensitive are the probabilities of determinacy and indeterminacy to the selection of some prior parameters, in particular the inflation target and its standard deviation. In the case of Canada, using an inflation target of 4% with a standard deviation of 2, I find the equilibrium to be indeterminate (see Table 1.7 left side). In contrast, when changing the inflation target to 2% with a standard deviation of 1.5, which are the actual parameters for Canada (actual target and standard deviation for the period), I find the probability of a equilibrium to be 1 (see Table 1.7 right side). For the particular case of Canada, not only the probabilities are dependent on prior parameters but also the sensitivity is high enough to change the result completely, turning an indeterminate equilibrium in to a determinate one. In the cases of Norway, New Zealand and Sweden, I also found a higher probability of determinacy when changing the target mean and standard deviation. Nevertheless, the changes were not as significant as in the case of Canada. On average, I see that the correct mean and standard deviation of the inflation target prior distribution results in a more accurate estimation of the model posterior parameters. This is not the case for developing economies, where we could not find a clear pattern of behavior of the probability of determinacy when testing for prior parameter sensitivity. In summary, I find that results should be interpreted carefully, considering its high sensitivity with respect to the selection of prior parameters.

1.7 Conclusion

I estimated a prototypical monetary DSGE model, allowing for both indeterminacy of equilibrium and sunspot fluctuations, for a set of sixteen countries explicitly following the inflation targeting rule. After the first approach, where I fitted the model for the full sample for each country, I could not find a pattern of results towards determinacy for developed economies with respect to developing economies as I initially expected. In a second stage, I shorten the sample for developed economies were I saw a generalized increase in the inflation rate and divided the sample in two periods for developing economies before re-running the model for each economy. I found a significant increase in the probability of determinacy for developed economies and the same pattern was found for the second period sample in developing economies. I propose that this could suggest some form of learning curve for the latter group. For the former group of countries, the early success is consistent with a higher level of central bank credibility, supported by a strong initial active monetary policy that leads to a fast adjustment of inflation levels towards the target. After inflation is controlled and adjusted downwards, these central banks are left with more degrees of freedom to tackle with objectives of secondary importance in the inflation targeting policy like the output gap adjustment. A better and more robust way to determine were to establish the division between periods of determinacy and indeterminacy is left as a topic for future research.

I also tested potential differences in the propagation of shocks under the unrestricted and restricted specifications. I find differences between results coming from different model specifications much more compelling for data consistent with indeterminacy (the Czech Republic in this case) than the ones coming from Iceland (representing data consistent with determinacy of equilibrium). Differences in the loss/gain in output for the Czech Republic appear higher than the one I found for Iceland, possibly suggesting an over/under estimation of policy cost when the model is misspecified. In principle, I would conclude that, forcing the model to estimate the parameters in the determinate region when in fact we already know the indeterminate region is relevant for the solution, creates some noise in the measure of output and inflation fluctuations.

Finally, I discuss the sensitivity of posterior parameter estimation with respect to prior selection (in particular the mean inflation targets selected and their standard deviations). I found that sensible changes in means and standard deviations of inflation targeting parameters dramatically affect model predictions.

My finding comes in line with Levin et al.(2004) in the sense that developing economies take longer to bring inflation to target. On the other hand, the contribution from Garín, Lester and Sims (2015) may shed some light on why developed economies seem to turn away from inflation targeting after having achieved their target.

Florio and Gobbi (2015) finding that developing economies may need a more aggressive activeness in monetary policy conduction looks compatible with my finding of indeterminacy of equilibrium for developing economies in the first period after the implementation of the inflation targeting rule.

In terms of policy implications, structural conditions at the time of inflation targeting implementation seem to be critical for the rule to be successful. However, these differences are inherent of developing and developed economies, and may not be solved by choosing a different date for policy implementation. Yet, that is a subject for future research.

In summary, when testing for determinacy of equilibrium, the selected economies under scrutiny show heterogeneous results. In principle, there is no clear indication that the inflation targeting rule leads towards determinacy. Second, I find that, on average, data coming from developed economies are consistent with determinacy in the first stage of inflation targeting implementation. In a second stage, once the target is achieved, the equilibrium turns indeterminate. In the same line, I find that developing economies, also on average, show the inverse behavior. While data in early stages of inflation targeting implementation is consistent with indeterminacy, a second period, after inflation peaks, shows an increase in the probability of determinacy. Finally, in the same line as Lubik and Schorfheide proved, I find that, allowing for an indeterminate region in the model solution affects the propagation of shocks, making possible some measure of the cost of policymaking in terms of inflation and output under different model specifications.

Table 1.1: Prior distributions for DSGE model parameters

Name	*Range*	*Density*	*Mean*	*Std.Dev.*	90% *Interval*
ψ_1	R^+	*Gamma*	1.10	0.50	$[0.33, 1.85]$
ψ_2	R^+	*Gamma*	0.25	0.15	$[0.06, 0.43]$
ρ_R	$[0,1)$	*Beta*	0.50	0.20	$[0.18, 0.83]$
π^*	R^+	*Gamma*	4.00	2.00	$[0.90, 6.91]$
r^*	R^+	*Gamma*	2.00	1.00	$[0.49, 3.47]$
κ	R^+	*Gamma*	0.50	0.20	$[0.18, 0.81]$
τ^{-1}	R^+	*Gamma*	2.00	0.50	$[1.16, 2.77]$
ρ_g	$[0,1)$	*Beta*	0.70	010	$[0.54, 0.86]$
ρ_z	$[0,1)$	*Beta*	0.70	0.10	$[0.54, 0.86]$
ρ_{gz}	$[-1,1]$	*Normal*	0.00	0.40	$[-0.65, 0.65]$
$M_{R\zeta}$	R	*Normal*	0.00	1.00	$[-1.64, 1.64]$
$M_{g\zeta}$	R	*Normal*	0.00	1.00	$[-1.64, 1.64]$
$M_{z\zeta}$	R	*Normal*	0.00	1.00	$[-1.64, 1.64]$
σ_R	R^+	*Inv.Gamma*	0.31	0.16	$[0.13, 0.50]$
σ_g	R^+	*Inv.Gamma*	0.38	0.20	$[0.16, 0.60]$
σ_z	R^+	*Inv.Gamma*	1.00	0.52	$[0.42, 0.67]$
σ_ζ	R^+	*Inv.Gamma*	0.25	0.13	$[0.11, 0.40]$

Table 1.2: Full sample results for Emerging and Developed economies

Full sample results			
Emerging economies		Probability	
	Prior	Determinate	Indeterminate
Brazil			
IT full sample	1	0.0000	1.0000
	2	0.0000	1.0000
	3	0.0256	0.9744
	4	0.7006	0.2994
Czech Rep.			
IT full sample	1	0.0012	0.9988
	2	0.0409	0.9591
	3	0.0017	0.9983
	4	0.7203	0.2797
Korea			
IT full sample	1	0.8013	0.1987
	2	0.6606	0.3394
	3	0.9038	0.0962
	4	0.3887	0.6113
Israel			
IT full sample	1	0.9999	0.0001
	2	0.5676	0.4324
	3	0.8017	0.1983
	4	0.1524	0.8476
Hungary			
IT full sample	1	0.0180	0.9820
	2	0.6217	0.3783
	3	0.0006	0.9994
	4	0.4888	0.5112
Colombia			
IT full sample	1	0.9966	0.0034
	2	0.9897	0.0103
	3	0.8661	0.1339
	4	0.9403	0.0597
Poland			
IT full sample	1	0.6543	0.3457
	2	0.5129	0.4871
	3	0.9999	0.0001
	4	0.8795	0.1205
Mexico			
IT full sample	1	0.9199	0.0801
	2	0.9705	0.0295
	3	0.8528	0.1472
	4	0.9676	0.0324
South Africa			
IT full sample	1	0.0000	1.0000
	2	0.0284	0.9716
	3	0.0000	1.0000
	4	0.2010	0.7990

Full sample results			
Developed economies		Probability	
	Prior	Determinate	Indeterminate
Sweden			
IT full sample	1	0.0000	1.0000
	2	0.0000	1.0000
	3	0.0000	1.0000
	4	0.1027	0.8973
New Zealand			
IT full sample	1	0.0000	1.0000
	2	0.6245	0.3755
	3	0.0000	1.0000
	4	0.9915	0.0085
Norway			
IT full sample	1	0.9917	0.0083
	2	0.9628	0.0372
	3	0.9583	0.0417
	4	0.9255	0.0745
Australia			
IT full sample	1	0.0000	1.0000
	2	0.0364	0.9636
	3	0.0000	1.0000
	4	0.1798	0.8202
Iceland			
IT full sample	1	0.6588	0.3412
	2	0.5831	0.4169
	3	0.0006	0.9994
	4	0.7722	0.2278
UK			
IT full sample	1	0.0000	1.0000
	2	0.5903	0.4097
	3	0.0000	1.0000
	4	0.5107	0.4893
Canada			
IT full sample	1	0.0000	1.0000
	2	0.0010	0.9990
	3	1.0000	0.0000
	4	0.5170	0.4830

Table 1.3: Partial sample results for Emerging and Developed economies

Partial sample results				
Emerging Economies			Probability	
		Prior	Determinate	Indeterminate
Brazil				
Period 1	2Q99-2Q02	1	0.9531	0.0469
Period 2	3Q99-4Q05	1	1.0000	0.0000
Czech Rep.				
Period 1	1Q98-1Q03	1	0.3554	0.6446
Period 2	2Q03-4Q05	1	0.5311	0.4689
Korea				
Period 1	1Q01-4Q03	1	0.6124	0.3876
Period 2	1Q04-4Q05	1	0.9840	0.0160
Israel				
Period 1	2Q97-2Q02	1	0.0000	1.0000
Period 2	3Q02-4Q05	1	0.0551	0.9449
Hungary				
Period 1	2Q01-2Q03	1	0.2058	0.7942
Period 2	3Q03-4Q05	1	0.8660	0.1340

Partial sample results			
Developed Economies		Probability	
	Prior	Determinate	Indeterminate
Sweden			
	1	0.9981	0.0019
IT partial sample	2	0.0000	1.0000
π = 2 - σ = 1 1Q93-4Q98	3	0.2179	0.7821
	4	0.2325	0.7675
New Zealand			
	1	1.0000	0.0000
IT partial sample	2	0.9997	0.0003
π = 2 - σ = 1 1Q90-3Q99	3	0.0000	1.0000
	4	0.9958	0.0042
Norway			
IT partial sample	1	0.9177	0.0823
π = 2.5 - σ = 1 1Q01-	2	0.9136	0.0864
1Q04	3	0.9383	0.0617
	4	0.6977	0.3023
Australia			
IT full sample	1	0.0000	1.0000
π = 4 - σ = 2 2Q93-	2	0.0364	0.9636
4Q05	3	0.0000	1.0000
	4	0.1798	0.8202
Iceland			
	1	0.7224	0.2776
IT partial sample	2	0.8959	0.1041
π = 2.5 - σ = 1 1Q01-4Q03	3	0.7883	0.2117
	4	0.8541	0.1459
UK			
	1	0.0000	1.0000
IT full sample	2	0.5903	0.4097
π = 2 - σ = 1 4Q92-4Q01	3	0.0000	1.0000
	4	0.5107	0.4893
Canada			
	1	1.0000	0.0000
IT full sample	2	0.0000	1.0000
π = 2 - σ = 1.5 1Q91-4Q05	3	1.0000	0.0000
	4	0.2931	0.7069

Table 1.4: Classification of adjustment toward inflation targeting

Classification of adjustment towards IT	1	2	3
Developing Economies			
Israel		X	
Czech Rep.			X
Poland		X	
Brazil			X
Colombia		X	
South Africa	w/o cat.		
Korea		X	
Mexico		X	
Hungary			X
Developed Economies			
New Zealand	X		
Canada	X		
UK	w/o cat.		
Sweden	X		
Australia	w/o cat.		
Iceland	X		
Norway	X		

1 - IT effective - quick adjustment of inflation near target

2 - IT effective but slow - Takes the hole sample to adjust inflation near the target

3 - IT ineffective in Period 1 - effective in Period2

Table 1.5: Iceland – restricted vs. unrestricted results

	Iceland							
	Unrestricted model				Restricted model			
Parameter Estimation Results								
	Mean	St. Dev.	CI (high)	CI (low)	Mean	St. Dev.	CI (high)	CI (low)
Y_1	1.20	0.38	0.57	1.77	1.28	0.38	0.66	1.88
Y_2	0.24	0.12	0.05	0.41	0.25	0.12	0.07	0.43
r_R	0.56	0.12	0.36	0.76	0.55	0.12	0.36	0.74
p^*	3.91	0.99	2.31	5.57	4.06	0.83	2.66	5.41
r^*	3.83	0.61	2.83	4.81	3.73	0.62	2.71	4.75
k	0.34	0.13	0.14	0.55	0.29	0.13	0.09	0.48
t^{-1}	1.93	0.52	1.07	2.71	1.95	0.50	1.15	2.76
r_g	0.75	0.06	0.65	0.86	0.75	0.06	0.65	0.86
r_z	0.62	0.09	0.48	0.77	0.65	0.09	0.50	0.81
$r_{g,z}$	0.97	0.03	0.93	1.00	0.95	0.05	0.87	1.00
M_{Rz}	-0.31	0.98	-1.79	1.42	0.00	0.00	0.00	0.00
M_{gz}	0.30	1.11	-1.48	2.12	0.00	0.00	0.00	0.00
M_{zz}	-0.01	0.81	-1.34	1.42	0.00	0.00	0.00	0.00
s_R	0.28	0.05	0.19	0.35	0.28	0.05	0.20	0.37
s_g	0.83	0.21	0.48	1.15	0.78	0.20	0.46	1.11
s_z	3.17	0.53	2.32	3.95	3.22	0.55	2.35	4.04
s_z	0.23	0.10	0.11	0.35	0.00	0.00	0.00	0.00

Figure 1.1: Iceland - Impulse response functions - Unrestricted model

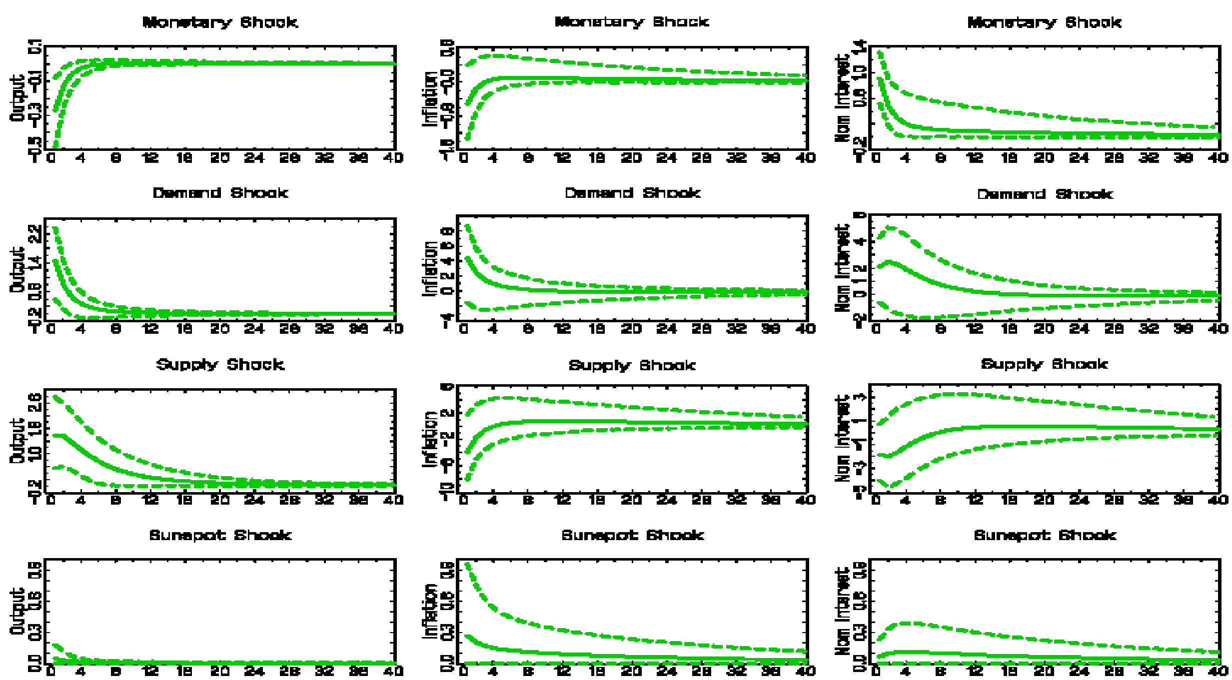


Figure 1.2: Iceland - Impulse response functions - Restricted model

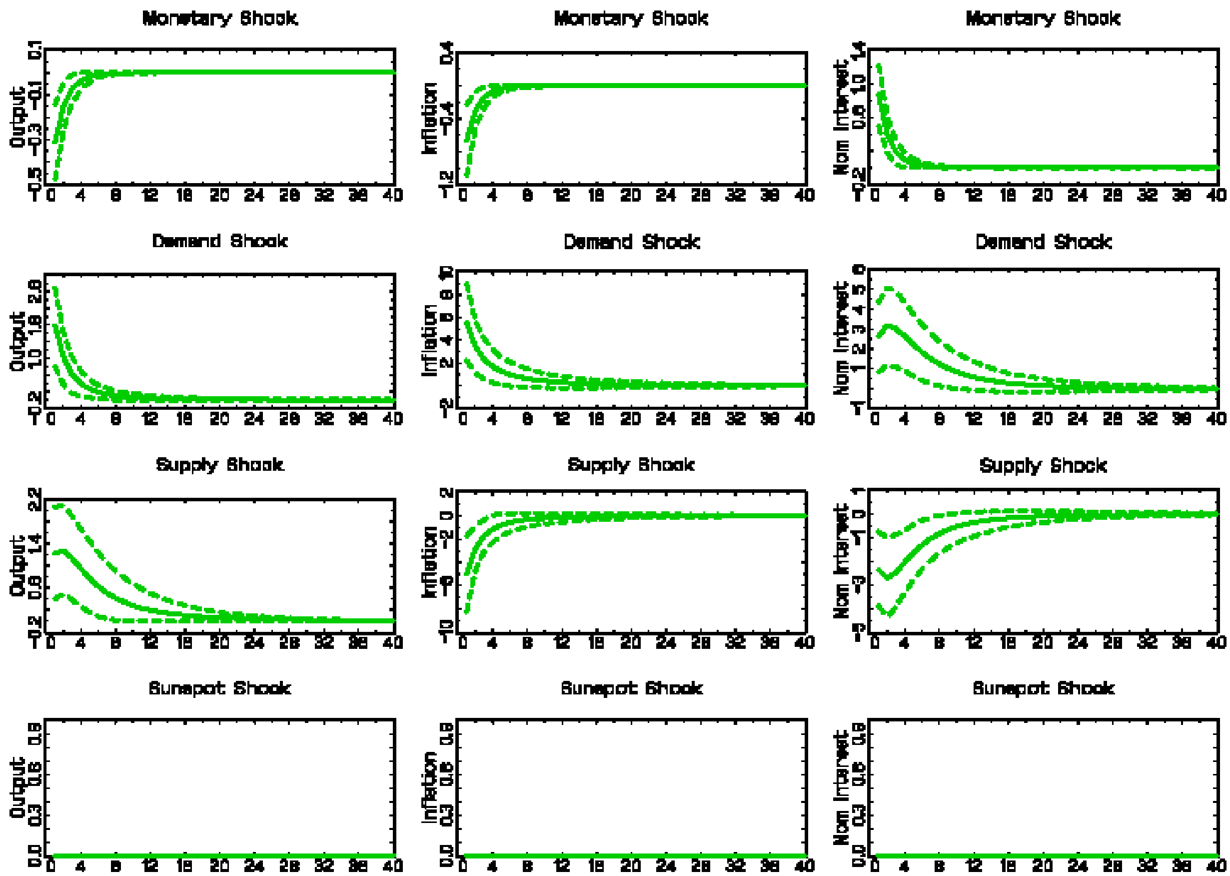


Table 1.6: Czech Rep. – restricted vs. unrestricted results

	Czech Republic							
	Irrestricted model				Restricted model			
Parameter Estimation Results								
	Mean	St. Dev.	CI (high)	CI (low)	Mean	St. Dev.	CI (high)	CI (low)
Y_1	0.89	0.07	0.80	1.00	1.12	0.22	0.77	1.47
Y_2	0.25	0.11	0.07	0.41	0.25	0.12	0.07	0.43
r_R	0.64	0.06	0.55	0.74	0.58	0.06	0.48	0.68
p^*	3.76	1.68	1.10	6.34	4.47	0.90	2.96	5.93
r^*	0.90	0.29	0.41	1.37	1.09	0.31	0.58	1.59
k	0.26	0.11	0.10	0.42	0.23	0.09	0.10	0.37
t^{-1}	2.03	0.49	1.22	2.79	2.14	0.51	1.29	2.95
r_g	0.68	0.06	0.59	0.78	0.80	0.05	0.73	0.87
r_z	0.74	0.10	0.60	0.90	0.76	0.07	0.65	0.88
$r_{g,z}$	0.02	0.29	-0.43	0.51	0.46	0.27	0.06	0.89
M_{Rz}	-0.77	0.62	-1.78	0.26	0.00	0.00	0.00	0.00
M_{gz}	1.98	0.57	1.06	2.88	0.00	0.00	0.00	0.00
M_{zz}	-0.72	0.21	-1.05	-0.39	0.00	0.00	0.00	0.00
s_R	0.17	0.02	0.13	0.21	0.19	0.03	0.15	0.23
s_g	0.37	0.07	0.25	0.48	0.24	0.06	0.15	0.32
s_z	0.91	0.22	0.58	1.23	0.84	0.17	0.56	1.10
s_z	0.21	0.05	0.12	0.29	0.00	0.00	0.00	0.00

Figure 1.3: Czech Rep. - Impulse response functions - Unrestricted model

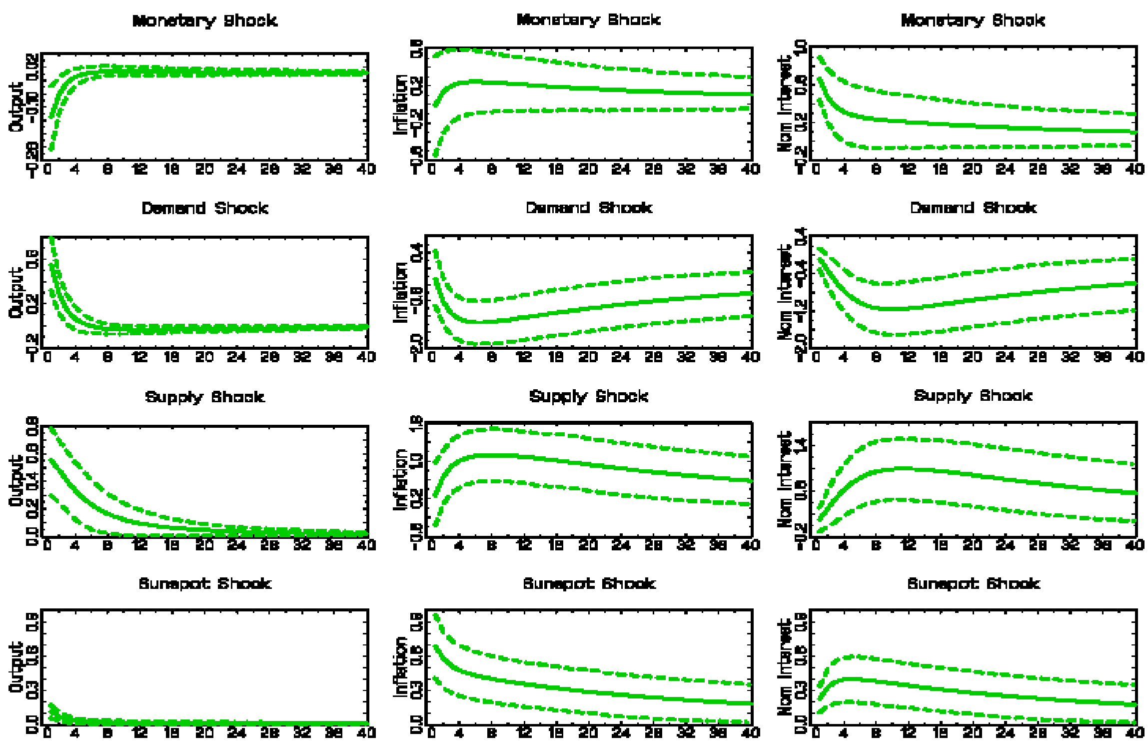


Figure 1.4: Czech Rep. - Impulse response functions - Restricted model

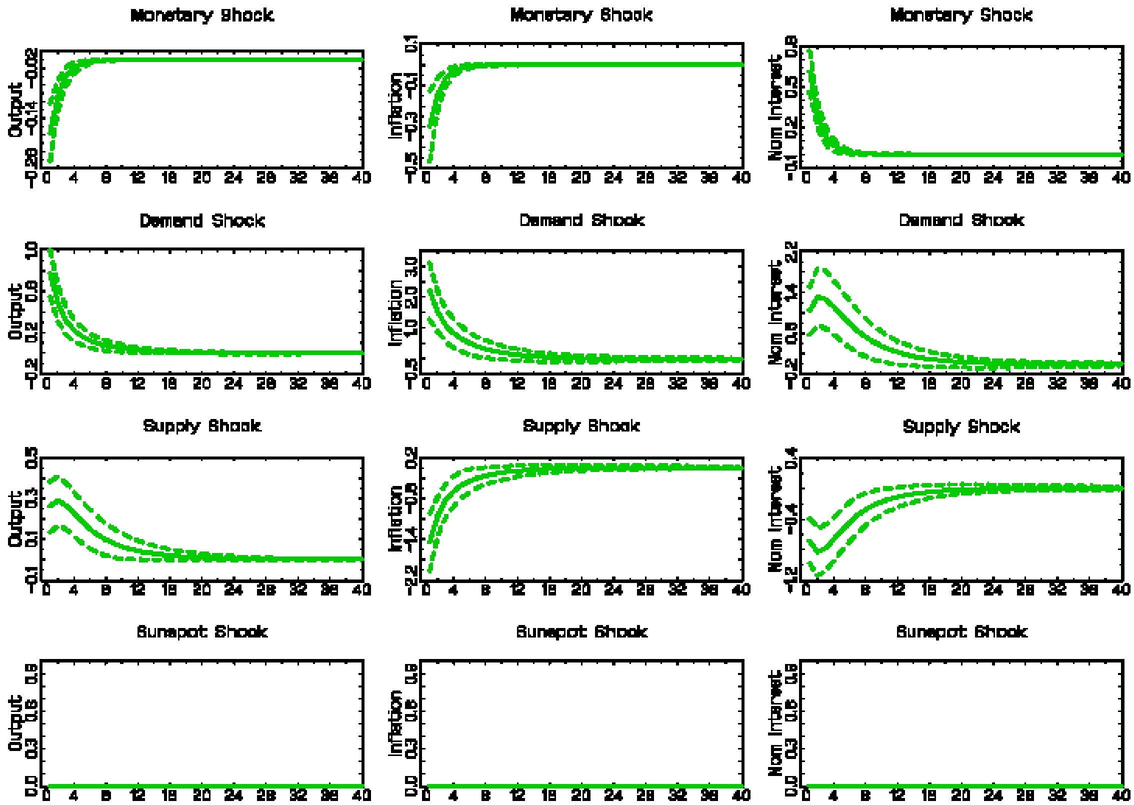


Table 1.7: Canada – Testing mean and standard deviation changes

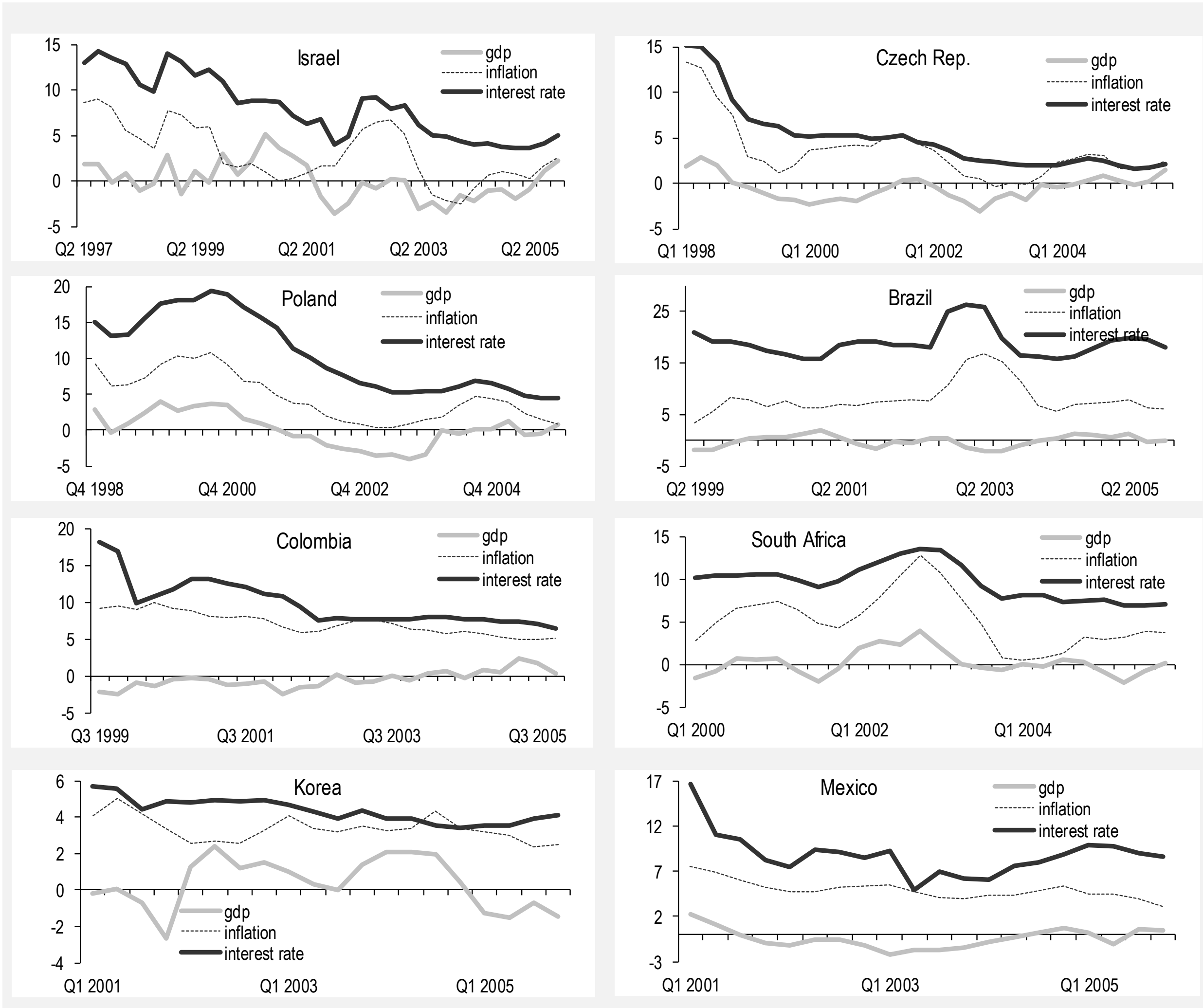
Selected Target					
Name	**Range**	**Density**	**Mean**	**Std.Dev.**	**90%Int.**
p^*	$\mathbb{R}^+$	Gamma	**4.00**	**2.00**	[0.90,6.91]

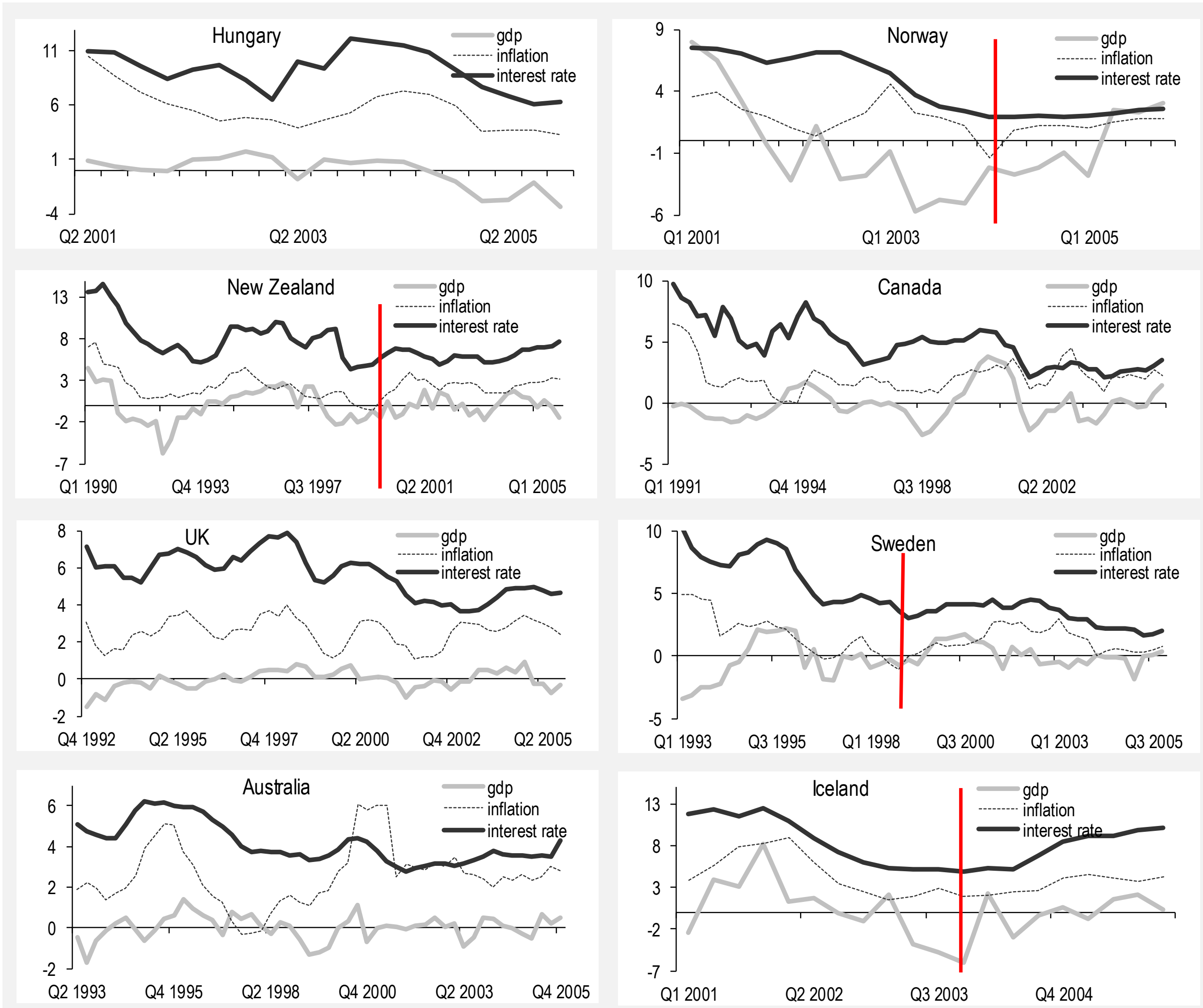
Canada				
Probability				
Determinat	0.0000		**π = 4 - σ = 2**	
Indetermin	1.0000		**1Q91-4Q05**	
Parameter Estimation Results				
	Mean	St. Dev.	CI (high)	CI (low)
Y_1	**0.9811**	0.0014	0.9793	0.9836
Y_2	0.1968	0.0023	0.1935	0.2003
r_R	0.7983	0.0464	0.726	0.8755
p^*	3.6418	1.8764	0.8998	6.6357
r^*	2.3886	0.0004	2.3881	2.3893
k	0.0718	0.0003	0.0713	0.0722
t^{-1}	2.6816	0.495	1.8488	3.4495
r_g	0.7413	0.0445	0.6661	0.8127
r_z	0.7152	0.0903	0.5695	0.8594
$r_{g,z}$	0.3584	0.2558	-0.0582	0.7891
M_{Rz}	-0.2836	0.3116	-0.7453	0.1969
M_{gz}	0.8379	0.4964	0.0439	1.6546
M_{zz}	-0.3681	0.1679	-0.6373	-0.108
s_R	0.2158	0.0203	0.1831	0.249
s_g	0.258	0.0457	0.185	0.3275
s_z	0.8965	0.2191	0.5369	1.2581
s_z	0.207	0.0421	0.1351	0.2742

Selected Target					
Name	**Range**	**Density**	**Mean**	**Std.Dev.**	**90%Int.**
p^*	$\mathbb{R}^+$	Gamma	**2.00**	**1.50**	[0.90,6.91]

Canada				
Probability				
Determi	1.0000		**π = 2 - σ = 1.5**	
Indetern	0.0000		**1Q91-4Q05**	
Parameter Estimation Results				
	Mean	St. Dev.	CI (high)	CI (low)
Y_1	**0.9849**	0.0006	0.984	0.9858
Y_2	0.1943	0.0019	0.1911	0.1977
r_R	0.7696	0.0346	0.7135	0.826
p^*	2.3053	0.4398	1.5931	3.0271
r^*	2.3925	0.0004	2.3918	2.393
k	0.0718	0.0001	0.0715	0.0719
t^{-1}	2.7681	0.5231	1.9318	3.6268
r_g	0.7443	0.0459	0.6704	0.8194
r_z	0.8533	0.0396	0.7888	0.9173
$r_{g,z}$	0.2365	0.1725	-0.0417	0.5284
M_{Rz}	0.0201	0.9934	-1.606	1.6384
M_{gz}	0.0379	1.0026	-1.6005	1.6931
M_{zz}	0.0197	1.0081	-1.6392	1.6795
s_R	0.229	0.0225	0.1924	0.2647
s_g	0.2527	0.0522	0.1693	0.3342
s_z	0.6821	0.1219	0.4847	0.873
s_z	0.2703	0.1553	0.1025	0.4532

Figure 1.6: GDP, inflation and interest rate series for selected economies





[9]

[9] Notation: Time series from the above figures were extracted from IFS (IMF) database. Output is expressed as percentage deviations from the trend (log real per capita GDP HP de-trended). Inflation is annualized percentage changes. The nominal interest rates are quarterly averages of the benchmark rate in each country.

1.8 Appendix I: The Underlying Model

The underlying model consists in a non-monetary economy with sticky prices (as proposed by Calvo 1983) and an infinite number of households (a continuum in the interval (0,1)). In this economy, the monetary authority observes the inflation rate and the output gap and decides over the short term nominal interest rate.

The households

The household i maximizes a separable utility function, where each household specializes in the production of one particular good with competitive financial markets. Each time, utility function shocks are the same for all households,

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t [U(c_{t,\xi_t}) - V(h_t(i), \xi_t)] \right\} \qquad (1.23)$$

where C_t represents the bundle of goods supplied, p_t is the price index for those goods, $h_t(i)$ is the only factor of production being the quantity of work supplied for the production of good i (where $i \in (0,1)$). V is the disutility of supplying work to produce good i while ξ_t represents a preference shock in t.

Using Dixit and Stiglitz aggregators (Dixit and Stiglitz, 1977) for C_t and P_t, the authors define the following indexes:

$$C_t \equiv \left[\int_0^1 c_t^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}} \qquad (1.24)$$

$$P_t \equiv \left[\int_0^1 p_t(i)^{1-\theta} di \right]^{\frac{\theta}{1-\theta}} \qquad (1.25)$$

where P_t is the minimum cost of a unit of the aggregate consumption, given prices $p_t(i)$.

The budget constraint for the household i in moment t is:

$$M_t + B_t \leq W_t + \left[\int_0^1 w_t(i) h_t(i) + \pi_t(i) di \right] - T_t - \int_0^1 p_t(i) c_t(i) di \qquad (1.26)$$

where, M_t, is the monetary base, B_t, denote the number of assets, T_t, taxes at the end of period t and W_t, is the wealth endowment at the beginning of t; $w_t(i)$ and $p_t(i)$ are wages and benefits related to good i respectively. Importantly, all variables are expressed in nominal terms.

The following condition must be satisfied so that consumption among different goods and periods is optimally chosen.

$$c_t(i) = C_t \left(\frac{p_t(i)}{P_t} \right)^{-\theta} \qquad (1.27)$$

Therefore, households will maximize their utility (1.23) subject to restrictions (1.24) and (1.28).

The firms

Firms have the following production function:

$$g_t(i) = A_t f(h_t(i)) \qquad A_t > 0 \tag{1.28}$$

The firm benefits can be summarized in:

$$\pi_t(i) = \pi(p_t(i); p_t^I; P_t; Y_t; \xi_t) \tag{1.29}$$

where $p_t(i)$ is the price of good i, p_t^I is a price index for industry I where i belongs, P_t is the price index of the full economy and Y_t is the aggregate demand. The industry is defined as a group of producers that change their price at the same time.

The first order conditions (FOC) gives us the equilibrium conditions.

From the consumers stand point, the path of aggregate consumption $(Y_t - G_t)$ and the price index P_t consistent with equilibrium must satisfy:

$$(1 + i_t) = \beta E_t \left\{ \frac{[U_c(Y_{t+1} - G_{t+1}; \xi_{t+1})]}{[U_c(Y_t - G_t; \xi_t)]} \frac{P_t}{P_{t+1}} \right\}^{-1} \quad \forall t \tag{1.30}$$

where i_t is the free risk short-term interest rate perceived for holding bonds, B_t. The authors define i_t^m as the interest rate the Central Bank pays for its monetary base, M_t. The rational is that, in order for families to be willing to hold money, an interest rate is needed for the monetary base since money is not inside the utility function. So, in equilibrium $i_t = i_t^m$.

The households' aggregate consumption and price level must also satisfy,

$$\sum_{T=t}^{\infty} \beta^T E_T[U_c(Y_T - G_T;\xi_T)(Y_T - G_T)] < \infty \qquad (1.31)$$

$$\lim_{\longrightarrow} E\left[U_c(Y_t - G_t;\xi_t)\frac{D_T}{P_T} \right] = 0 \qquad (1.32)$$

given the evolution of government spending G_t and debt D_t, which will depend on the monetary and fiscal policy.

Equations (1.31) and (1.32) rule out over-accumulation of aggregate consumption or wealth (see Kamihigashi, 2006). The intuition could be that the present discounted value of aggregate consumption or the overall production of the economy should be a finite number. Or wealth cannot growth faster than its marginal value $\beta^T E_T[U_c(Y_T - G_T;\xi_T)]$.

In the particular case where a proportion α of prices remain unchanged, meaning that firms take some time to adjust their prices to a new steady state level, equation (1.25) can be re-written in the following way:

$$P_t = \rho(p_t, P_{t-1}) \equiv [(1-\alpha)p_t^{1-\theta} + \alpha P_{t-1}^{1-\theta}]^{\frac{1}{1-\theta}} \qquad \theta > 1 \quad (1.33)$$

The optimality condition for the representative firm is therefore:

$$E_t\left\{ \sum_{T=t}^{\infty} (\alpha\beta)^{T-1} U_c(Y_t - G_t;\xi_t)\, P_T^{-1} \pi(p_{t,i}^*, p_{t,-i}^*, P_T; Y_T; \xi_T) \right\} = \quad (1.34)$$

Where $p_{t,i}^*$ and $p_{t,-i}^*$ are the optimal price decisions for firms i and the rest of the $i-1$ firms which can modify their prices at time t. The FOC of equation

(1.33) shows that, if possible, a firm i will choose an optimal price $p^*_{t,i}$ at time t in order to maximize its benefits taken all other prices, including the ones of its own industry ($p^*_{t,-i}$) as given.

Equations (1.33) and (1.34) jointly determine the evolution of $p^*_{t,i}$ and the price level P_t taken as given the aggregate demand Y_t and the disturbances ξ_t. These are the supply side of the model.

The Taylor Rule, which only depend on endogenous variables P_t and Y_t:

$$i_t = \Phi\left(\frac{\pi_t}{\pi^*_t};Y_t;\upsilon_t\right), \tag{1.35}$$

being π^*_t the long-term inflation and υ_t, an white noise error term that could be interpreted as a Central Bank control error.

1.9 Appendix II: Solution of the model

The solution of the model and characterization of the set of solutions follows Lubik and Schorfheide (2004) and Sims (2002)[10]. For the sake of simplicity, they assume the matrix Γ_0 in equation (1.6) is invertible. The system can be re-written as:

$$s_t = \Gamma_1^*(\theta)s_{t-1} + \Psi^*(\theta)\epsilon_t + \Pi^*(\theta)\eta_t \qquad (1.36)$$

Replacing Γ_1^* for is Jordan decomposition $J\Delta J^{-1}$, where J is the matrix of left eigenvectors and Δ is a block diagonal matrix with one different block for every distinct eigenvalue. Define the vector $w_t = J^{-1}s_t$. Let the $i'th$ element of w_t be $w_{i,t}$ and denoting the $i'th$ row of $J^{-1}\Pi^*$ and $J^{-1}\Psi^*$ by $[J^{-1}\Pi^*]_i$ and $[J^{-1}\Psi^*]_i$, respectively, obtains:

$$w_{i,t} = \lambda_i w_{i,t-1} + [J^{-1}\Psi^*]_i\epsilon_t + [J^{-1}\Pi^*]_i\eta_t \qquad (1.37)$$

Define the set of stable processes AR(1) as:

$$I_s(\theta) = \{i \in \{1,\ldots\ldots,n\} | \lambda_i(\theta)| \leq 1\} \qquad (1.38)$$

and let $I_x(\theta)$ its complement. Let, Ψ_x^J and Π_x^J be the matrices composed of the row vectors $[J^{-1}\Pi^*]_i$ and $[J^{-1}\Psi^*]_i$ that correspond to unstable eigenvalues. To

[10] This method is an extension from Blanchard and Kahn (1980). First, BC assume regularity conditions that are not met by some models encountered in practice. Second, BC solution method needs to separate between jump and predetermined variables. Sims realizes that the structure of the coefficients in the matrices pin down the solution and determines endogenously the linear combinations of variables that have to be endogenous for a solution to exist.

ensure stability of s_t, the forecast errors, η_t, must satisfy:

$$\Psi_x^J \epsilon_t + \Pi_x^J \eta_t = 0 \qquad \forall t \tag{1.39}$$

This equation has either no solution, multiple solutions (indeterminacy) or one solution (determinacy). The authors restrict the parameter space of Θ to make sure that at least one solution exists (only determinacy and indeterminacy of solutions are possible). To solve potentially undetermined system of equations for η_t, Lubik and Schorfheide (2004) proceed to with a singular value decomposition of the matrix Π_x^J :

$$\Pi_x^J = [U_1 \quad U_2] \begin{bmatrix} D_{11} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \underbrace{U}_{mxm} \underbrace{D}_{mxk} \underbrace{V'}_{kxk} = \underbrace{U_1}_{mxr} \underbrace{D_{11}}_{rxr} \underbrace{V}_{rxk} \tag{1.40}$$

where D_{11} is a diagonal matrix and U and V are orthonormal matrices. Here they used m to denote the number of unstable eigenvalues and r is the number of non-zero singular values of Π_x^J. Recall that k is the dimension of the vector of forecast errors η_t and l denotes the number of exogenous shocks.

Let p be the dimension of the sunspot shock ς_t.

Proposition 1 (from Lubik and Schorfheide (2004)): *If there exists a solution to equation (38.1) that expresses the forecast errors as a function of the fundamental shocks ϵ_t and sunspot shocks ς_t , it is of the form:*

$$\eta_t = \eta_1 \epsilon_t + \eta_2 \varsigma_t$$

$$= \left(-V_1 D_{11}^{-1} U_1' \Psi_x^J + V_2 \widetilde{M}\right)\epsilon_t + V_2 M_\varsigma \varsigma_t \tag{1.41}$$

Proposition *Where* $\widetilde{M}$ *is a* $(k-r)$ *x* l *matrix,* M_ς *is a* $(k-r)$ *x* p *matrix, and the dimension of* V_2 *is* k *x* $(k-r)$. *The solution is unique if* $k = r$ *and* V_2 *is zero. The proof of this proposition can be found in Lubik and Schorfheide (2004).*

If they replace equation (1.41) in (1.36), the previous representation of the rational expectations forecast errors leads to the following law of motion for s_t :

$$s_t = \Gamma_1^*(\theta)s_{t-1} + [\Psi^*(\theta) - \Pi^*(\theta)V_1(\theta)D_{11}^{-1}(\theta)U_1'(\theta)\Psi_x^J(\theta)]\epsilon_t + \Pi^*(\theta)V_2(\theta)\left(\widetilde{M}\epsilon_t + M_\varsigma \varsigma_t\right) \tag{1.42}$$

Under determinacy V_2 is zero and the third term of equation (1.42) drops out. In this case the dynamics of s_t is only dependent on the vector of parameters θ. Indeterminacy introduces additional parameters and changes the nature of the solution in two dimensions. First, the propagation of the structural shocks ϵ_t is not uniquely determined as it depends on the matrix $\widetilde{M}$. Second, the dynamics of s_t is potentially affected ($M_\varsigma \neq 0$) by the sunspot shocks ς_t. In the monetary DSGE model we derive above the degree of indeterminacy $k - r$ is at most 1. Hence we set $p = 1$ and impose the normalization $M_\varsigma = 1$.

Since it is not possible to identify the covariances of the sunspot shock with the fundamental shocks in addition to $\widetilde{M}$, Lubik and Schorfheide (2004) use the normalization $E[\epsilon_t \varsigma_t] = 0$.

They also reparametrized the indeterminacy solutions by $\widetilde{M} = 1 + M,$ such that $M = 0$ corresponded to $y_t = \epsilon_t$. While they consider all possible values of $\widetilde{M}$ in our estimation procedure, They specify a prior distribution that is centered around a particular solution. They do this by replacing $\widetilde{M}$ with $M^*(\theta) + M$ and setting the prior mean for M equal to zero. Lubik and Schorfheide (2004) find desirable to choose $M^*(\theta)$ such that the impulse responses $\frac{\partial s_t}{\partial \epsilon_t'}$ are continuous at the boundary between the determinacy and indeterminacy region. According to the selected prior mean, small changes of θ do not lead to drastic changes in the propagation of fundamental shocks.

One candidate for $M^*(\theta)$ is the minimal-state-variable considered above. Considering a model with a one-dimensional indeterminacy, it is possible to identify an eigenvalue function $\lambda_{i^*}(\theta)$ such that $|\lambda_{i^*}(\theta)|$ is greater than one in the determinacy region $(\theta \in \Theta^D)$ and less than one in the indeterminacy region $(\theta \in \Theta^I)$. In this case a baseline solution could be constructed by solving the system of equations:

$$[J^{-1}\Pi^*]_i \epsilon_t + [J^{-1}\Psi^*]_i \eta_t = 0 \tag{1.43}$$

for $i \in I_x(\theta)$ and $i = i^*$.

While it is possible to define and track eigenvalue functions in the presented DSGE model, it is difficult to do so in larger systems. Hence, Lubik and Schorfheide (2004) proceed with an alternative method. For every vector $\theta \in \Theta^I$

they construct a vector $\widetilde{\theta} = g(\theta)$ that lies on the boundary of the determinacy region and choose $M^*(\theta)$ such that the response of s_t to ϵ_t conditional on θ mimics the response conditional on $\widetilde{\theta}$. Thus, we compare

$$\frac{\partial s_t}{\partial \epsilon_t'}(\theta, M) = \Psi^*(\theta) - \Pi^*(\theta)V_1(\theta)D_{11}^{-1}(\theta)U_1'(\theta)\Psi_x^J(\theta) + \Pi^*(\theta)V_2(\theta)\widetilde{M}B_1(\theta) + B_2(\theta \quad (1.44)$$

to

$$\frac{\partial s_t}{\partial \epsilon_t'}(g(\theta), .) = B_1(g(\theta)) \quad (1.45)$$

In their application, Lubik and Schorfheide minimize the discrepancy using a least squares criterion and choose

$$M^*(\theta) = [B_2(\theta)'B_2(\theta)]^{-1}B_2(\theta)' * [B_1(g(\theta)) - B_1(\theta)] \quad (1.46)$$

The function $g(\theta)$ is obtained by replacing ψ_1 in the vector θ with,

$$\widetilde{\psi_1} = 1 - \frac{\beta\psi_2}{\kappa}(\frac{1}{\beta} - 1) \quad (1.47)$$

which marks the boundary between the determinacy and indeterminacy region for the presented model. They will refer to the solution $\widetilde{M} = M^*(\theta)$ as baseline indeterminacy solution.

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